JÜLICH FORSCHUNGSZENTRUM

The four-pion decays of η' and η

May 31, 2012 | **Andreas Wirzba**

in collaboration with

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Outline:

1. Introduction
2. Anomalous processes
3. Charged final states: $\eta' \to \mathbf{2}(\pi^+\pi^-)$, $\eta' \to \pi^+\pi^-\mathbf{2}\pi^\mathbf{0}$
   - a. chiral perturbation theory and vector-meson dominance
   - b. predictions for the branching ratios
4. Neutral final states: $\eta, \eta' \to \mathbf{4}\pi^\mathbf{0}$
   - a. CP-conserving mechanism; branching ratios
   - b. CP-violation for $\eta \to \mathbf{4}\pi^\mathbf{0}$ through the QCD θ term
5. Conclusions

Feng-Kun Guo, Bastian Kubis & A.W., Phys. Rev. D **85**, 014014 (2012) [arXiv:1111.5949]

see also: Andrzej Kupść & AW, J. Phys. Conf. Ser. **335**, 012017 (2011) [arXiv:1103.3860]

Introduction (1)

- What do we know about these four-pion decays? Very little...

η'(958) DECAY MODES	Fraction (Γ_i/Γ)		Confidence level	p (MeV/c)
$2(\pi^+\pi^-)$	< 2.4	$\times 10^{-4}$	90%	372
$\pi^+\pi^-2\pi^0$	< 2.6	$\times 10^{-3}$	90%	376
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- A little puzzle from the PDG:

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- If you know any theoretical calculations for these, please tell us!

  e.g.: quark-model calculation of D. Parashar (1979) violates $\eta' \to 2(\pi^+\pi^-)$ bound

Introduction (2)

In principle, these are not terribly forbidden . . .

- not isospin-forbidden, not electromagnetic

. . . except for:

- phase space

$$M_{\eta'} - 4M_\pi = 399.5\,\text{MeV} \cdots 417.9\,\text{MeV}$$
$$M_\eta - 4M_{\pi^0} = 7.9\,\text{MeV}\ , \quad M_\eta - 2\big(M_{\pi^\pm} + M_{\pi^0}\big) = -1.2\,\text{MeV}$$

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- odd number of pseudoscalars
  $\longrightarrow$ process of odd intrinsic parity, "anomalous"
- Wess–Zumino–Witten (WZW) term in QCD induces
  - ▷ triangle-anomaly: $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma \ldots$
  - ▷ box-anomaly: $\gamma\pi \to \pi\pi$, $\eta \to \pi\pi\gamma \ldots$
  - ▷ pentagon anomaly – where?

Anomalous processes

- amplitudes of anomalous/odd-intrinsic-parity processes involve totally antisymmetric $\epsilon_{\mu\nu\alpha\beta}$ tensor

  e.g. $$\mathcal{A}_{\mathrm{WZW}}(K^+K^- \to \pi^+\pi^-\pi^0) = \frac{3}{4\pi^2 F_\pi^5}\,\epsilon_{\mu\nu\alpha\beta}\, p^\mu_{\pi^+} p^\nu_{\pi^-} p^\alpha_{K^+} p^\beta_{K^-}$$

  $\Rightarrow \mathcal{O}(p^4)$ in chiral counting; strength fixed by F_π
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- $\eta' \to 2(\pi^+\pi^-)$, $\eta' \to \pi^+\pi^- 2\pi^0$ P-wave-dominated
- $\eta' \to 4\pi^0$, $\eta \to 4\pi^0$: Bose symmetry forbids P-wave $\Rightarrow$ D-waves
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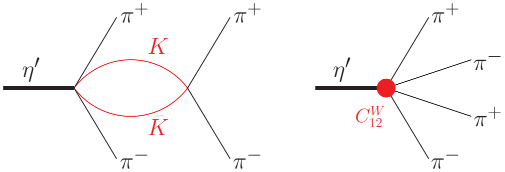
- flavour structure of WZW term: pentagon anomaly genuinely SU(3), doesn't work without kaons: $\pi^+\pi^-\pi^0 K\bar{K}$ $\quad \eta\pi\pi K\bar{K}$ $\quad \eta(K\bar{K})^2$

$\eta' \to 2(\pi^+\pi^-)$, $\eta' \to \pi^+\pi^-2\pi^0$ in ChPT

- leading contribution to $\eta' \to \pi^+(p_1)\pi^-(p_2)\pi^+(p_3)\pi^-(p_4)$ at $\mathcal{O}(p^6)$! (we assume standard $\eta\eta'$ mixing):



