HADRON–HADRON SCATTERING: LESSONS from CHIRAL SYMMETRY +

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

Supported by DFG, SFB/TR-16

and by EU, I3HP EPOS

and by BMBF 06BN9006

and by HGF VIQCD VH-VI-417

CONTENTS

- **Intro: Why hadron-hadron scattering?**
- **Lesson 1: Pion-pion scattering**
- **Lesson 2: Pion-kaon scattering**
- **Lesson 3: Pion-nucleon scattering**
- **Lesson 4: Kaon-nucleon scattering**
- **Lesson 5: Goldstone boson scattering off $D, D^\star$-mesons**
- **Short summary & outlook**

Introduction

WHY HADRON-HADRON SCATTERING?

- Weinberg's 1966 paper "Pion scattering lengths"

Weinberg, Phys. Rev. Lett. **17** (1066) 616

– pion scattering on a target with mass m_t and isospin T_t:

$$a_T = -\frac{L}{1 + M_\pi/m_t}\,[T(T+1) - T_t(T_t+1) - 2]$$

– pion scattering on a pion ["the more complicated case"]:

$$a_0 = \frac{7}{4}L\,, \quad a_2 = -\frac{1}{2}L$$

$$L = \frac{g_V^2 M_\pi}{2\pi F_\pi^2} \simeq 0.1\,M_\pi^{-1}$$

- amazing predictions - witness to the power of chiral symmetry
- what have we learned since then?

CHIRAL SYMMETRY of QCD

- Three flavor QCD: Fritzsch, Gell-Mann, Leutwyler (1973)

$$\boxed{\mathcal{L}_{\rm QCD} = \mathcal{L}^0_{\rm QCD} - \bar{q}\mathcal{M}q}\,, \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}\,, \quad \mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

- $\mathcal{L}^0_{\rm QCD}$ is invariant under **chiral** $SU(3)_L \times SU(3)_R$ (split off U(1)'s)

$$\mathcal{L}^0_{\rm QCD}(G_{\mu\nu}, q', D_\mu q') = \mathcal{L}^0_{\rm QCD}(G_{\mu\nu}, q, D_\mu q)$$

$$q' = RP_Rq + LP_Lq = Rq_R + Lq_L \quad R, L \in SU(3)_{R,L}$$

- conserved L/R-handed [vector/axial-vector] Noether currents:

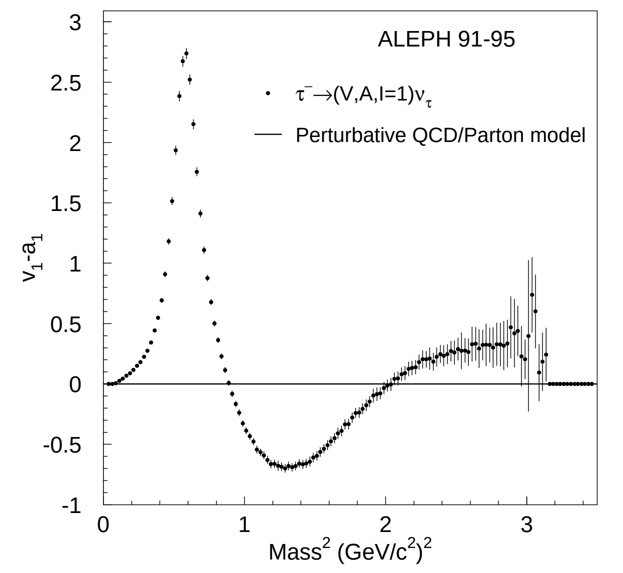
$$J^{\mu,a}_{L,R} = \bar{q}_{L,R}\gamma^\mu \frac{\lambda^a}{2} q_{L,R}\,, \quad a = 1, \ldots, 8$$

$$\partial_\mu J^{\mu,a}_{L,R} = 0 \quad [\text{or } \; V^\mu = J^\mu_L + J^\mu_R\,, \;\; A^\mu = J^\mu_L - J^\mu_R]$$

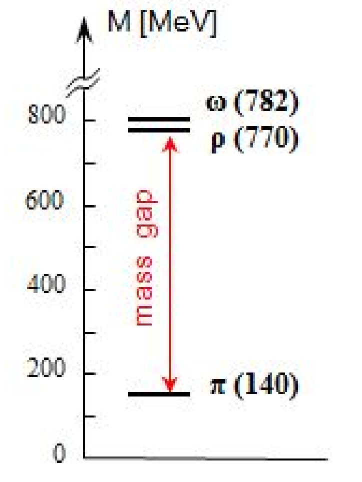
- Is this symmety reflected in the vacuum structure/hadron spectrum?

THE FATE of QCD's CHIRAL SYMMETRY

- the chiral symmetry is not "visible" (spontaneously broken)
  - no parity doublets
  - $\langle 0|AA|0\rangle \neq \langle 0|VV|0\rangle$
  - scalar condensate $\bar{q}q$ acquires v.e.v.
  - Vafa-Witten theorem
  - (almost) massless pseudoscalar bosons



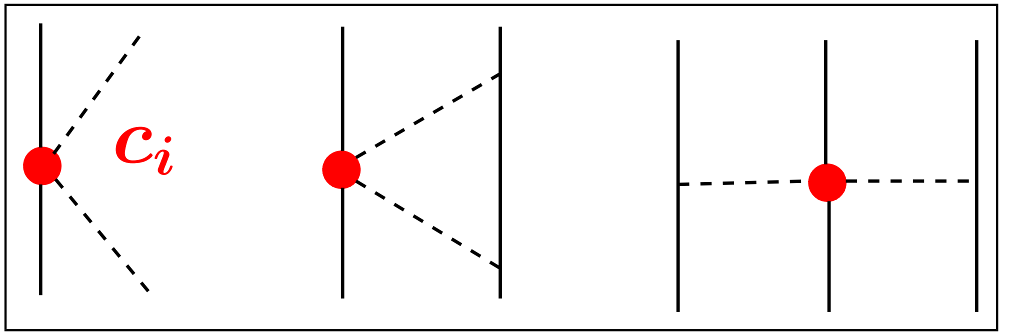
- the chiral symmetry is realized in the Nambu-Goldstone mode
  - weakly interacting massless pseudoscalar excitations
  - approximate symmetry (small quark masses)
    $\longrightarrow \pi, K, \eta$ as Pseudo-Goldstone Bosons
  - calls for an effective field theory
    $\Rightarrow$ Chiral Perturbation Theory (CHPT)



