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FACULTY OF SCIENCES

Bayesian inference of the resonance content of $p(\gamma, K^+)\Lambda$

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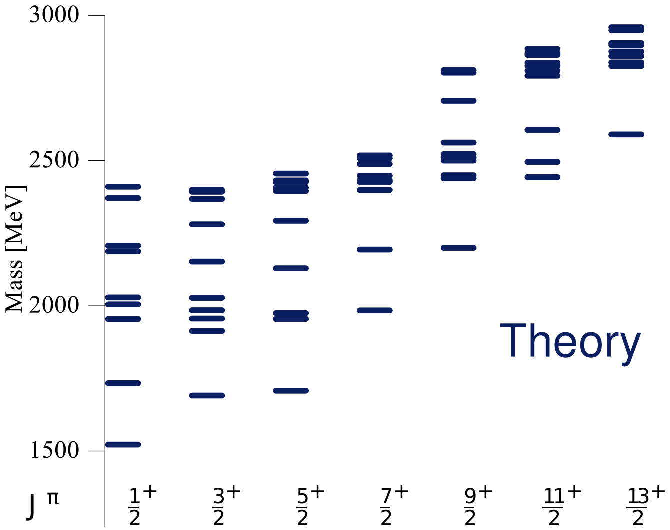
MESON 2012

12th International Workshop on Meson Production, Properties & Interaction
Kraków, Poland, 31 May - 5 June 2012

The nucleon spectrum as we know it



Bonn constituent-quark model U. Loering et al., EPJA 10 (2001) 395

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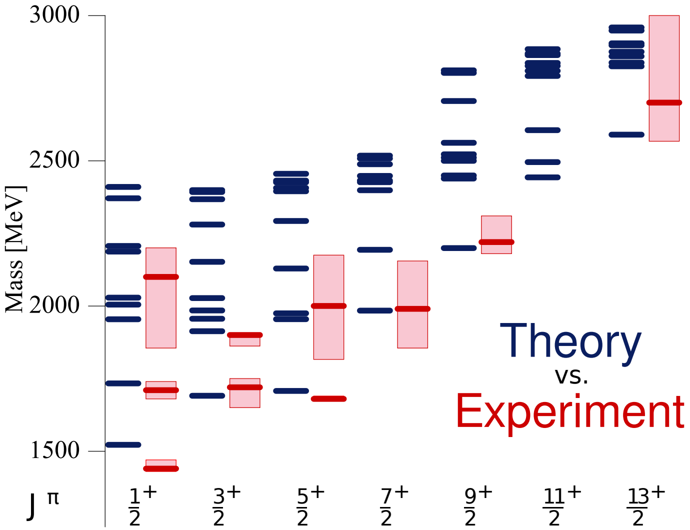
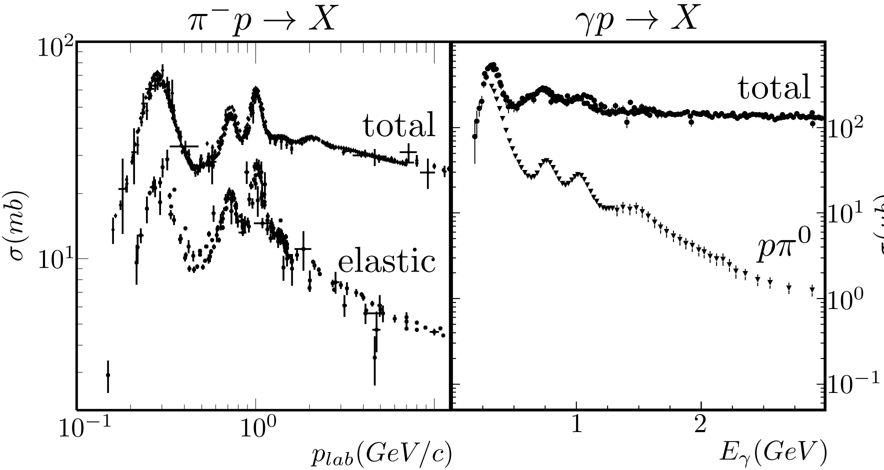


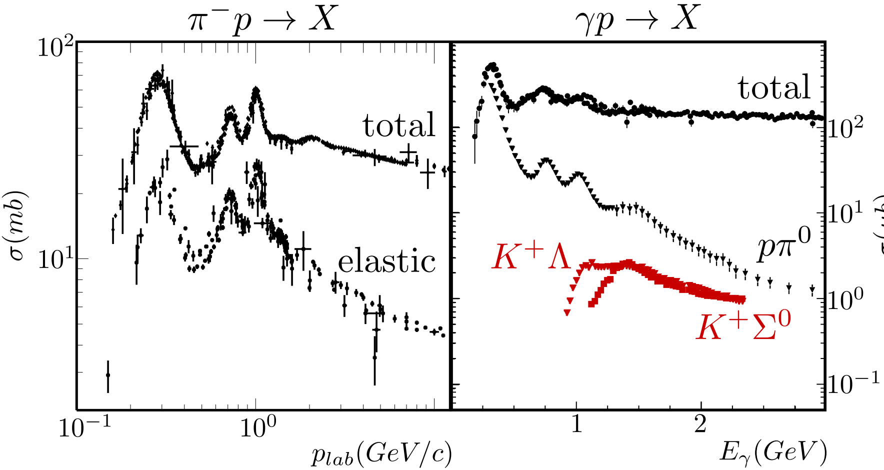
Bonn constituent-quark model U. Loering et al., EPJA 10 (2001) 395

An indirect glimpse inside the nucleon



Structures are manifestations of resonances

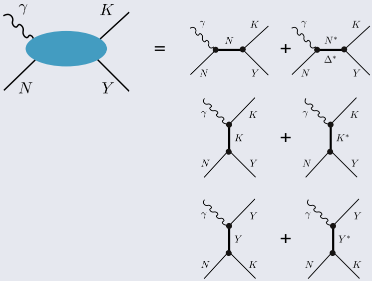
An indirect glimpse inside the nucleon



Focus on weaker channels to hunt for missing resonances

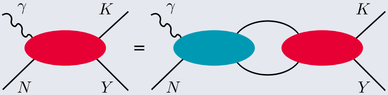
Many analyses of $p(\gamma, K^+)\Lambda$

Single-channel



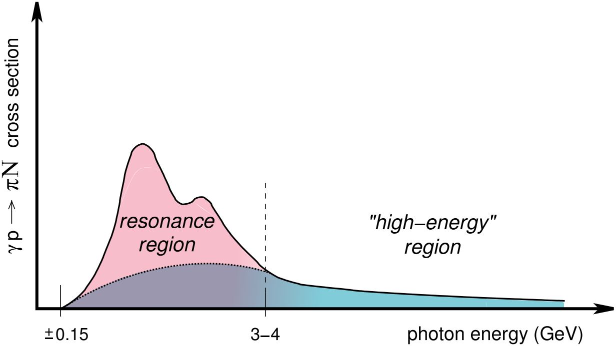
- Saclay-Lyon David et al., PRC 53 (1996) 2613
- VGL Vanderhaeghen et al., PRC 57 (1998) 1454
- KaonMAID Mart and Bennhold, PRC 61 (2000) 012201
- Gent-Isobar Ireland et al., NPA 740 (2004) 147
- RPR-2007 Corthals et al., PLB 656 (2007) 186
- RPR-2011 De Cruz et al., PRL 108 (2012) 182002

Coupled-channel



- Bonn-Gatchina Anisovich et al., EPJA 48 (2012) 15
- DCC-EBAC Julia-Diaz et al., PRC 73 (2006) 055204
- Giessen Shklyar et al., PRC 72 (2005) 015210