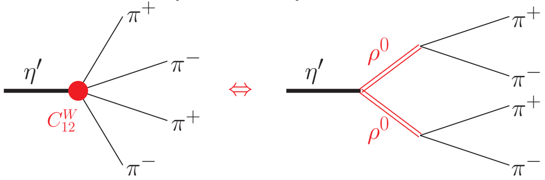
$$\mathcal{A} = \frac{\epsilon_{\mu\nu\alpha\beta}}{\sqrt{3}F_\pi^5} p_1^\mu p_2^\nu p_3^\alpha p_4^\beta \left\{F(s_{12}) + F(s_{34}) - F(s_{14}) - F(s_{23})\right\}, \quad \left[s_{ij} \equiv (p_i + p_j)^2\right]$$

$$F(s) = \frac{1}{8\pi^2 F_\pi^2}\left\{(s - 4M_K^2)\bar{J}_{KK}(s) - \frac{s}{16\pi^2}\left(\log\frac{M_K^2}{\mu^2} + \frac{1}{3}\right)\right\} - 16\, C_{12}^{Wr}(\mu)\, s$$

- $\eta' \to \pi^+\pi^-2\pi^0$ amplitude the same
- $\mathcal{O}(p^6)$ counterterm Lagrangian $\propto C_i^W$ canceling μ-dependence known
  Bijnens, Girlanda, Talavera 2002
- How to estimate finite counterterm contribution $\propto C_{12}^{Wr}(\mu)$?

Resonance saturation via HLS model (1)

- estimate counterterms via resonance saturation
  here: vector mesons (P-waves!)



- has been studied for anomalous sector — Kampf, Novotný 2011
- here: simpler, but more predictive framework:
  hidden local symmetry (HLS) — Bando, Kugo, Yamawaki 1988
  - ▷ extension of chiral perturbation theory with vectors as gauge bosons of enlarged symmetry group
  - ▷ effectively only 3 couplings in the anomalous sector; need 2:

$$c_1 - c_2 \approx c_3 \approx 1 \qquad \text{or} \qquad c_1 - c_2 \approx 1.21\,,\; c_3 \approx 0.93$$

Benayoun et al. 2010

Resonance saturation via HLS model (2)

- HLS estimate for $\mathcal{O}(p^6)$ couplings contributing to PPPPP:

$$C_1^{Wr}(M_\rho) = -2C_{12}^{Wr}(M_\rho) = \frac{3(c_1 - c_2 + c_3)}{128\pi^2 M_\rho^2}$$

- relative importance of ρ vs. kaon loop contributions:

$$F'(0) = \frac{1}{8\pi^2(4\pi F_\pi)^2}\left\{\underbrace{3(c_1 - c_2 + c_3)\frac{(4\pi F_\pi)^2}{2M_\rho^2}}_{\approx 6.7} - \underbrace{\left(1 + 2\log\frac{M_K}{M_\rho}\right)}_{\approx 0.1}\right\}$$

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- kinematically accessible in $\eta' \to 4\pi$ decays:

$$\sqrt{s_{ij}} \le M_{\eta'} - 2M_\pi \approx 680\,\text{MeV}$$

compared to $M_\rho = 775\,\text{MeV}$, $\Gamma_\rho = 149\,\text{MeV}$

  - ▷ retain full ρ propagators for phenomenologically reliable description
  - ▷ takes care of P-wave $\pi\pi$ final-state interactions

Results: branching ratios

- calculate branching ratios as functions of HLS couplings
- isospin limit:

$$2 \times \mathrm{BR}\left(\eta' \to 2(\pi^+\pi^-)\right) = \mathrm{BR}\left(\eta' \to \pi^+\pi^- 2\pi^0\right)$$

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- results:

$$\mathrm{BR}\left(\eta' \to 2(\pi^+\pi^-)\right) = \left[0.15\,(c_1-c_2)^2 + 0.47\,(c_1-c_2)c_3 + 0.37\,c_3^2\right] \times 10^{-4} = \left\{1.0,\ 1.1\right\} \times 10^{-4}$$

$$\mathrm{BR}\left(\eta' \to \pi^+\pi^-2\pi^0\right) = \left[0.35\,(c_1-c_2)^2 + 1.09\,(c_1-c_2)c_3 + 0.87\,c_3^2\right] \times 10^{-4} = \left\{2.3,\ 2.5\right\} \times 10^{-4}$$

- remember: $\mathrm{BR}_{\mathrm{PDG}}\left(\eta' \to 2(\pi^+\pi^-)\right) < 2.4 \times 10^{-4}$

$$\mathrm{BR}_{\mathrm{PDG}}\left(\eta' \to \pi^+\pi^-2\pi^0\right) < 2.6 \times 10^{-3}$$

$\longrightarrow$ there is room for (experimental) improvement!