THE ESSENCE of CHIRAL PERTURBATION THEORY

- explores the consequences of chiral symmetry breaking in QCD
- has an underlying power counting, based on the scale separation
- relates *many* processes
- increasing number of LECs w/ higher orders, but: NOT prolific for basic processes

  e.g. $\pi\pi \to \pi\pi$ at one loop:
  has 4 LECs
  $\pi\pi \to \pi\pi$ at two loops
  has only 2 new LECs



$\pi N \to \pi N$ $\quad NN \to NN$ $\quad 3N$-force

- and the **+** : predictions can be sharpened by combining with dispersion relations, coupled-channels, lattice, . . ., but: there is no free lunch!
- and now lets see how this works and what we can learn...

Lesson 1

ELASTIC PION-PION SCATTERING

- Purest process in two-flavor chiral dynamics (really light quarks)

- scattering amplitude at threshold: two numbers (a_0, a_2)

- History of the prediction for a_0:

LO (tree):	$a_0 = 0.16$	Weinberg 1966
NLO (1-loop):	$a_0 = 0.20 \pm 0.01$	Gasser, Leutwyler 1983
NNLO (2-loop):	$a_0 = 0.217 \pm 0.009$	Bijnens et al. 1996

- even better: match 2-loop representation to Roy equation solution

Roy + 2-loop:	$a_0 = 0.220 \pm 0.005$	Colangelo et al. 2000

$\Rightarrow$ this is an *amazing* prediction!

- same precision for a_2, but corrections very small . . .

HOW ABOUT EXPERIMENT?

- Kaon decays (K_{e4} and $K^0 \to 3\pi^0$): most precise
- Lifetime of pionium: experimentally more difficult

Kaon decays:

$$a_0^0 = 0.2210 \pm 0.0047_{\rm stat} \pm 0.0040_{\rm sys}$$

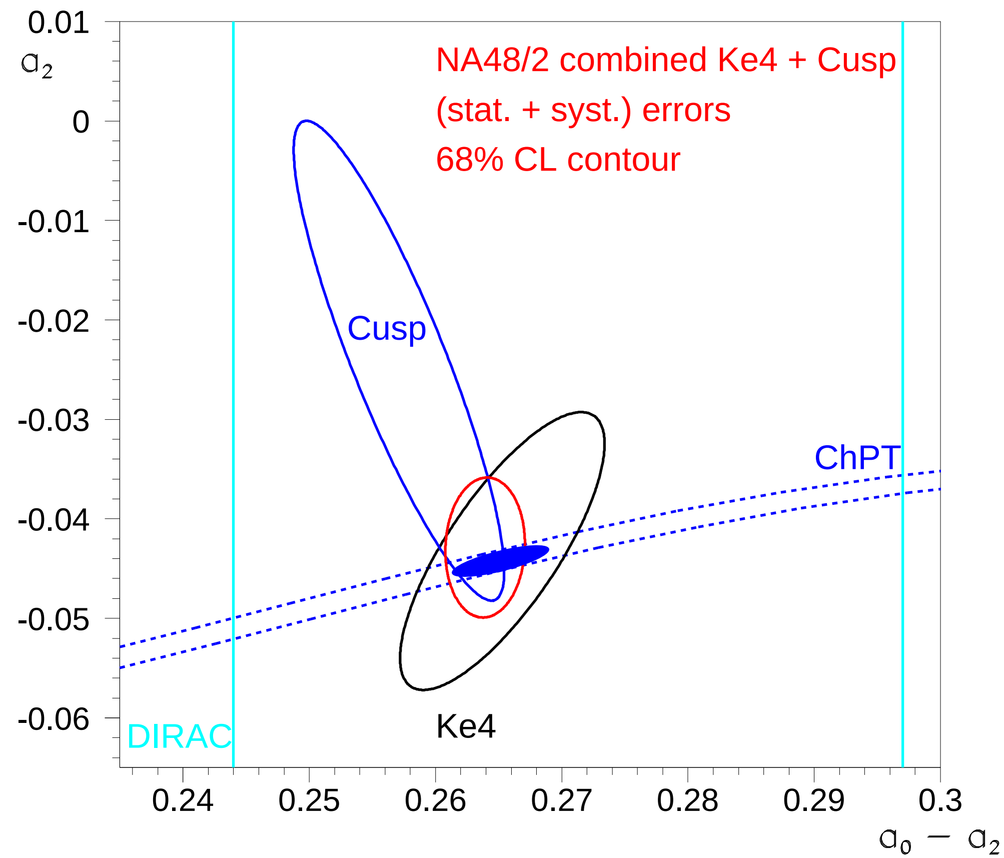
$$a_0^2 = -0.0429 \pm 0.0044_{\rm stat} \pm 0.0028_{\rm sys}$$

J. R. Batley et al. [NA48/2 Coll.] EPJ C 79 (2010) 635

Pionium lifetime:

$$|a_0^0 - a_0^2| = 0.264^{+0.033}_{-0.020}$$

B. Adeva et al. [DIRAC Coll.] PL B 619 (2005) 50

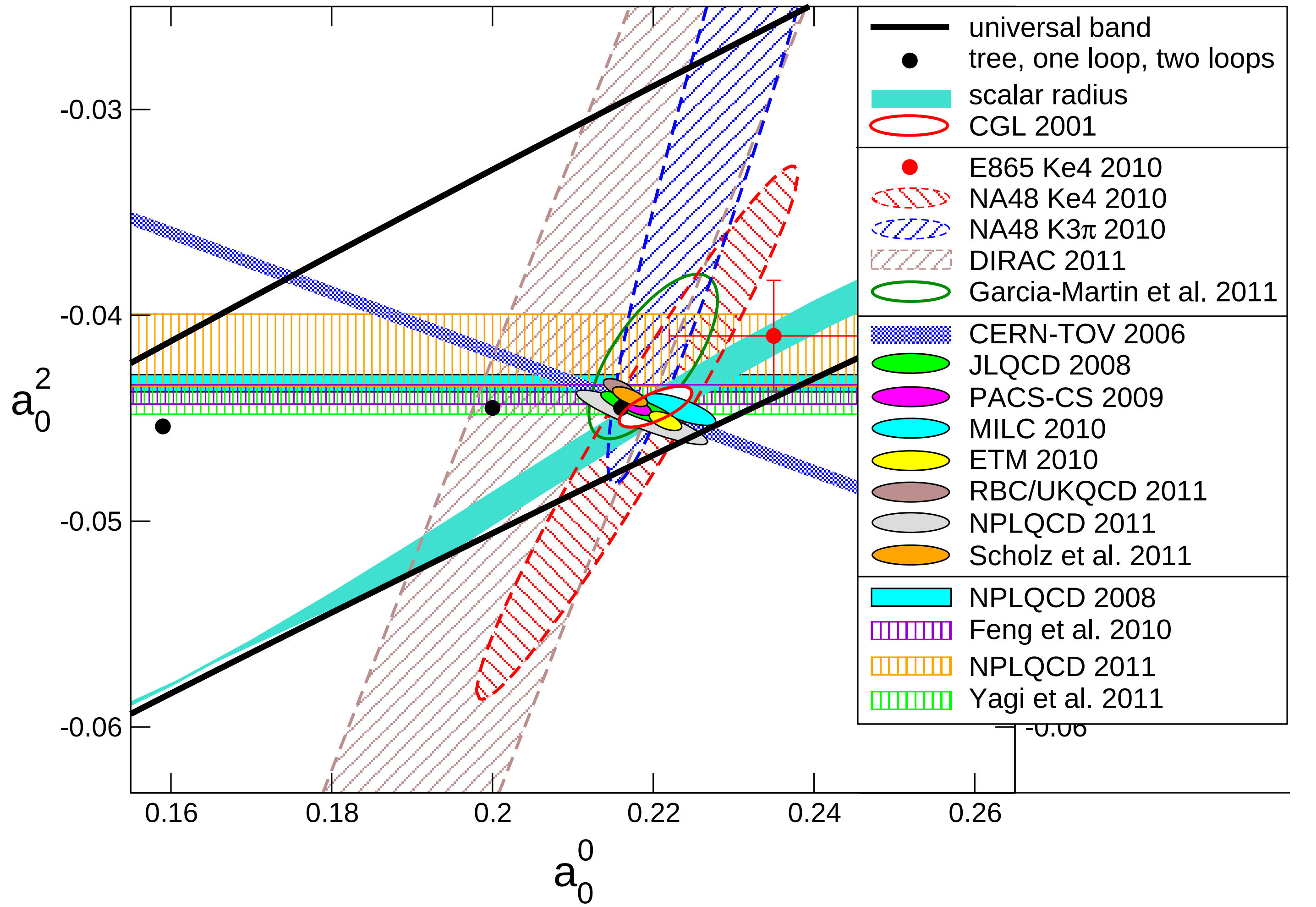


- and how about the lattice?

$\Rightarrow$ direct and indirect determinations of the scattering lengths

THE GRAND PICTURE

Fig. courtesy Heiri Leutwyler 2012



• one of the finest tests of the Standard Model (but: direct lattice a_0 missing)

Lesson 2

STRANGE QUARK MYSTERIES

- Is the strange quark really light? $\boxed{m_s \sim \Lambda_{\rm QCD}}$

$\longrightarrow$ expansion parameter: $\xi_s = \frac{M_K^2}{(4\pi F_\pi)^2} \simeq 0.18$ [SU(2): $\xi = \frac{M_\pi^2}{(4\pi F_\pi)^2} \simeq 0.014$]

- many predictions of SU(3) CHPT work quite well, but:

$\hookrightarrow$ indications of bad convergence in some recent lattice calculations:

$\star$ masses and decay constants — Allton et al. 2008

$\star$ $K_{\ell 3}$-decays — Boyle et al. 2008

$\hookrightarrow$ suppression of the three-flavor condensate?

$\star$ sum rule: $\Sigma(3) = \Sigma(2)[1 - 0.54 \pm 0.27]$ — Moussallam 2000

$\star$ lattice: $\Sigma(3) = \Sigma(2)[1 - 0.23 \pm 0.39]$ — Fukuya et al. 2011

ELASTIC PION-KAON SCATTERING

- Purest process in three-flavor chiral dynamics
- scattering amplitude at threshold: two numbers $(a_0^{1/2}, a_0^{3/2})$
- History of the chiral predictions:

	CA [1]	1-loop [2]	2-loop [3]
$a_0^{1/2}$	0.14	0.18 ± 0.03	0.220 [0.17 . . . 0.225]
$a_0^{3/2}$	-0.07	-0.05 ± 0.02	$-0.047[-0.075 \ldots -0.04]$

