Meson spectroscopy from lattice QCD: progress and challenges

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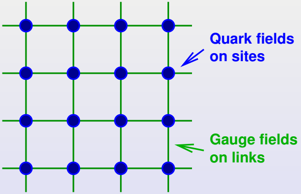
Overview

- Introduction
- New techniques in lattice spectroscopy
- Results
    - Isovector mesons
    - Isoscalar mesons
    - Charmonium
- Scattering and resonances
    - $\pi\pi$-scattering ($I = 2$)
    - $\pi\pi$-scattering ($I = 1$) [Graz]
- Conclusions

Lattice regularisation

- Lattice provides a **non-perturbative, gauge-invariant** regulator for QCD



- Quarks live on sites
- Gluons live on links
- lattice spacing: $a \sim 0.1$ fm
- Nielson-Ninomiya: no chirally symmetric quarks
- In a finite volume $V = L^4$, finite number of degrees of freedom
- Minkowski$\rightarrow$ Euclid allows efficient Monte Carlo

Finite V and a: path-integral is an ordinary (but large) integral.
Make predictions from the QCD lagrangian by **Monte Carlo**

Spectroscopy in lattice QCD

- Energies of colourless QCD states can be extracted from **two-point functions** in Euclidean time

$$C(t) = \langle 0|\, \Phi(t)\Phi^\dagger(0)\, |0\rangle$$

- Euclidean time: $\Phi(t) = e^{Ht}\Phi e^{-Ht}$ so $C(t) = \langle\Phi|e^{-Ht}|\Phi\rangle$. Insert a complete set of energy eigenstate and:

$$C(t) = \sum_{k=0}^{\infty} |\langle\Phi|k\rangle|^2 e^{-E_k t}$$

- $\lim_{t\to\infty} C(t) = Ze^{-E_0 t}$, so if observe large-t fall-off, then **energy of ground-state** is measured.

Euclidean metric very useful for spectroscopy; it provides a way of isolating and examining low-lying states

Excited states

- **Excited-state** energies can be measured by correlating between operators in a bigger set, $\{\Phi_1, \Phi_2, \ldots, \Phi_N\}$

$$C_{ij}(t) = \langle 0| \, \Phi_i(t)\Phi_j^\dagger(0) \, |0\rangle$$

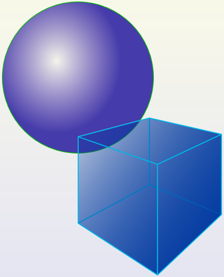
- Solve generalised eigenvalue problem:

$$C(t_1) \, \underline{v} = \lambda \, C(t_0) \, \underline{v}$$

  for different t_0 and t_1 [Lüscher & Wolff, C. Michael]
- Then $\lim_{(t_1-t_0)\to\infty} \quad \lambda_n = e^{-E_n(t_1-t_0)}$
- Method constructs optimal ground-state creation operator, then builds orthogonal states.

Excited states accessed if basis of creation operators is used and the matrix of correlators can be computed

Spin on the lattice



- Lattice breaks $O(3) \to O_h$
- Lattice states classified by quantum letter, $R \in \{A_1, A_2, E, T_1, T_2\}$.
- Continuum: subduce $O(3)$ irreps $\to O_h$

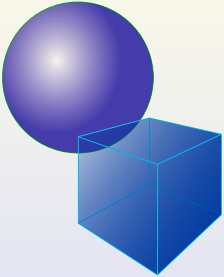
- Continuum spins can appear in more than one lattice irrep
- Degeneracies in continuum limit?
- Problem: Spin 4 has same pattern as $0 \oplus 1 \oplus 2$

	A_1	A_2	E	T_1	T_2
0	×				
1				×	
2			×		×
3		×		×	×
4	×		×	×	×

Lattice regulator breaks continuum rotation group, so states on the lattice classified by a "quantum letter" $R \in \{A_1, A_2, E, T_1, T_2\}$