Chiral counting for $\eta' \to 4\pi^0$

- remember: $\pi^0\pi^0$ pairs have to emerge in relative D-waves
- find: this increases the chiral power of $\eta' \to 4\pi^0$ to $\mathcal{O}(p^{10})$!

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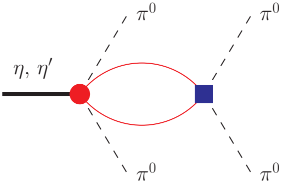

- ▷ ● vertex needs to be $\mathcal{O}(p^6)$, as WZW term does not contain 5-meson-vertices with 2 π^0
- ▷ ■ vertex has to be D-wave, that is at least $\mathcal{O}(p^4)$
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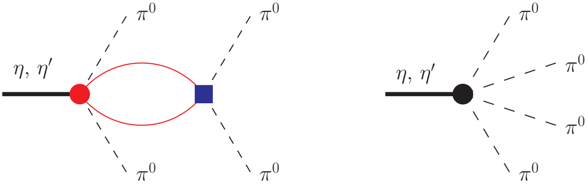

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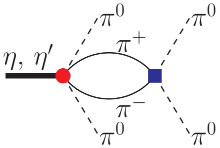
  - ▷ one-loop + $\mathcal{O}(p^6)$ vertex + $\mathcal{O}(p^4)$ vertex $\Rightarrow \mathcal{O}(p^{10})$
  - ▷ $\epsilon_{\mu\nu\alpha\beta}\, p_1^\mu p_2^\nu p_3^\alpha p_4^\beta \times P(s_{ij})$: requires polynomial P of at least 3rd power in s_{ij} to yield totally symmetric amplitude
- we are not going to do an $\mathcal{O}(p^{10})$ [= three-loop] calculation...

Something you *can calculate for* $\eta' \to 4\pi^0$



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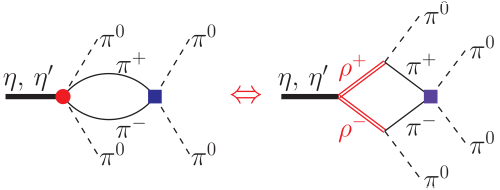

- can calculate the complete imaginary part at $\mathcal{O}(p^{10})$ (given by charged-pion intermediate states)
- use the full vector-meson-dominated $\eta' \to \pi^+\pi^- 2\pi^0$ amplitude

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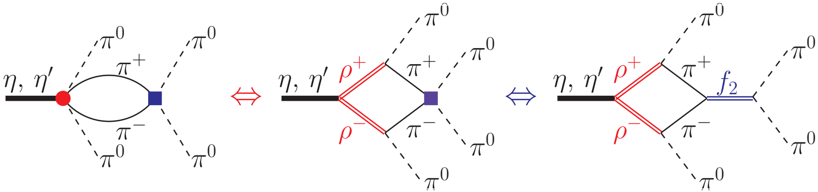

- can calculate the complete imaginary part at $\mathcal{O}(p^{10})$ (given by charged-pion intermediate states)
- use the full vector-meson-dominated $\eta' \to \pi^+\pi^- 2\pi^0$ amplitude
- "trick" to reconstruct the full $\pi\pi$ D-wave final-state amplitude via Omnès function (neglecting any crossed-channel effects):

at threshold: $$\mathrm{Im}\,\Omega_2^0(s) \approx \sqrt{1-\frac{4M_\pi^2}{s}} \times t_2^0(s), \qquad t_2^0 : \pi\pi \text{ partial wave}$$

f_2 dominance: $$\Omega_2^0(s) \approx \frac{M_{f_2}^2}{M_{f_2}^2 - s}$$

- note: *full* result far from f_2 dominance (as $\rho\rho$ not "short-ranged")

Branching ratios for $\eta' \to 4\pi^0$, $\eta \to 4\pi^0$

- results for branching ratios:

$$\begin{aligned}
\mathrm{BR}\left(\eta' \to 4\pi^0\right) &= \left[0.4\,(c_1-c_2)^2 + 1.6\,(c_1-c_2)c_3 + 1.7\,c_3^2\right]\times 10^{-8} \\
&= \left\{3.7,\ 3.9\right\}\times 10^{-8} \\
\mathrm{BR}\left(\eta \to 4\pi^0\right) &= \left[0.4\,(c_1-c_2)^2 + 1.1\,(c_1-c_2)c_3 + 1.0\,c_3^2\right]\times 10^{-30} \\
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- conclusions...
  - ▷ ...for the η':