[1] Weinberg 1966, Griffith 1969

[2] Bernard, Kaiser, UGM 1990

[3] Bijnens, Dhonte, Talavera 2004

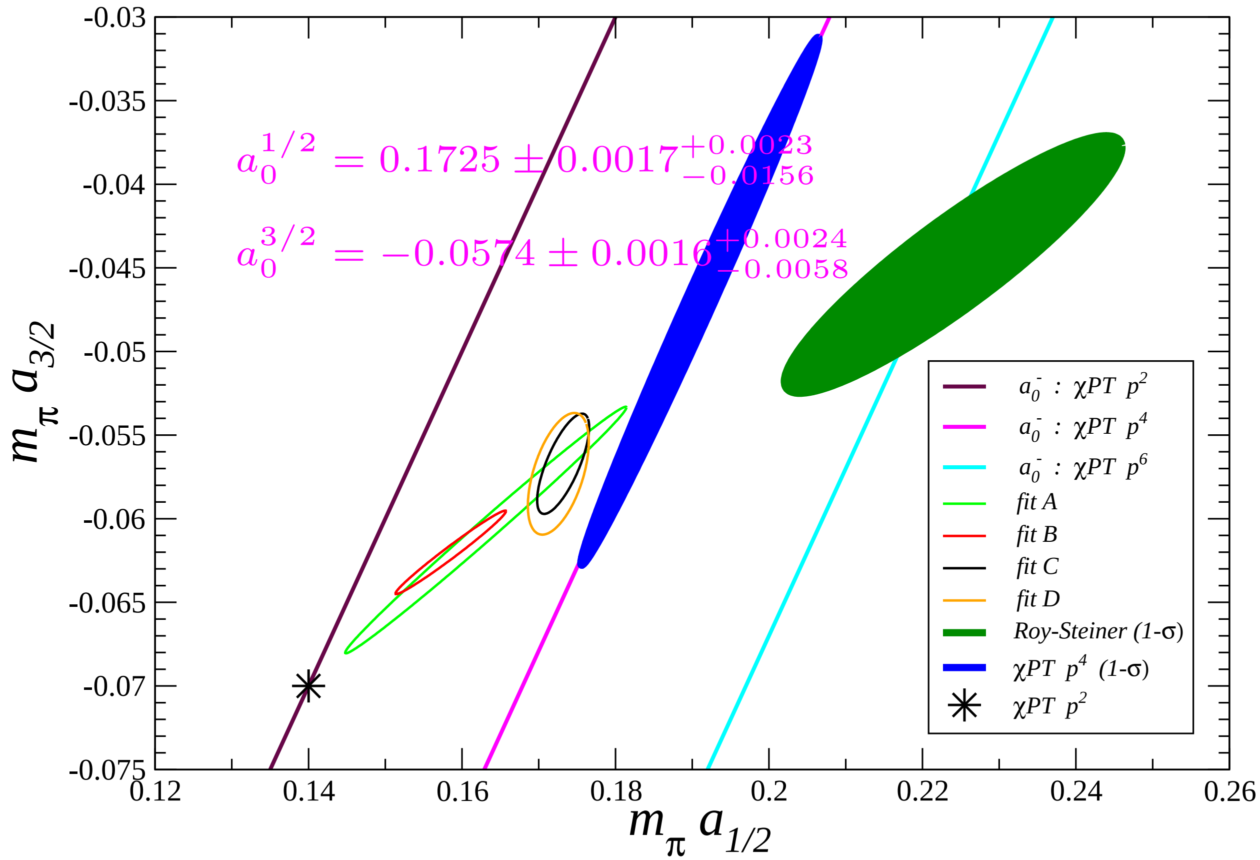
- match 1-loop representation to Roy-Steiner equation solution

$$a_0^{1/2} = 0.224 \pm 0.022\ , \quad a_0^{3/2} = -0.0448 \pm 0.0077$$

Büttiker et al. 2003

THE GRAND PICTURE

Fig. courtesy Silas Beane



- tension between lattice and Roy-Steiner (loophole: inconsistencies)
- need improved lattice results (direct calculations) $\Rightarrow$ work required

Lesson 3

PION-NUCLEON SCATTERING

- simplest scattering process involving nucleons
- intriguing LO prediction for isoscalar/isovector scattering length:

$$a^+_{\rm CA} = 0, \quad a^-_{\rm CA} = \frac{1}{1+M_\pi/m_p}\frac{M_\pi^2}{8\pi F_\pi^2} = 79.5\cdot 10^{-3}/M_\pi,$$

- chiral corrections:
  - chiral expansion for a^- converges fast — Bernard, Kaiser, UGM 1995
  - large cancellations in a^+, even sign not known from scattering data

	$\mathcal{O}(q)$	$\mathcal{O}(q^2)$	$\mathcal{O}(q^3)$	$\mathcal{O}(q^4)$
fit to KA85	0.0	0.46	-1.00	-0.96
fit to EM98	0.0	0.24	0.49	0.45
fit to SP98	0.0	1.01	0.14	0.27

Fettes, UGM 2000

A WONDERFUL ALTERNATIVE: HADRONIC ATOMS

- Hadronic atoms: bound by the static Coulomb force (QED)

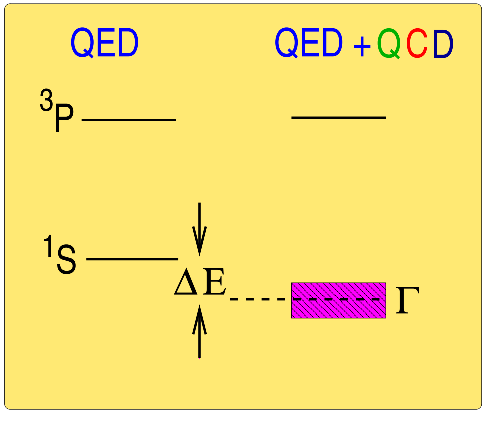
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\pi^- d$, $K^- p$, $K^- d$, ...