The conventional picture



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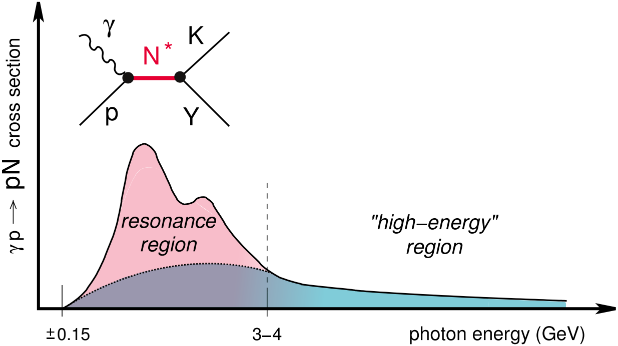
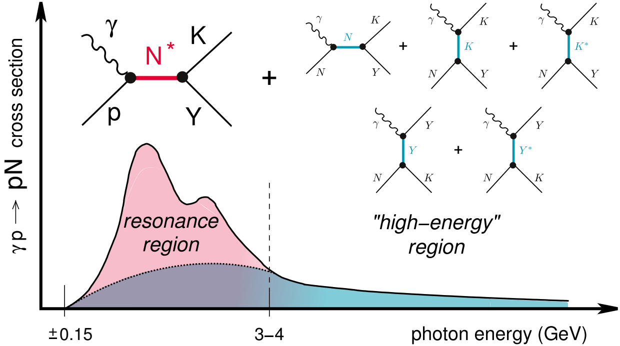


Isobar model

- Focus on resonance region
- Dominated by resonant contributions

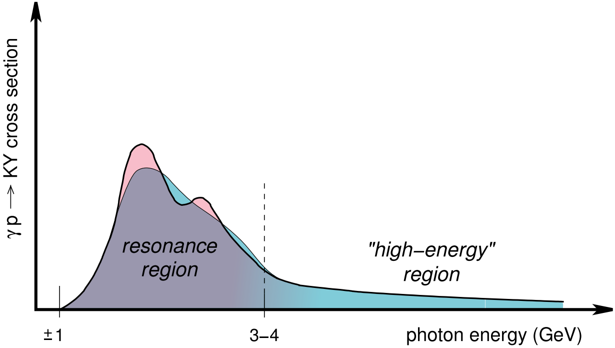
The conventional picture



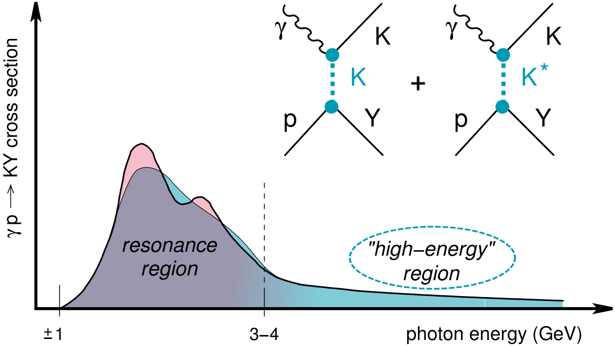
Isobar model

- Focus on resonance region
- Dominated by resonant contributions
- Many non-resonant contributions ⇒ background

The Regge-plus-resonance approach



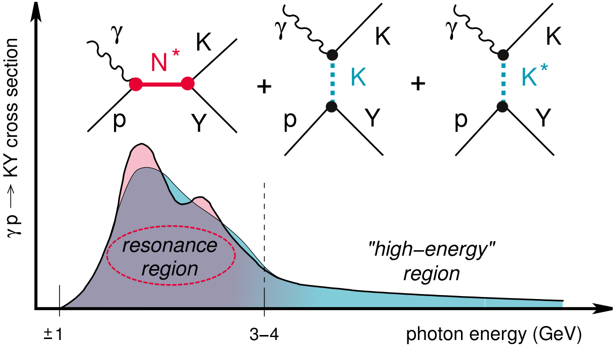
The Regge-plus-resonance approach



Background contributions Guidal, Laget and Vanderhaeghen, NPA 627 (1997) 645

- Exchange of $K(494)$ and $K^*(892)$ Regge trajectories in t channel
- Only 3 parameters
- Parametrizes non-resonant diagrams in resonance region

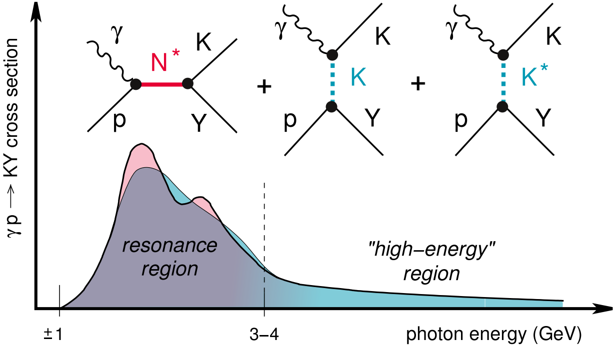
The Regge-plus-resonance approach



Resonant contributions

- enrich Regge background with nucleon resonances
- spin-1/2 resonance $\rightarrow$ 1 parameter
- spin-3/2 & -5/2 resonances $\rightarrow$ 2 parameters

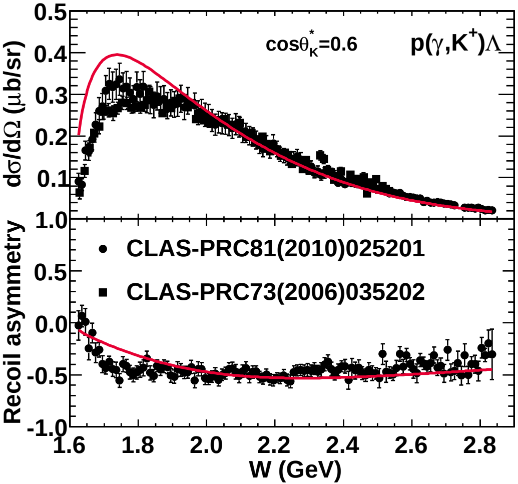
The Regge-plus-resonance model



Our strategy Corthals et al., PRC 73 (2006) 045207

1. - Construct Regge model (=background)
   - Fit parameters to high-energy data
2. - Add resonance contributions
   - Fit parameters to resonance region data

Regge-2011: results

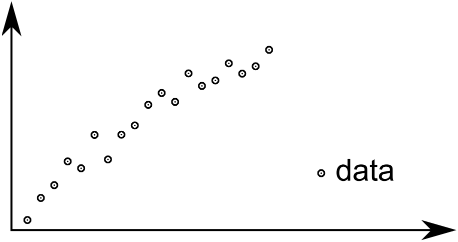


Regge model with 3 parameters

Extracting resonance content from data

Conventional approach

- Minimize χ^2
- Compare χ^2_{min}



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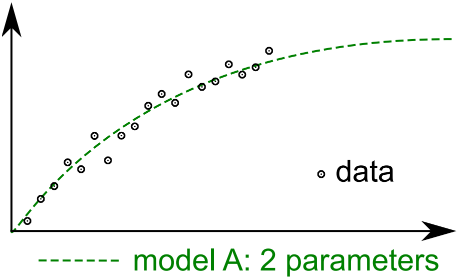

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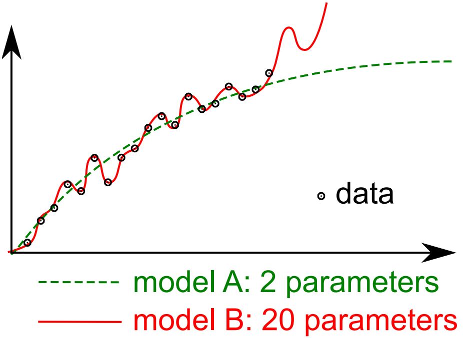

Extracting resonance content from data

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- Minimize χ^2
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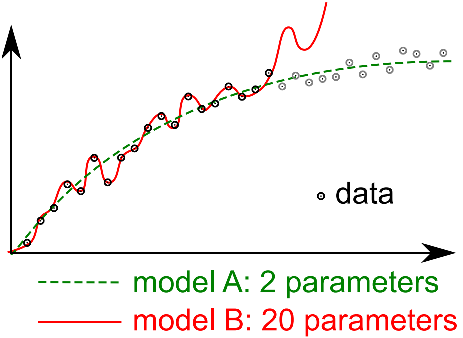
Problem

- More parameters
  $\Rightarrow$ lower χ^2_{min}
- Adding resonance
  $\Rightarrow$ improved model (?)