Progress - new techniques

Spin on the lattice



- Lattice states classified by quantum letter, $R \in \{A_1, A_2, E, T_1, T_2\}$.
- Start with continuum: $\bar{\psi}\Gamma D_i D_j \dots \psi$ and subduce $O(3)$ irreps $\to O_h$
- Example:
  $\Phi_{ij} = \bar{\psi}\left(\gamma_i D_j + \gamma_j D_i - \frac{2}{3}\delta_{ij}\gamma \cdot D\right)\psi$
- Lattice: substitute $D \to D_{latt}$
- Now have a reducible representation:

$$\Phi^{T_2} = \{\Phi_{12}, \Phi_{23}, \Phi_{31}\} \;\&\; \Phi^{E} = \left\{\frac{1}{\sqrt{2}}(\Phi_{11} - \Phi_{22}), \frac{1}{\sqrt{6}}(\Phi_{11} + \Phi_{22} - 2\Phi_{33})\right\}$$

- Look for signature of continuum symmetry:

$$\langle 0|\Phi^{T_2}|2^{++(T_2)}\rangle = \langle 0|\Phi^{E}|2^{++(E)}\rangle$$

Remnants of continuum spin can be found on the lattice. Build operators in continuum and measure overlaps to find patterns

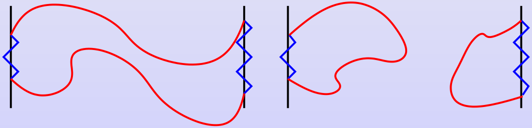
Isoscalar meson correlation functions

- Isovector mesons: Wick contraction gives

- Isoscalar meson correlator has extra diagram. Wick contraction:

$$\langle \psi_i \bar{\psi}_j \psi_k \bar{\psi}_l \rangle = M^{-1}_{ij} M^{-1}_{kl} - M^{-1}_{il} M^{-1}_{jk}$$

$$\begin{aligned} \langle 0|\Phi^{(I=0)}(t)\Phi^{\dagger(I=0)}(0)|0\rangle &= \\ \langle 0|\Phi^{(I=1)}(t)\Phi^{\dagger(I=1)}(0)|0\rangle &- \langle 0|\mathrm{Tr}\, M^{-1}\Gamma(t)\mathrm{Tr}\, M^{-1}\Gamma(0)|0\rangle \end{aligned}$$

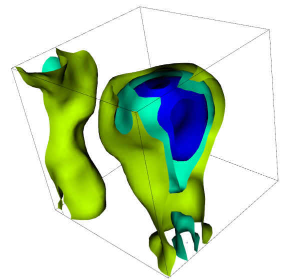


Measuring isoscalar meson correlation functions means also computing the disconnected Wick graphs by Monte Carlo.

New methods: distillation

- We can ameliorate the problems coming from measuring quark propagation by looking more carefully at how hadrons are constructed most efficiently
- **Smeared fields:** determine $\tilde{\psi}$ from the "raw" field in the path-integral, ψ:

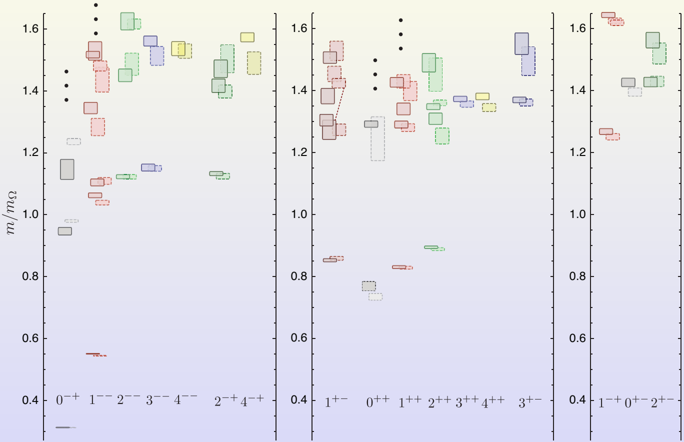
$$\tilde{\psi}(t) = \square[U(t)]\psi(t)$$