- Observable effects of QCD: strong interactions as **small** perturbations

  ⋆ energy shift $\boldsymbol{\Delta E}$

  ⋆ deacy width $\boldsymbol{\Gamma}$

  ⇒ access to scattering at zero energy!
  = S-wave scattering lengths



- can be analyzed in suitable NREFTs

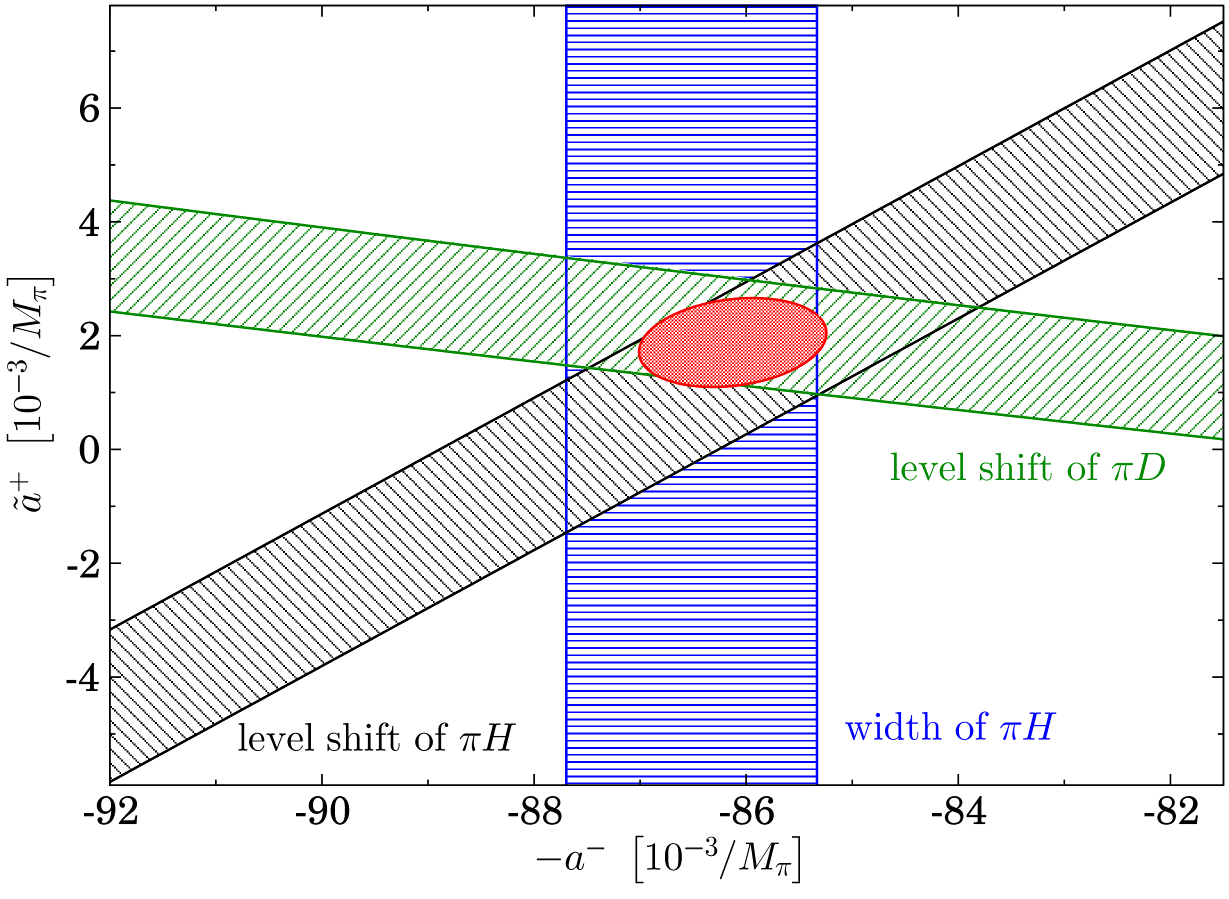
  Pionic hydrogen — Gasser, Rusetsky, . . . 2002

  Pionic deuterium — Baru, Hoferichter, Kubis . . . 2011

PION–NUCLEON SCATTERING LENGTHS

- superbe experiments preformed at PSI

Gotta et al.



$$a^+ = (7.6 \pm 3.1) \cdot 10^{-3}/M_\pi$$

$$a^- = (86.1 \pm 0.9) \cdot 10^{-3}/M_\pi$$

GMO sum rule:

$$\frac{g^2_{\pi N}}{4\pi} = 13.69(12)(15)$$

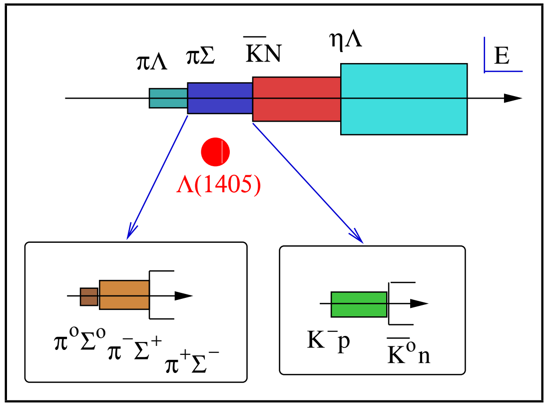
$\Rightarrow$ very precise value for a^- & first time definite sign for a^+

Lesson 4

ANTIKAON-NUCLEON SCATTERING

- $K^- p \to K^- p$: fundamental scattering process with strange quarks

- coupled channel dynamics



- dynamic generation of the $\Lambda(1405)$

  Dalitz, Tuan 1960

- major playground of **unitarized CHPT**

- chiral Lagrangian + unitarization leads to generation of certain resonances

  like e.g. the $\Lambda(1405)$, $S_{11}(1535)$, $S_{11}(1650)$, ...

  Kaiser, Siegel, Weise, Oset, Ramos, Oller, UGM, Lutz, ...

- loopholes: convergence a posteriori, crossing symmetry, on-shell approximation, unphysical poles, ...

A PUZZLE RESOLVED

- DEAR data inconsistent with scattering data

  UGM, Raha, Rusetsky 2004

$\Rightarrow$ vaste number of papers . . .