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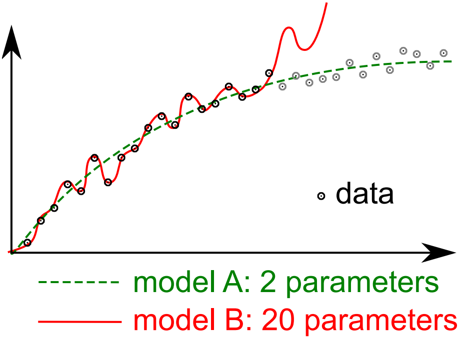
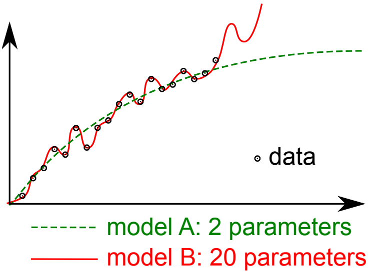
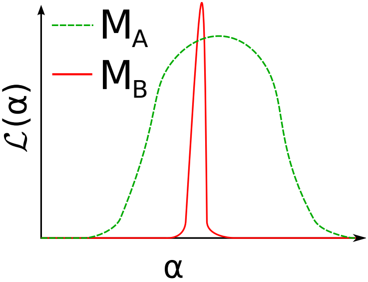


What is a good model?

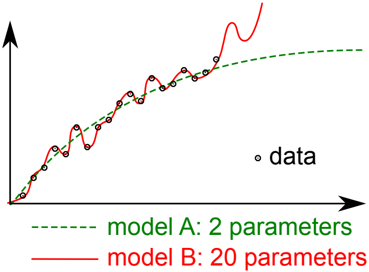
- High predictive power!
- Parsimony principle: *Occam's razor*.

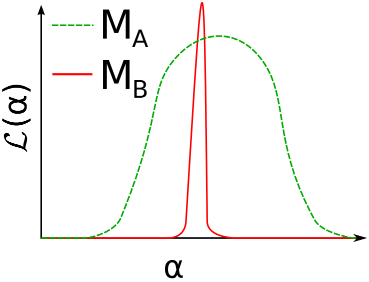
Model selection





Model selection





Are we asking the right question?

- Which model has the highest maximum likelihood?
- What is the probability of the model, given the data?

$$\boxed{P\,(\,\text{Model}\mid\text{Data}\,)}$$

Bayesian model selection

- $P\,(\text{Model}|\text{Data}) \propto$ Bayesian evidence $\mathcal{Z}$

$$\mathcal{Z} = \int \underbrace{\mathcal{L}(\alpha)}_{\text{Likelihood}} \underbrace{\pi(\alpha)}_{\text{Prior}} d\alpha$$

Bayesian model selection

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- Absolute $\mathcal{Z}$ has no meaning, only ratios do

$$\frac{\mathcal{Z}_A}{\mathcal{Z}_B} = \frac{P\left(M_A|D\right)}{P\left(M_B|D\right)}$$

Bayesian model selection

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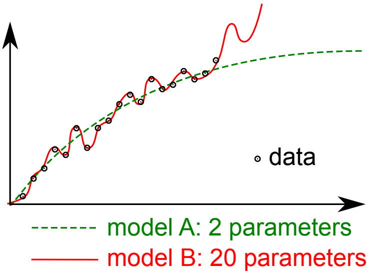
- Absolute $\mathcal{Z}$ has no meaning, only ratios do

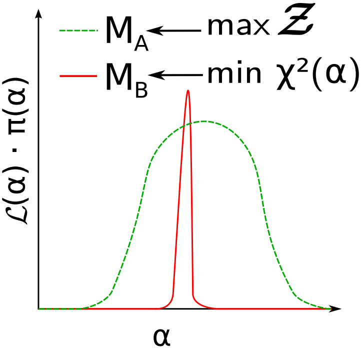
$$\frac{\mathcal{Z}_A}{\mathcal{Z}_B} = \frac{P\,(M_A|D)}{P\,(M_B|D)}$$

- Model comparison $\Rightarrow \Delta \ln \mathcal{Z} \equiv \ln \mathcal{Z}_A/\mathcal{Z}_B$

	$\lvert\Delta \ln \mathcal{Z}\rvert$	< 1	Not worth more than a bare mention.
$1 <$	$\lvert\Delta \ln \mathcal{Z}\rvert$	< 2.5	Significant.
$2.5 <$	$\lvert\Delta \ln \mathcal{Z}\rvert$	< 5	Strong to very strong.
$5 <$	$\lvert\Delta \ln \mathcal{Z}\rvert$		Decisive.

Bayesian model selection





- $\mathcal{Z}$ is not obvious to calculate
- Need genetic algorithms + MINUIT/MINOS + VEGAS integration

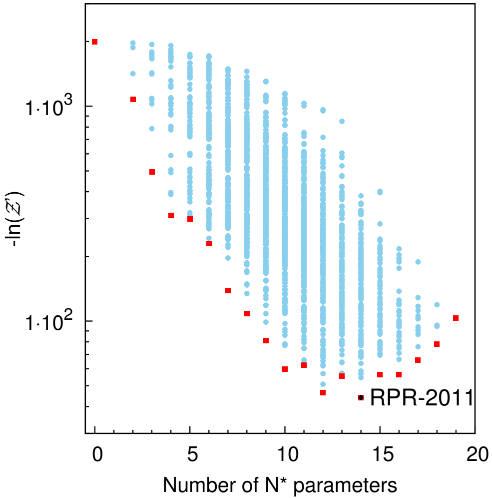
For many details, see arXiv:1205.2195

Bayesian evidence map for 2048 models

Possible resonances

- $S_{11}(1535)$ ⋆⋆⋆⋆
- $S_{11}(1650)$ ⋆⋆⋆⋆
- $D_{15}(1675)$ ⋆⋆⋆⋆
- $F_{15}(1680)$ ⋆⋆⋆⋆
- $D_{13}(1700)$ ⋆⋆⋆
- $P_{11}(1710)$ ⋆⋆⋆
- $P_{13}(1720)$ ⋆⋆⋆⋆
- $D_{13}(1900)$ *m*
- $P_{13}(1900)$ ⋆⋆
- $P_{11}(1900)$ *m*
- $F_{15}(2000)$ ⋆⋆⋆