- Extract confinement-scale degrees of freedom while preserving symmetries
- Build creation operators on smeared fields
- Re-define smearing to be a projection operator into a small vector space smooth fields: **distillation**

Results:
meson excitation spectra

Isovector meson spectroscopy



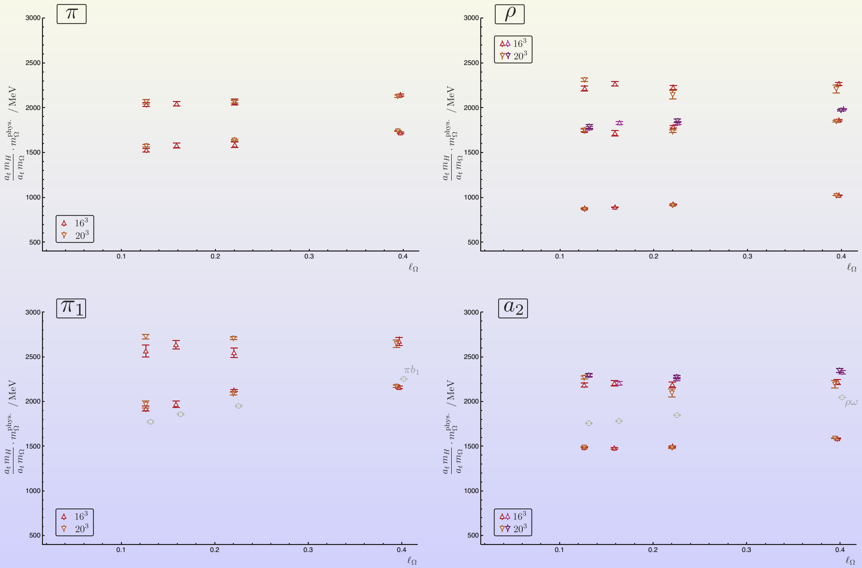
- $m_\pi = 400$ MeV
- No 2-meson operators

Should be a dense spectrum of two-meson states:

— Not seen at all

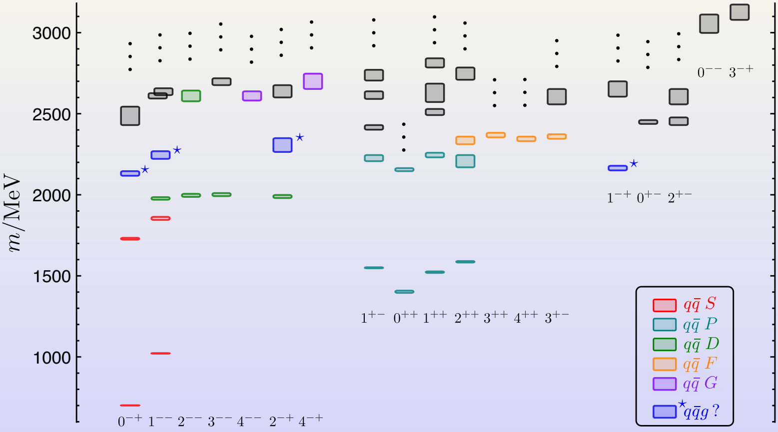
[Dudek et.al. Phys.Rev.D82:034508,2010]

Light quark mass dependence



[Dudek et.al. Phys.Rev.D82:034508,2010]

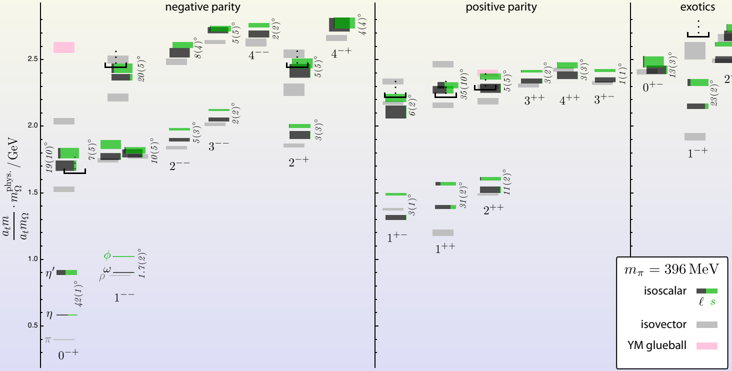
Hybrid excitations?