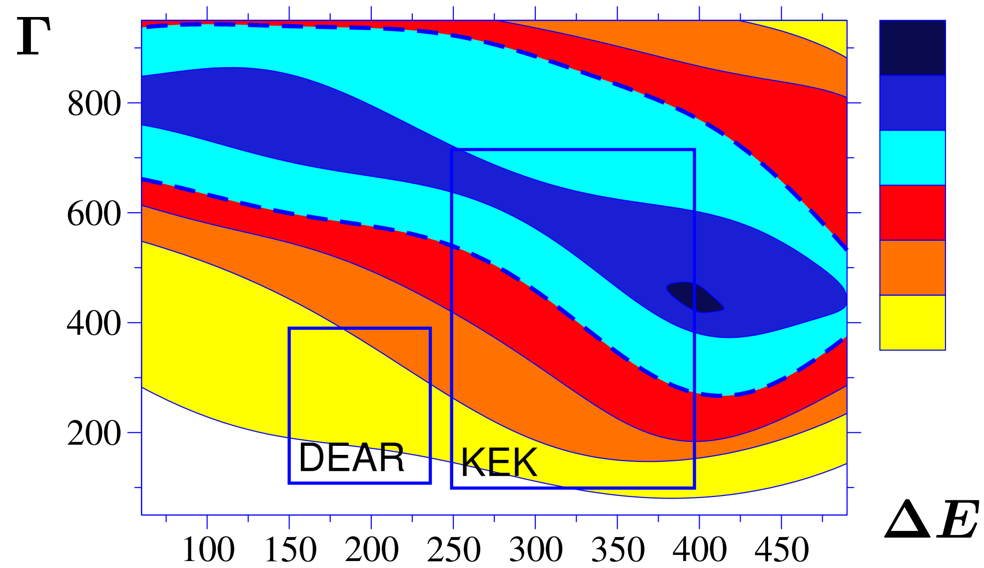
- SIDDHARTA to the rescue

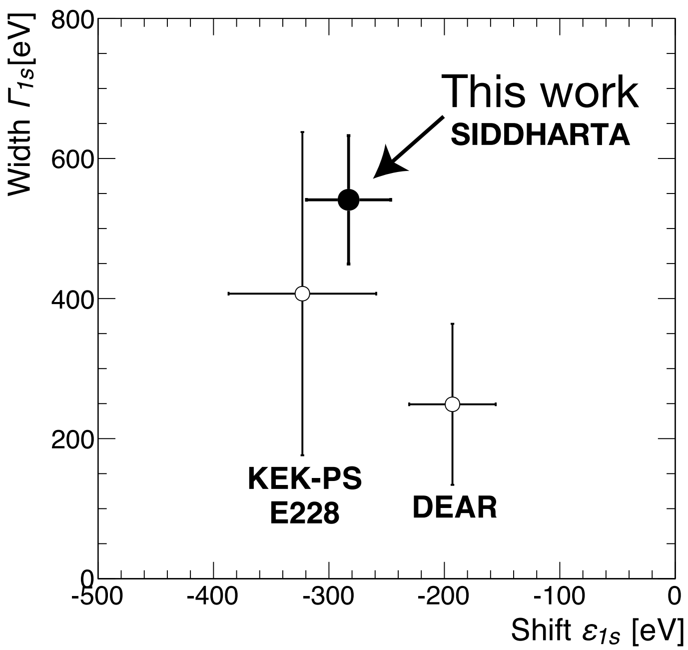
  Bazzi et al. 2011

$\Rightarrow$ more precise, consistent with KpX

$$\epsilon_{1s} = -283 \pm 36(\text{stat}) \pm 6(\text{syst})\ \text{eV}$$

$$\Gamma_{1s} = 541 \pm 89(\text{stat}) \pm 22(\text{syst})\ \text{eV}$$





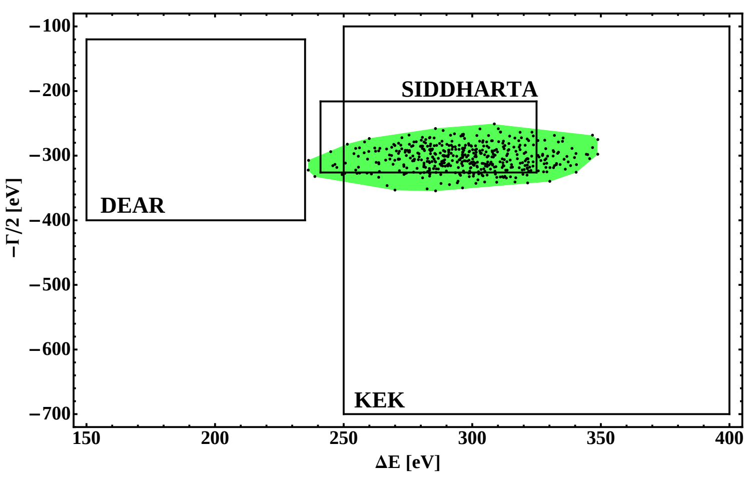
CONSISTENT ANALYSIS

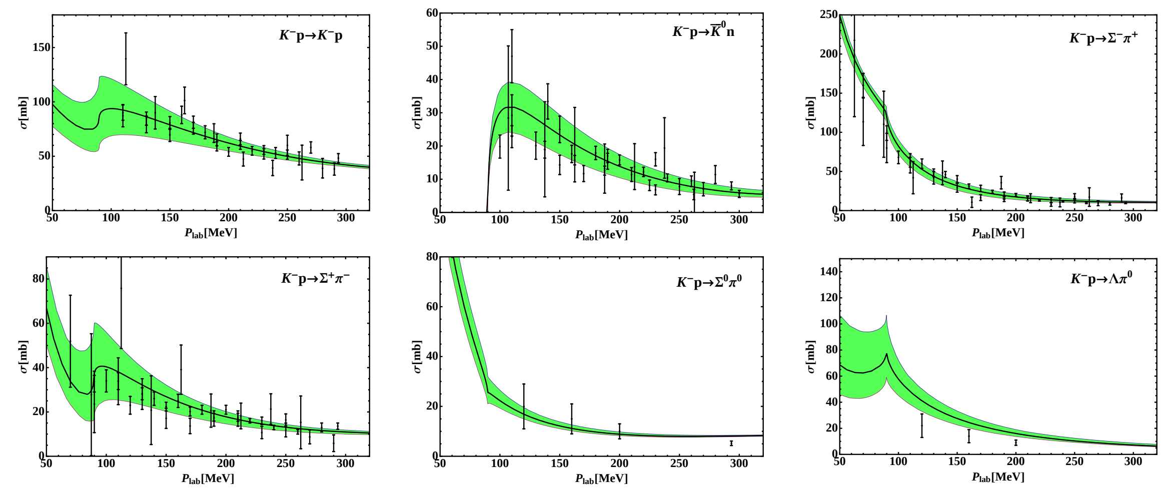
- kaonic hydrogen + scattering data can now be analyzed consistently
- use the chiral Lagrangian at NLO, two groups (different schemes)