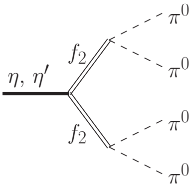
    D-wave mechanism suppressed $\eta' \to 4\pi^0$ by 3–4 orders of magnitude compared to charged-pion final states

  - ▷ ...for the η:

    D-wave plus tiny phase space suppresses this enormously $\longrightarrow$ any signal indeed sign of CP-violation

Suppression of double-f_2 mechanism

An alternative decay mechanism via two virtual f_2 mesons



- is formally also of chiral order $\mathcal{O}(p^{10})$
- but is heavily suppressed

$$\mathrm{BR}\big(\eta' \to f_2 f_2 \to 4\pi^0\big) \approx 4 \times 10^{-14}\,, \quad \mathrm{BR}\big(\eta \to f_2 f_2 \to 4\pi^0\big) \approx 3 \times 10^{-35}$$

versus

$$\mathrm{BR}\big(\eta' \to \rho\rho \to 4\pi^0\big) \approx 4 \times 10^{-8}\,, \quad \mathrm{BR}\big(\eta \to \rho\rho \to 4\pi^0\big) \approx 3 \times 10^{-30}$$

CP-violating $\eta \to 4\pi^0$ decay via the θ-term

- CP-violating term in QCD: θ-term, linked to $U(1)_A$ anomaly
- can be treated on effective Lagrangian level

Crewther et al. 1980; Pich, de Rafael 1991

- induces e.g. neutron electric dipole moment and $\eta \xrightarrow{\not{C}P} 2\pi$

  ...but also CP-violating S-wave $\eta \to 4\pi^0$ amplitude:

$$\mathcal{A}\left(\eta \xrightarrow{\not{C}P} 4\pi^0\right) = -\sqrt{\frac{2}{3}\frac{M_{\eta'}^2}{3F_\pi^3}} \times \bar{\theta}_0$$

- resulting branching ratio:

$$\mathrm{BR}\left(\eta \xrightarrow{\not{C}P} 4\pi^0\right) = 5 \times 10^{-5} \times \bar{\theta}_0^2 \,, \qquad \left[\mathrm{BR}\left(\eta' \xrightarrow{\not{C}P} 4\pi^0\right) = 9 \times 10^{-2} \times \bar{\theta}_0^2\right]$$

$\longrightarrow$ if $\bar{\theta}_0$ were $\mathcal{O}(1)$, this would demonstrate the enhancement of the CP-violating S-wave mechanism

- current limits from neutron electric dipole moment: $\bar{\theta}_0 \lesssim 10^{-11}$

Ottnad et al. 2009

Summary / Conclusions

Analysis of (yet unmeasured) $\eta, \eta' \to 4\pi$ decays:

- $\eta' \to 2(\pi^+\pi^-)$, $\eta' \to \pi^+\pi^-2\pi^0$:

  P-wave / $\rho\rho$ dominated; predictions (uncertainty $\sim \mathcal{O}(1/N_c)$)

  $$\mathrm{BR}\big(\eta' \to 2(\pi^+\pi^-)\big) \approx (1.0\pm0.3)\times10^{-4}, \quad \mathrm{BR}\big(\eta' \to \pi^+\pi^-2\pi^0\big) \approx (2.4\pm0.7)\times10^{-4}$$

- $\eta' \to 4\pi^0$, $\eta \to 4\pi^0$: chirally suppressed,

  D-wave dominated; prediction via $\rho\rho + f_2$ mechanism:

  $$\mathrm{BR}\big(\eta' \to 4\pi^0\big) \approx 4\times10^{-8}, \quad \mathrm{BR}\big(\eta \to 4\pi^0\big) \approx 3\times10^{-30}$$

  $\longrightarrow$ any excess probably sign of CP violation

- CP violation for $\eta \to 4\pi^0$ via QCD θ term:

  $$\mathrm{BR}\big(\eta \xrightarrow{\not{C}\!P} 4\pi^0\big) \approx 5\times10^{-5}\times\bar\theta_0^2 \quad (\text{but } \bar\theta_0 \lesssim 10^{-11})$$

More details: F.-K. Guo, B. Kubis & A.W., Phys. Rev. D **85**, 014014 (2012) [arXiv:1111.5949]