- $m_\pi = 700$ MeV
- Complete hybrid supermultiplet seen

[J.Dudek, Phys.Rev.D84 (2011) 074023]

Isoscalar mesons



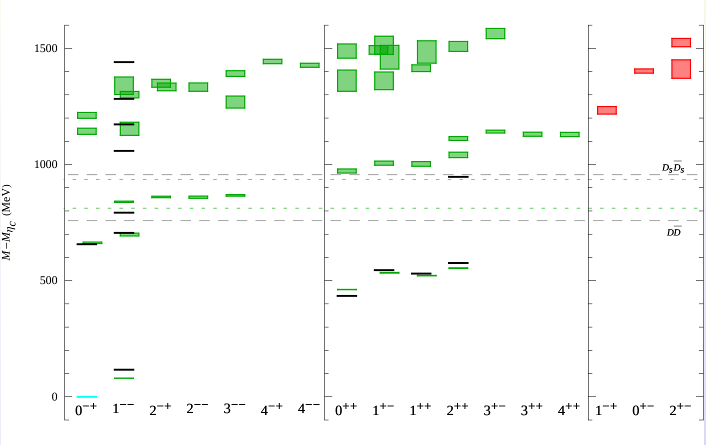
- $m_\pi = 400$ MeV, finite a
- No 0^{++} data presented
- No glueball or two-meson operators

Statistical precision:

η	0.5 %
η'	1.9 %

[Dudek et.al. Phys.Rev.D83:111502,2011]

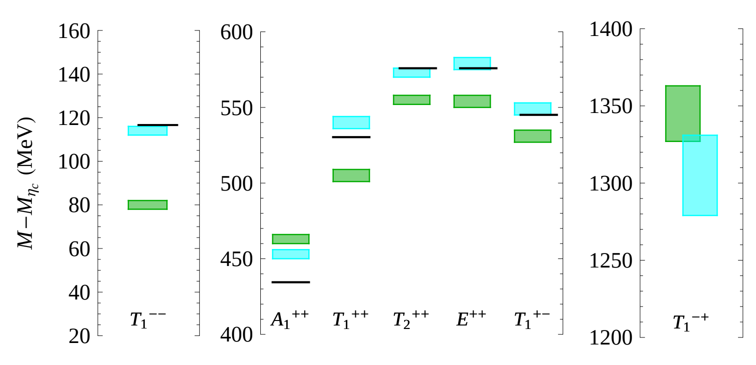
Excitation spectrum of charmonium



- Quark model: 1S, 1P, 2S, 1D, 2P, 1F, 2D, . . . all seen.
- Not all fit quark model: spin-exotic (and non-exotic) hybrids seen

[Liu et.al. arXiv:1204.5425]

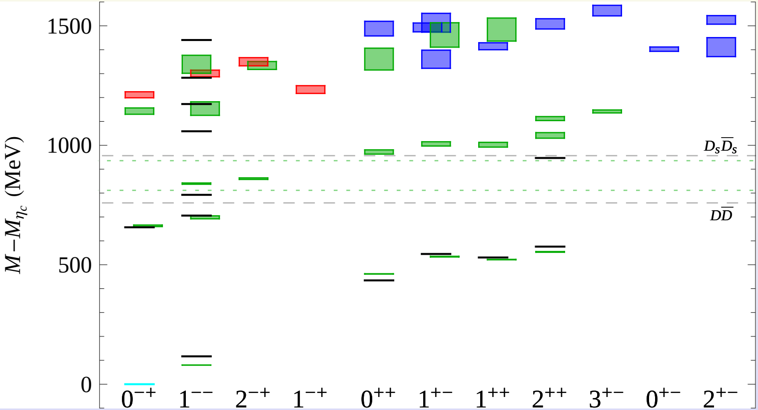
Lattice artefacts in charmonium



- Hyperfine structure sensitive to lattice artefacts. Boost co-efficient of action term to suppress these.
- green $\to$ light blue. Shifts are ≈ 40 MeV.