Ikeda, Hyodo, Weise 2011; UGM, Mai 2012

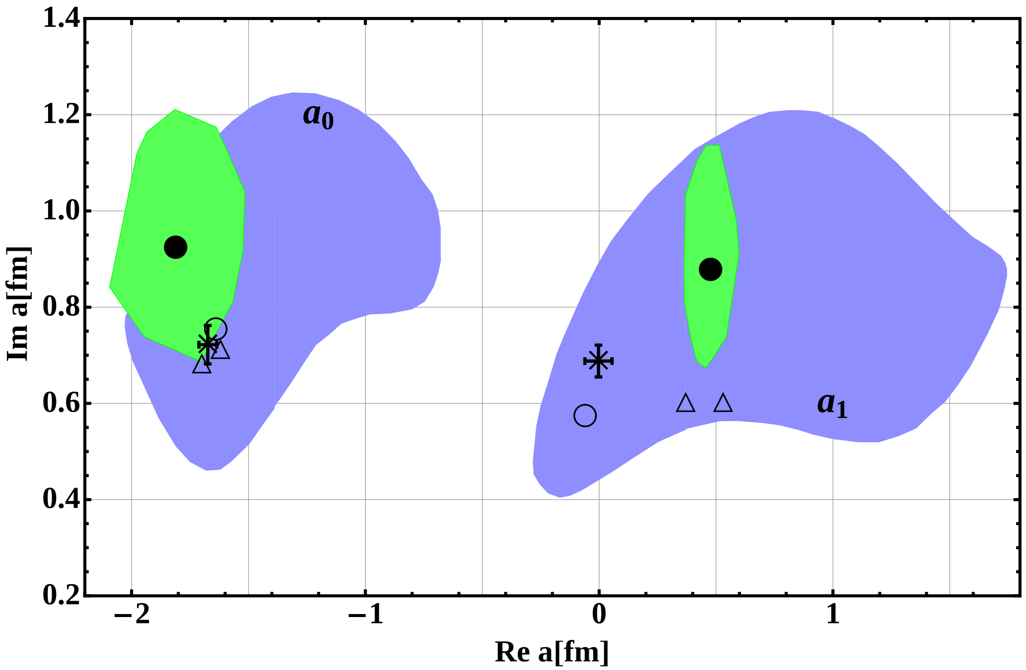
- 14 LECs and 3 subtraction constants to fit

$\Rightarrow$ simultaneous description of the SIDDHARTA and the scattering data





KAON–NUCLEON SCATTERING LENGTHS



$$a_0 = -1.81^{+0.30}_{-0.28} + i\,0.92^{+0.29}_{-0.23}\ \text{fm}$$

$$a_1 = +0.48^{+0.12}_{-0.11} + i\,0.87^{+0.26}_{-0.20}\ \text{fm}$$

$$a_{K^-p} = -0.68^{+0.18}_{-0.17} + i\,0.90^{+0.13}_{-0.13}\ \text{fm}$$

SIDDHARTA only:

$$a_{K^-p} = -0.65^{+0.15}_{-0.15} + i\,0.81^{+0.18}_{-0.18}\ \text{fm}$$

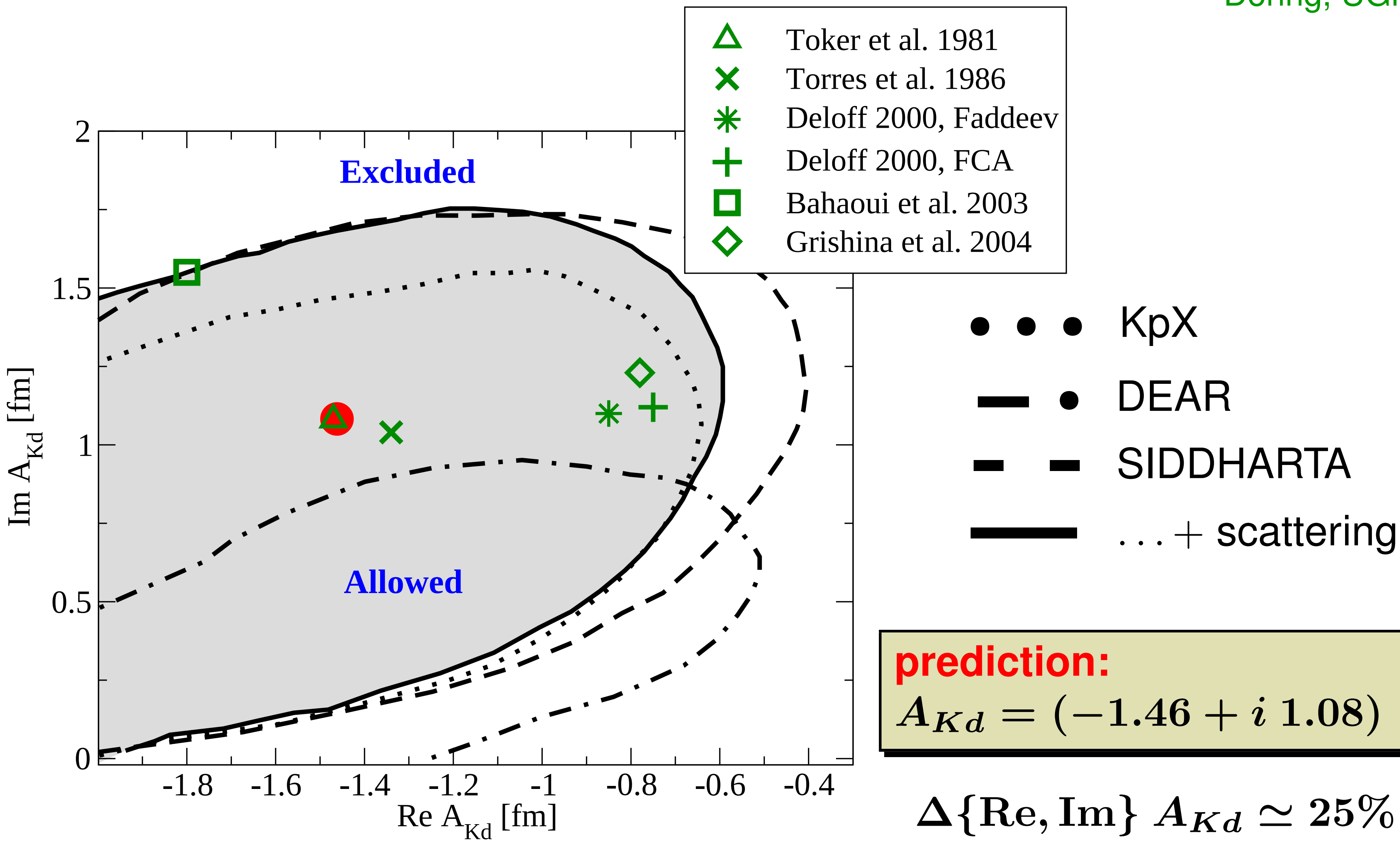
- clear improvement compared to scattering data only