Bayesian evidence map for 2048 models

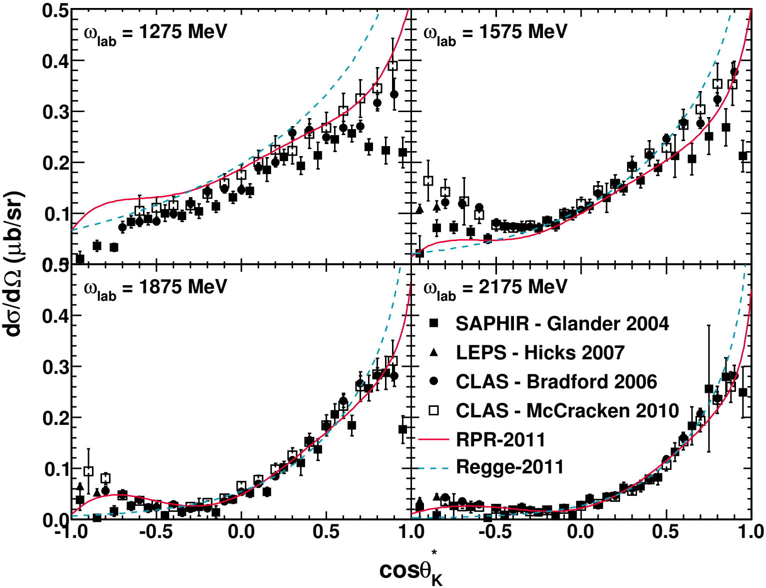


RPR-2011

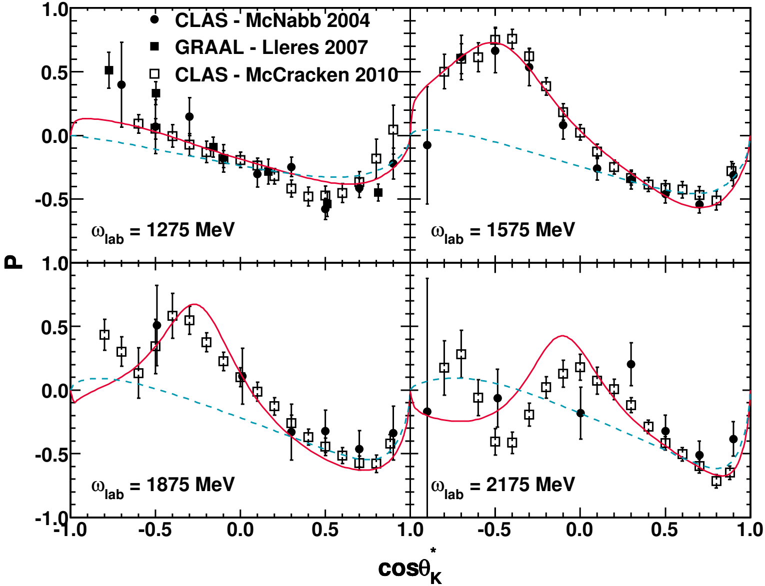
- $S_{11}(1535)$ ★★★★
- $S_{11}(1650)$ ★★★★
- $D_{15}(1675)$ ★★★★
- $F_{15}(1680)$ ★★★★
- $D_{13}(1700)$ ★★★
- $P_{11}(1710)$ ★★★
- $P_{13}(1720)$ ★★★★
- $D_{13}(1900)$ m
- $P_{13}(1900)$ ★★
- $P_{11}(1900)$ m
- $F_{15}(2000)$ ★★★

PRL 108 (2012) 182002

The RPR-2011 model - Differential cross section



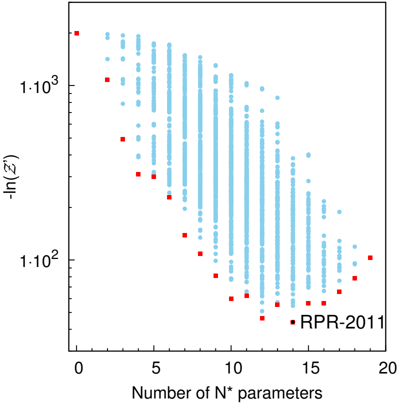
The RPR-2011 model - Recoil polarisation



Resonant contributions to $p(\gamma, K^+)\Lambda$

	$S_{11}(1535)$	$S_{11}(1650)$	$D_{15}(1675)$	$F_{15}(1680)$	$D_{13}(1700)$	$P_{11}(1710)$	$P_{13}(1720)$	$D_{13}(1900)$	$P_{13}(1900)$	$P_{11}(1900)$	$F_{15}(2000)$	$J \geq 7/2$
Bonn-Gatchina	✓	✓	✓	✓		✓	✓					✓
DCC-EBAC	✓	✓						✓	✓			
Gent-Isobar		✓				✓	✓			✓		
Giessen		✓				✓	✓		✓			
KaonMAID		✓				✓	✓	✓				
RPR-2007		✓				✓	✓	✓	✓			
RPR-2011	✓	✓		✓			✓	✓	✓	✓	✓	
Saclay-Lyon		✓	✓			✓	✓					
SAID	✓	✓	✓	✓			✓				✓	✓

Probability of a resonance?



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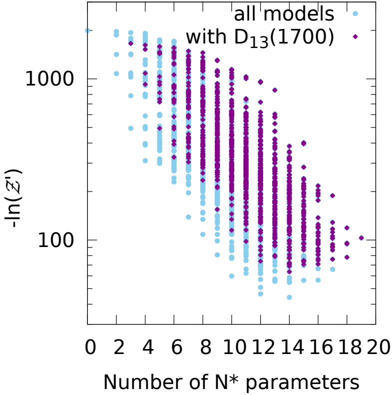

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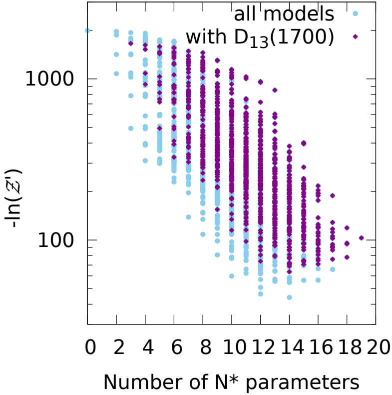

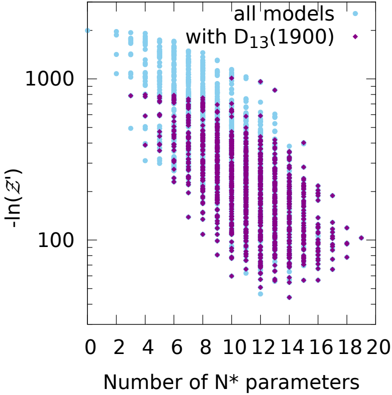

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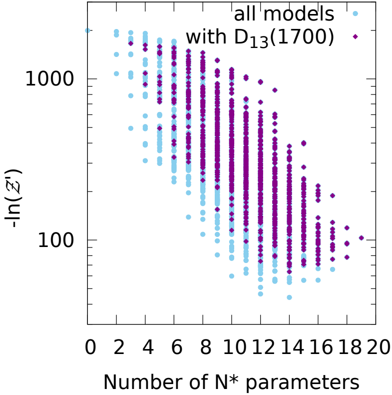
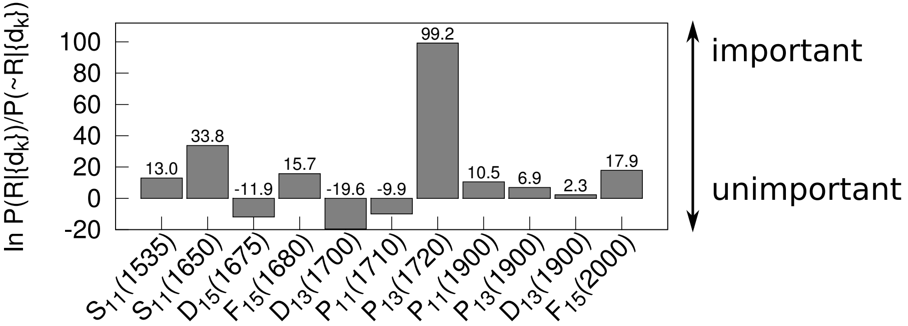


Bayesian evidences allow to determine $P(R|D)$ and $P(\neg R|D)$.