[Liu et.al. arXiv:1204.5425]

Gluonic excitations in charmonium?



- See states created by operators that excite intrinsic gluons
- two- and three-derivatives create states in the open-charm region.

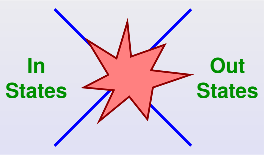
[Liu et.al. arXiv:1204.5425]

Challenges:
scattering and resonance

Scattering

Scattering matrix elements not directly accessible from Euclidean QFT [Maiani-Testa theorem]

- Scattering matrix elements: asymptotic $|\mathrm{in}\rangle, |\mathrm{out}\rangle$ states. $\langle \mathrm{out}\, |e^{i\hat{H}t}|\, \mathrm{in}\rangle \rightarrow \langle \mathrm{out}\, |e^{-\hat{H}t}|\, \mathrm{in}\rangle$
- Euclidean metric: project onto ground-state



- **Lüscher's formalism:** information on elastic scattering inferred from **volume dependence** of spectrum
- Requires precise data, resolution of two-hadron and excited states.

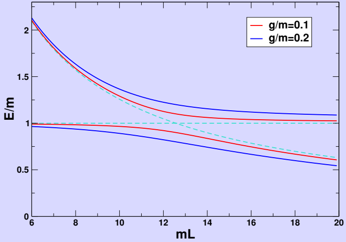
Hadrons in a finite box: scattering

- On a finite lattice with periodic b.c., hadrons have quantised momenta; $\underline{p} = \frac{2\pi}{L}\left\{n_x, n_y, n_z\right\}$
- Two hadrons with total $P = 0$ have a discrete spectrum
- These states can have same quantum numbers as those created by $\bar{q}\Gamma q$ operators and QCD can mix these

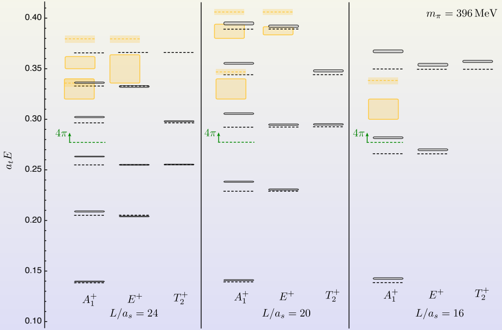
- This leads to shifts in the spectrum in finite volume
- This is the same physics that makes resonances in an experiment
- Lüscher's method - relate elastic scattering to energy shifts