Role of individual resonances in $p(\gamma, K^+)\Lambda$



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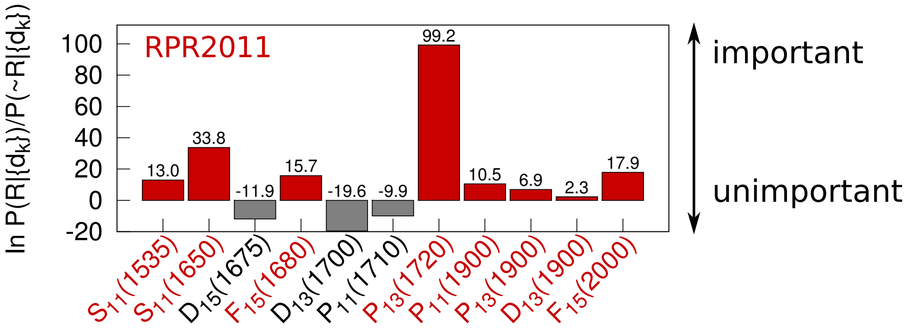
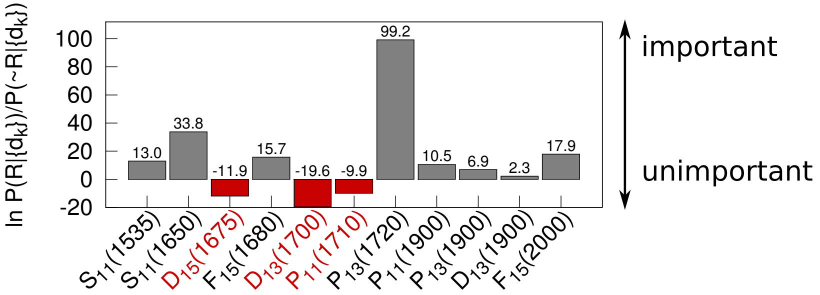


Role of individual resonances in $p(\gamma, K^+)\Lambda$



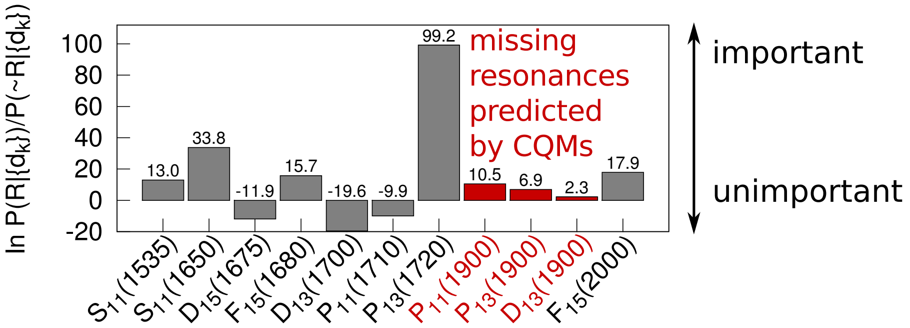
No evidence for...

$D_{15}(1675)$
- features in B-G, S-L, SAID
- doesn't couple to $K\Lambda$

$D_{13}(1700)$
- $***$ in PDG, absent in SAID
- couples mainly to $\pi\pi N$ in B-G

$P_{11}(1710)$
- $***$ in PDG, absent in SAID
- couples strongly (25%) to $K\Lambda$ in B-G

Role of individual resonances in $p(\gamma, K^+)\Lambda$



In the important 1900-MeV region...

- Evidence for 3 states → disfavors quark-diquark models
- $D_{13}(1900)$: evidence is significant, not decisive

Conclusions

- Strangeness production is challenging!
  - Background dominated
  - Many overlapping resonances
  - Conventional isobar approach less appropriate

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- Regge-plus-resonance (RPR) approach
  - reggeizes background and constrains it at high energies
  - adds N^*'s and Δ^*'s in the resonance region
  - valid threshold $\leq E_\gamma^{lab} \leq 16\,\text{GeV}$
  - economical description, i.e. limited number of parameters

Conclusions

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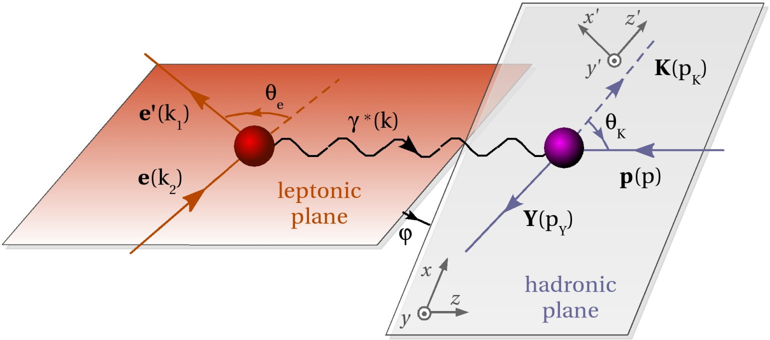
- RPR-2011 model
  - describes $p(\gamma, K^+)\Lambda$ world data
  - Bayesian methodology as ultimate tool for model selection
  - Evidence for $P_{13}(1900)$ ($**$), $P_{11}(1900)$ (missing) and $D_{13}(1900)$ (missing)

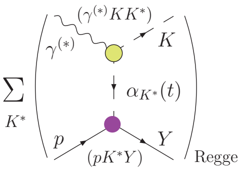
BACKUP

Kinematics



$$\frac{d\sigma}{d\Omega_K^*} = \frac{1}{64\,\pi^2}\,\frac{|\vec{p}_K^{\,*}|}{\omega^*}\,\frac{1}{(\omega^* + E_p^*)^2}\,\overline{\sum}_{\lambda,\lambda_i,\lambda_f} |\mathcal{M}_\lambda^{\lambda_i,\lambda_f}|^2$$

Background contributions: the Regge model



M. Guidal et al., PRC 68, 058201 (2003)

Background part of the amplitude contains exchanged K^* or Y^* states (t or u channel)at forward (backward) angles

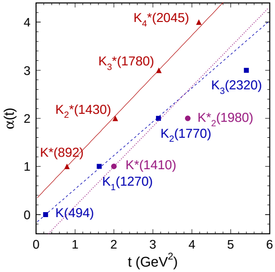
- we focus on K^* exchange $\rightsquigarrow$ forward kaon scattering angles
- instead of individual hadrons, entire families of hadrons are exchanged: **"Regge trajectories"**

Regge trajectories

Hadrons belong to classes with:

- same internal quantum numbers, but different spins J
- linear relation between squared mass (m_i^2) and spin (J_i) of members of a class

$\rightsquigarrow$ "Regge trajectory" $\alpha(t)$ with $\alpha(t = m_i^2) = J_i$



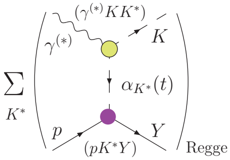
Modeling Regge-trajectory exchange

Modify the intermediate-particle propagator

Isobar: $\mathcal{P}^{K^*}_{isobar}(t) = \dfrac{1}{t - m^2_{K^*}}$

$\downarrow$

Regge: $\mathcal{P}^{K^*}_{Regge}(s,t) = \dfrac{s^{\alpha_{K^*}(t) - \alpha_{K^*,0}}}{\sin[\pi\alpha_{K^*}(t)]} \left\{ \begin{array}{c} 1 \\ e^{-i\pi\alpha_{K^*}(t)} \end{array} \right\} \dfrac{\pi\alpha'_{K^*}}{\Gamma[1 + \alpha_{K^*}(t) - \alpha_0]}$



$$s = (p_p + p_\gamma)^2 = W^2$$
$$t = (p_p - p_K)^2 = p^2_{K^*}$$
$$\alpha_{K^*}(t) = \alpha_{K^*,0} + \alpha'_{K^*}(t - m^2_{K^*})$$

isobar propagator $\longleftrightarrow$ **Regge propagator**

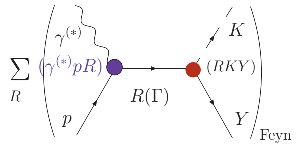
- single pole in t
- cross sections increase unrealistically with energy
- purely real

- series of poles, one per trajectory member
- s-dependence leads to cross sections decreasing with energy
- either constant or rotating phase

Resonance contributions

- Resonance decay accounted for through substitution in propagators:

$$s - m_R^2 \longrightarrow s - m_R^2 + i m_R \Gamma_R$$

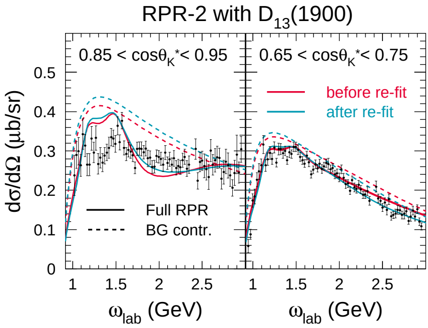


- Regularization of RKY vertex: Gaussian form factors (resonance contributions vanish at high energies)

$$F_{Gauss}(s) = \exp\left\{ -\frac{(s - m_R^2)^2}{\Lambda_{res}^4} \right\}$$

- Electromagnetic form factors for $\gamma^* pN^*$ and $\gamma^* p\Delta^*$ vertices: computed in Bonn constituent-quark model
R. Ricken et al., EPJA 9, 221 (2000); U. Loering et al., EPJA 10, 395 (2001); T. Van Cauteren et al., EPJ. A 26, 339 (2005)

The issue of double counting...



Phys. Lett. **B656**, 186 (2007)

Duality

energy-averaged sum over all N^*'s equals the sum over all t-channel Regge-trajectory echanges

Evaluate double counting

- Refit BG and resonances simultaneously
- effect on BG and full RPR is modest
- estimated effect on resonance parameters is 20 %

The RPR-2007 model

. . . a model for $K^+\Lambda$ and $K^+\Sigma^0$ production

High-energy region

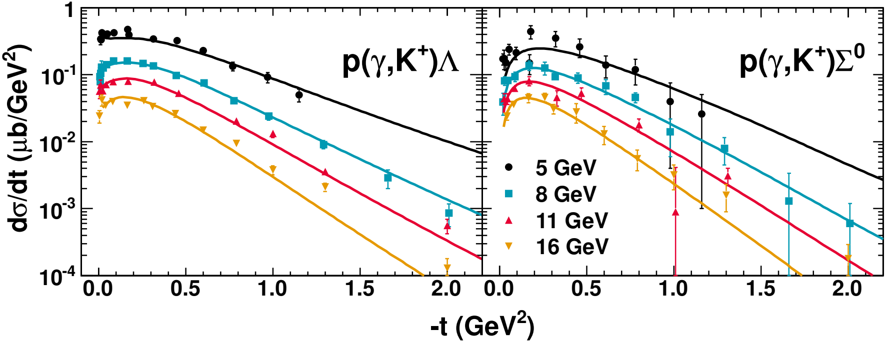
- $K(494)$ and $K^*(892)$ Regge-trajectory exchange
- Fitting database:
  - $K^+\Lambda$: 72 data points pre-1980
  - $K^+\Sigma^0$: 57 data points pre-1980

Resonance region

PRC73(2006)045207, PRC75(2007)045204, PLB656(2007)186

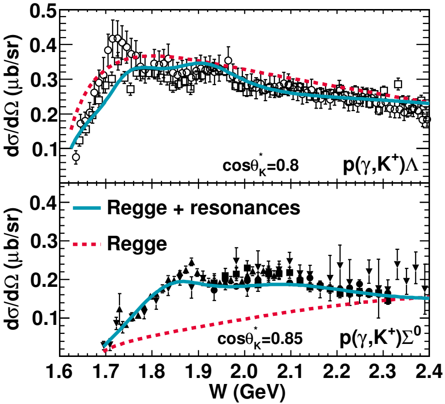
- Fixed set of established PDG resonances
- Investigate 3 possible missing resonances at $M_R = 1900\,\text{MeV}$
- Inconsistent Rarita-Schwinger couplings for $J = 3/2$ resonances
- Fitted to $K^+\Lambda$ and $K^+\Sigma^0$ world data pre-2007

RPR-2007 results at high energies



Regge model with 3 parameters

RPR-2007 results



$K^+\Lambda$ channel PRC73(2006)045207

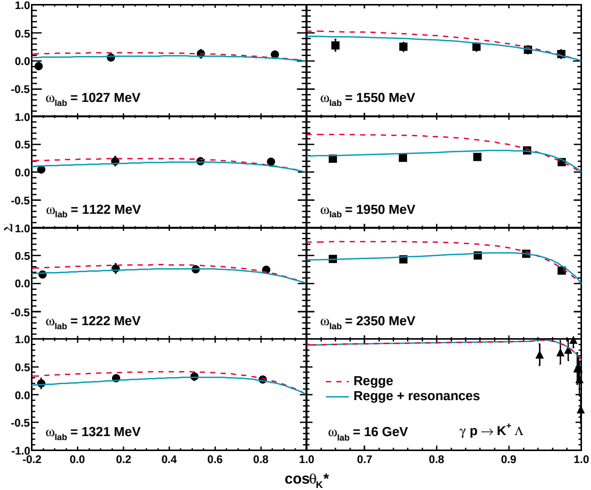
- $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$, $P_{13}(1900)$
- *missing* $D_{13}(1900)$

$K^+\Sigma^0$ channel PRC75(2007)045204

- $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$, $P_{13}(1900)$
- $D_{33}(1700)$, $S_{31}(1900)$, $P_{31}(1910)$, $P_{33}(1920)$

- Good description of data
- Resonances compatible with constituent-quark model

RPR-2007 results: photoproduction



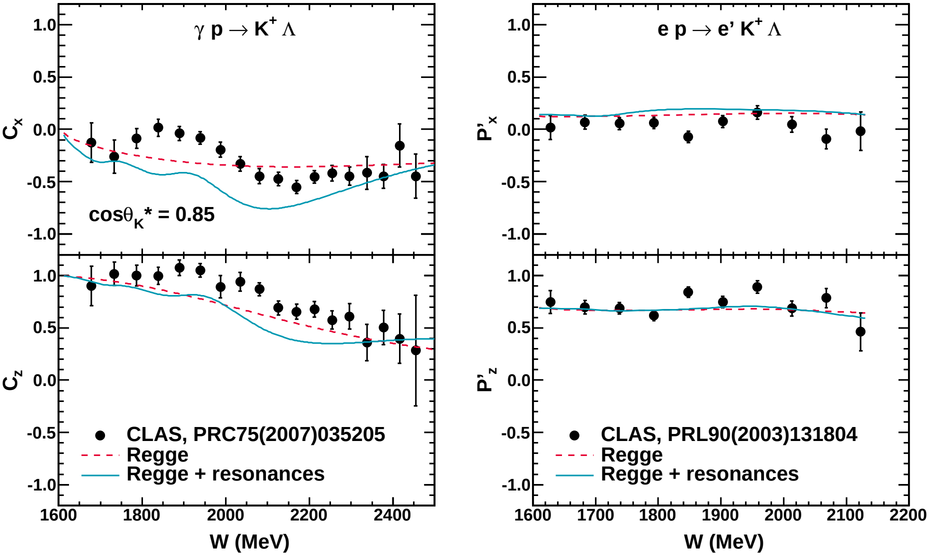
$K^+\Lambda$ amplitude

- K-traj.
- $K^*(892)$-traj.
- $S_{11}(1650)$
- $P_{11}(1710)$
- $P_{13}(1720)$
- $P_{13}(1900)$
- $D_{13}(1900)$