Toy model

$$H = \begin{pmatrix} m & g \\ g & \frac{4\pi}{L} \end{pmatrix}$$



$\mathrm{I} = 2 \;\; \pi - \pi$ phase shift

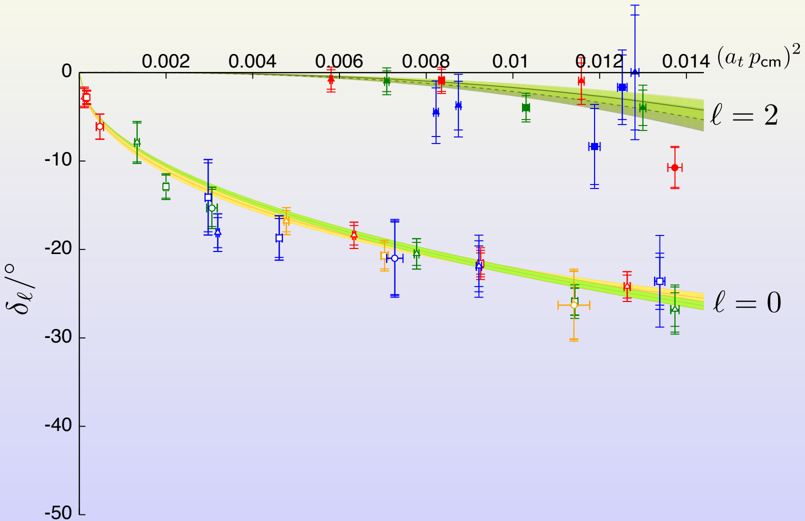


- Lüscher's method: first determine energy shifts as volume changes
- Data for $\mathrm{L} = 16\mathrm{a_s}, 20\mathrm{a_s}, 24\mathrm{a_s}$
- Small energy shifts are resolved

- Measured δ_0 and δ_2 (δ_4 is very small)
- $\mathrm{I} = 2$ a useful first test - simplest Wick contractions

Dudek et.al. [Phys.Rev.D83:071504,2011, `arXiv:1203.6041`]

I = 2 $\pi - \pi$ phase shift

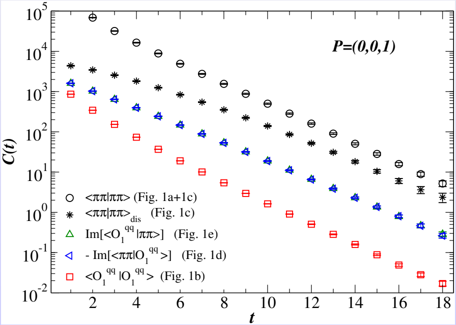


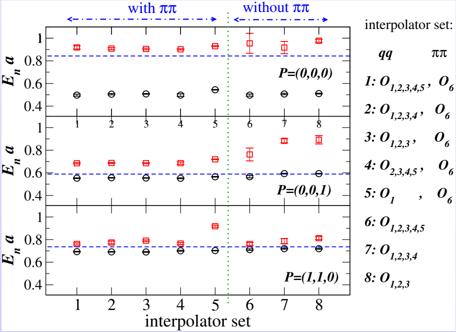
Dudek et.al. [Phys.Rev.D83:071504,2011, arXiv:1203.6041]

$I = 1$ scattering using distillation

[C.Lang et.al. arXiv:1105.5636]

- Number of groups have measured Γ_ρ on the lattice.
- Need non-zero relative momentum of pions in final state (P-wave decay)
- New calculation using distillation

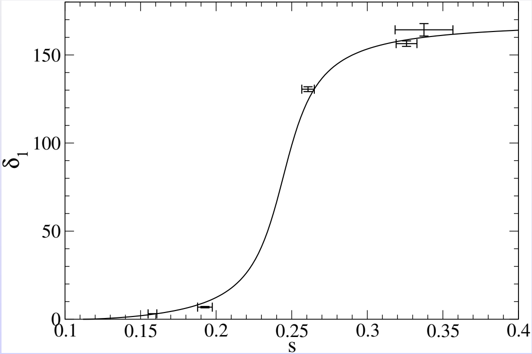




$I = 1$ $\pi\pi$ phase shift

[C.Lang et.al. arXiv:1105.5636]

- $m_\pi \approx 266$ MeV
- Better resolution by studying moving ρ as well
- ρ resonance resolved clearly, with $m_\rho = 792(7)(8)$ MeV
- $g_{\rho\pi\pi} = 5.13(20)$



- Meson spectroscopy from lattice QCD continues to make progress and face challenges …

Progress

☐ New methods have enabled:

- variational calculations to study excitations
- reliable spin identification
- isoscalar meson spectroscopy
- multi-meson states

☐ Good resolution in light and charmonium sectors, with more results on the way

Challenges

☐ Can we use these methods for tetraquarks, molecules … ?

☐ Studying resonances in Euclidean space-time is challenging

☐ The two-meson states are not seen with local operators

☐ Lüscher's technique - studies beginning and making progress

☐ What happens above inelastic thresholds?

- … expect more results soon.