Eur. Phys. J. **A31**, 79 (2007)
Phys. Rev. Lett. **91**, 092001 (2003)
Phys. Rev. **D20**, 1553 (1979)

RPR provides an efficient description of the world data from threshold up to $\omega_{\text{lab}} = 16\,\text{GeV}$

RPR-2007 predictions: photo- and electroproduction



The RPR-2011 model

...a model for $K^+\Lambda$ production

High-energy region PLB 694 (2010) 33

- $K(494)$ and $K^*(892)$ Regge-trajectory exchange
- Fitting database:
  - 262 data points from latest CLAS publication ($W > 2.6$ GeV)

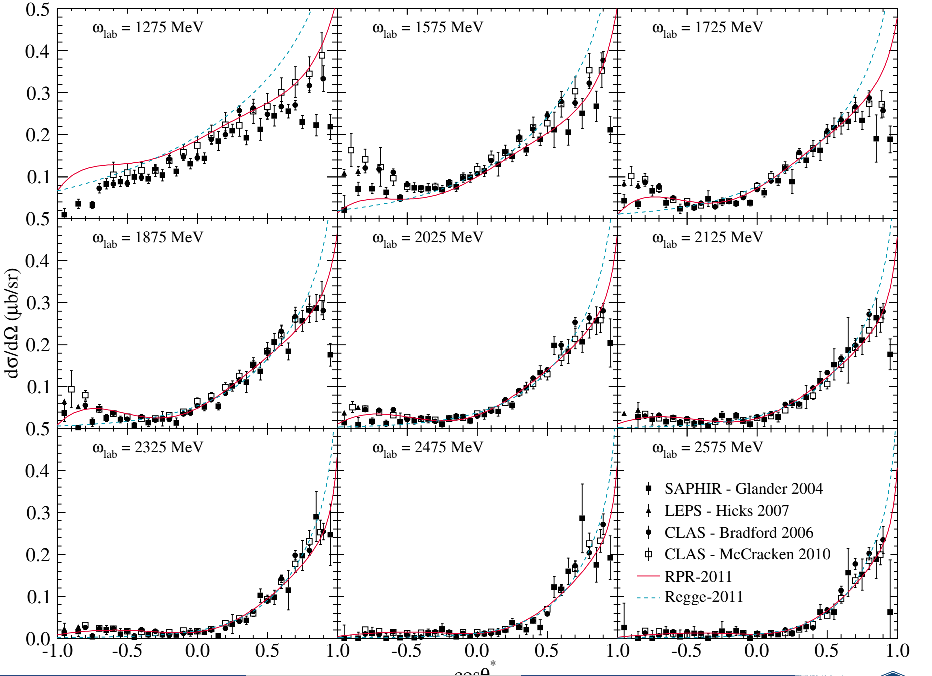
    McCracken et al., PRC81(2010)025201

Resonance region PRL 108 (2012) 182002

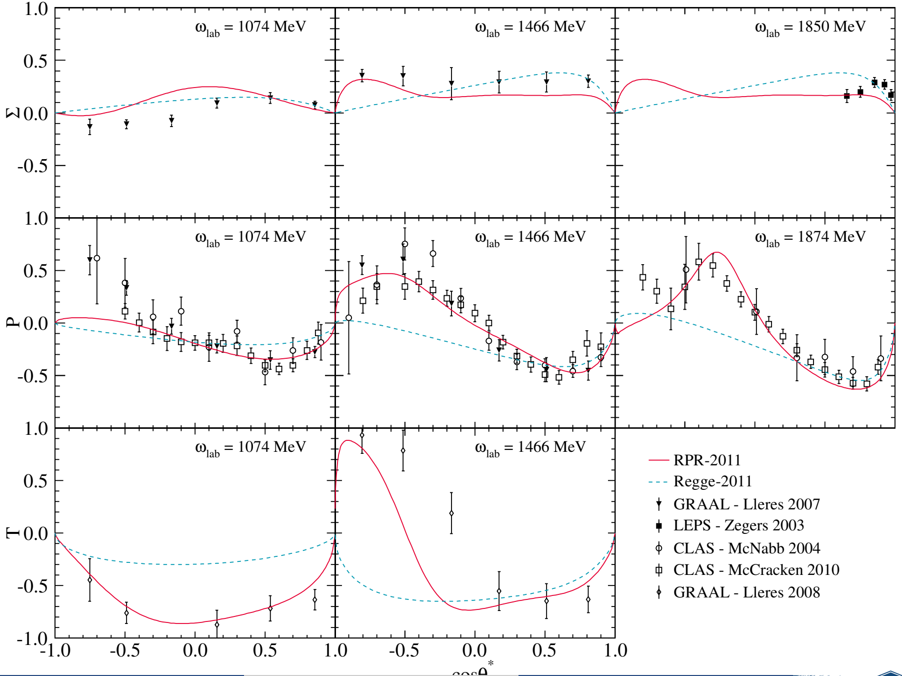
- Investigate 11 possible resonances
- Consistent couplings for $J = 3/2$ and $J = 5/2$ resonances

  Vrancx et al., PRC84(2011)045201
- Fitted to up-to-date world data

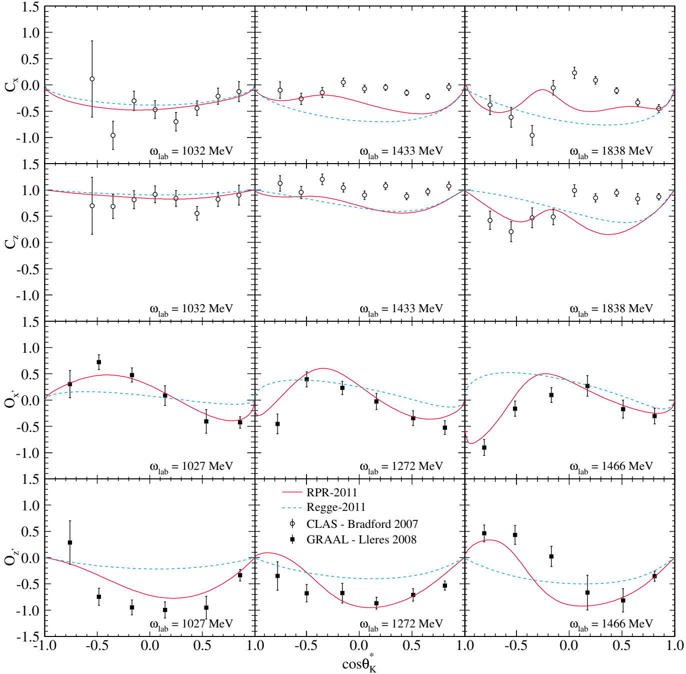
Observable	#data	Experiment	Year	Reference
$\frac{d\sigma}{d\Omega}$	56	SLAC	1969	Boyarski *et al.*
	720	SAPHIR	2004	Glander *et al.*
	1377	CLAS	2006	Bradford *et al.*
	12	LEPS	2007	Hicks *et al.*
	2066	CLAS	2010	McCracken *et al.*
Σ	9	SLAC	1979	Quinn *et al.*
	45	LEPS	2003	Zegers *et al.*
	54	LEPS	2006	Sumihama *et al.*
	4	LEPS	2007	Hicks *et al.*
	66	GRAAL	2007	Lleres *et al.*
T	3	BONN	1978	Althoff *et al.*
	66	GRAAL	2008	Lleres *et al.*
P	7	DESY	1972	Vogel *et al.*
	233	CLAS	2004	McNabb *et al.*
	66	GRAAL	2007	Lleres *et al.*
	1707	CLAS	2010	McCracken *et al.*
C_x , C_z	320	CLAS	2007	Bradford *et al.*
$O_{x'}$, $O_{z'}$	132	GRAAL	2008	Lleres *et al.*



ωlab = 1275 MeV
ωlab = 1575 MeV
ωlab = 1725 MeV
ωlab = 1875 MeV
ωlab = 2025 MeV
ωlab = 2125 MeV
ωlab = 2325 MeV
ωlab = 2475 MeV
ωlab = 2575 MeV
dσ/dΩ (μb/sr)
cosθ*
SAPHIR - Glander 2004
LEPS - Hicks 2007
CLAS - Bradford 2006
CLAS - McCracken 2010
RPR-2011
Regge-2011



ωlab = 1074 MeV
ωlab = 1466 MeV
ωlab = 1850 MeV
ωlab = 1874 MeV
Σ
P
T
1.0
0.5
0.0
-0.5
-1.0
-0.5
1.0
RPR-2011
Regge-2011
GRAAL - Lleres 2007
LEPS - Zegers 2003
CLAS - McNabb 2004
CLAS - McCracken 2010
GRAAL - Lleres 2008
cosθ*



C_x
C_z
$O_{x'}$
$O_{z'}$
ω_{lab} = 1032 MeV
ω_{lab} = 1433 MeV
ω_{lab} = 1838 MeV
ω_{lab} = 1027 MeV
ω_{lab} = 1272 MeV
ω_{lab} = 1466 MeV
RPR-2011
Regge-2011
CLAS - Bradford 2007
GRAAL - Lleres 2008
$\cos\theta_K^*$

Interacting Rarita-Schwinger fields

On-shell case

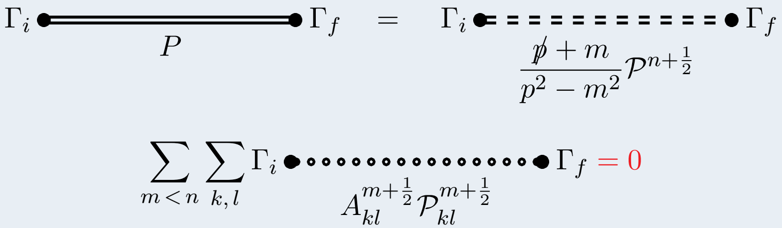
- On-shell R-S field is described by R-S spinor
- Unphysical components of R-S spinor decouple from the interaction

Off-shell case

- Off-shell R-S field is described by R-S propagator
- Unphysical components of R-S propagator do not decouple a priori
- Consistent interaction should be invariant under certain **local gauge**

Consistency and locality of the interaction

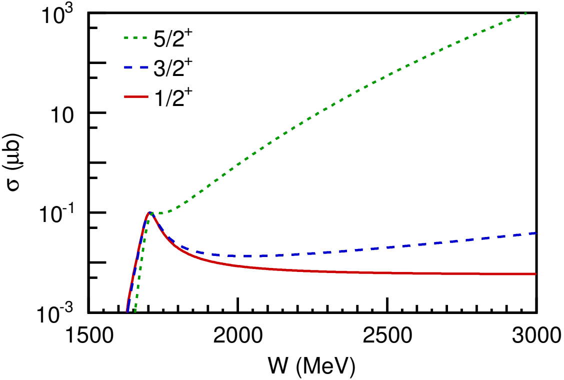
Consistent interaction for $\psi^*_{\mu_1\ldots\mu_n}$



Interpretation

- Interaction is mediated purely by physical component of R-S field

Inconsistency of standard hadronic form factors



Toy N^*

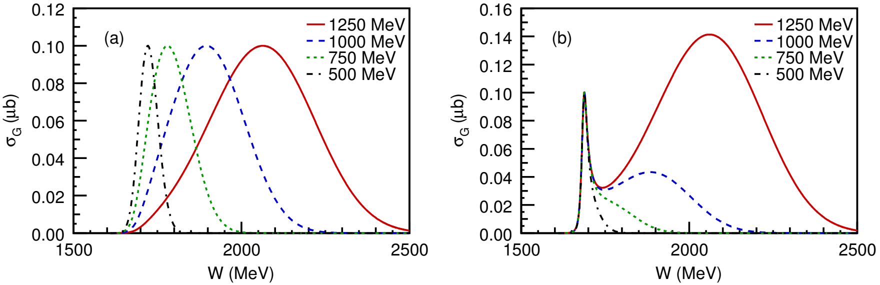
$m_R = 1700$ MeV

$\Gamma_R = 50$ MeV

$J_R^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$

Hadronic form factor required to suppress high-energy behavior

Inconsistency of standard hadronic form factors



Remarks

- Lowering Λ_R results in shift of artificial bump towards W_0
- Lowering Λ_R only effective when Γ_R is "small"
  - practically all N^*'s listed by PDG have "large" Γ_R

The multidipole-Gauss form factor

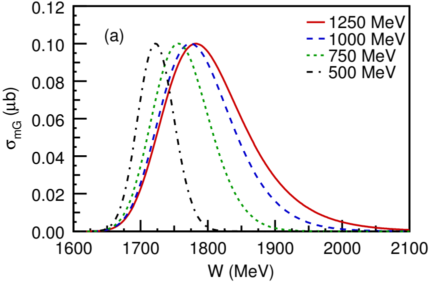
Multidipole-Gauss form factor

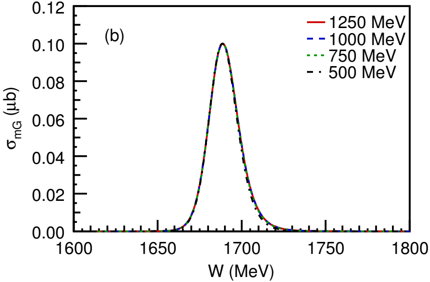
$$F_{mG}(s; m_R, \Lambda_R, \Gamma_R, J_R) = \left(\frac{m_R^2 \widetilde{\Gamma}_R^2(J_R)}{(s - m_R^2)^2 + m_R^2 \widetilde{\Gamma}_R^2(J_R)} \right)^{J_R - \frac{1}{2}} \exp\left(- \frac{(s - m_R^2)^2}{\Lambda_R^4} \right)$$

- Dipole part of F_{mG} raises multiplicity of propagator pole
- Modified decay width

$$\widetilde{\Gamma}_R(J_R) = \frac{\Gamma_R}{\sqrt{2^{\frac{1}{2J_R}} - 1}}$$

The multidipole-Gauss form factor





Remarks

- Artificial bump is removed and resonance peak is restored
- Threshold effects for $m_R - \frac{\Gamma_R}{2} \approx W_0$
  - Peak position not at $W = m_R$
  - Peak position and width are function of Λ_R