$\Rightarrow$ fundamental parameters to within about 15% accuracy

KAON–DEUTERON SCATTERING LENGTH

- analyze $K^- d$, imposing consistency with the $\bar{K}N$ scattering lengths

Döring, UGM 2011



prediction:
$$A_{Kd} = (-1.46 + i\,1.08)\ \text{fm}$$

$$\Delta\{\text{Re},\text{Im}\}\ A_{Kd} \simeq 25\%$$

Lesson 5

EFFECTIVE LAGRANGIAN for $\phi D \to \phi D$

- Goldstone boson octet (π, K, η) scatters off D-meson triplet (D^0, D^+, D_s^+)

- multi-scale/multi-faceted problem:

  – light particles, chiral symmetry $\longrightarrow$ chiral expansion in (p, m_q)

  – heavy particles, heavy quark symmetry $\longrightarrow$ expansion in $1/m_c$

  – isospin-violation $\longrightarrow$ strong = quark mass difference $m_d \neq m_u$

  $\longrightarrow$ electromagnetic = quark charge difference $q_u \neq q_d$

- 16 channels with different total strangeness and isospin

  – some are perturbative

  – some are non-perturbative, require resummation $\longrightarrow$ possible molecules

RESULTS for $\phi D \to \phi D$

- $T(\phi D \to \phi D)$ depends on two LECs at NLO, called h_3 and h_5:

  h_3 the mass of the $D^*_{s0}(2317)$ as a DK molecule

  h_5 from naturalness, $h_5/M_D^2 \in [-1, +1]$

  $\Rightarrow \boxed{\Gamma(D^*_{s0}(2317)^+ \to D_s^+\pi^0) = (180 \pm 110)\,\text{keV}}$ testable prediction

  note: much smaller in quark models (a few keV)

- expectation for the scattering length for $DK(I=0)$ in the molecular picture:

$$a_{DK}^{I=0} = -g_{\text{eff}}^2 \Delta_{DK} = -\frac{1}{2\sqrt{\mu_{DK}\varepsilon}} \simeq 1\,\text{fm}$$

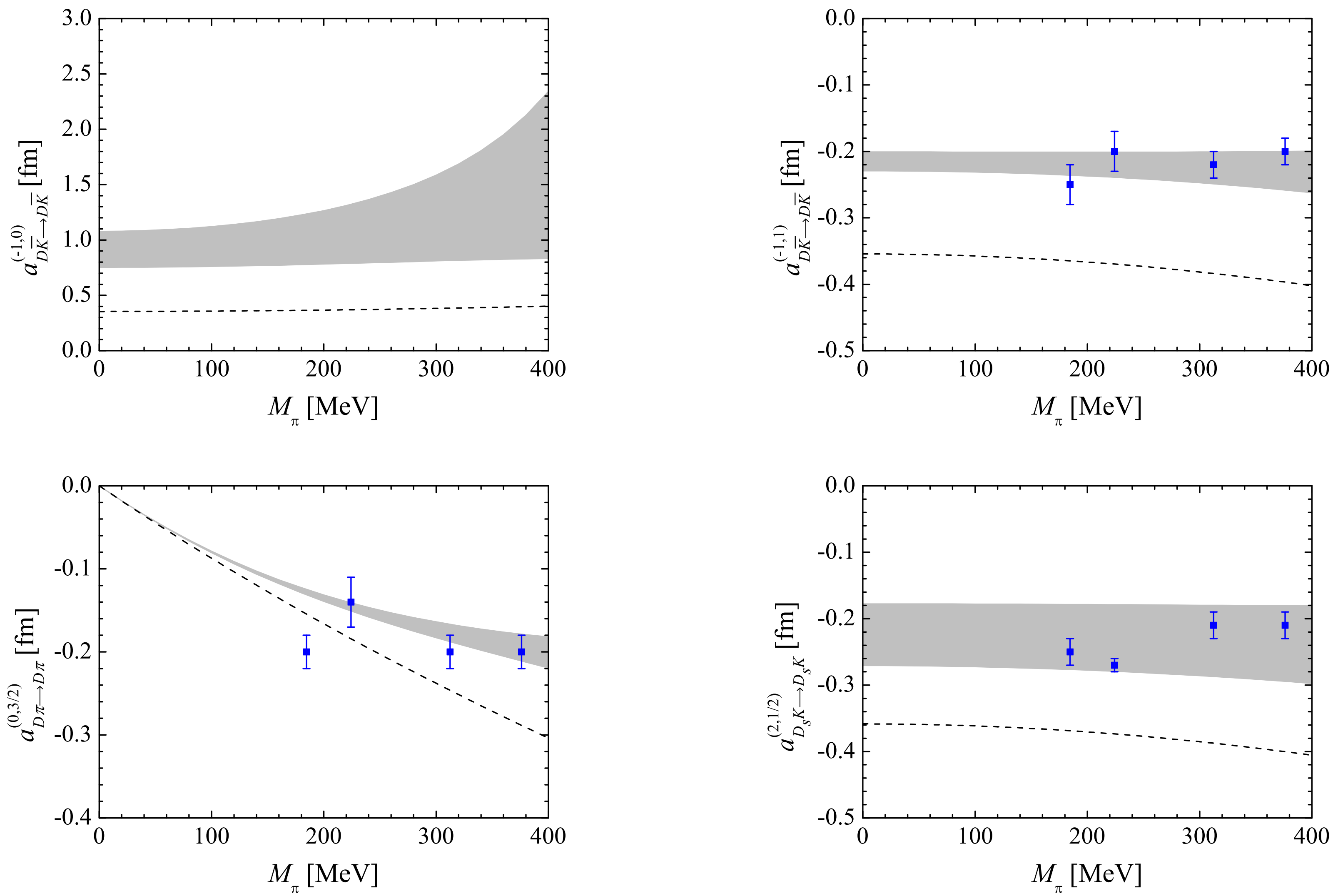
- no data, but first lattice investigations at varying quark masses

Liu, Lin, Orginos, PoS LATTICE **2008**, 112 and more to come!

QUARK MASS DEPENDENCE

- *predictions:* channels with no poles

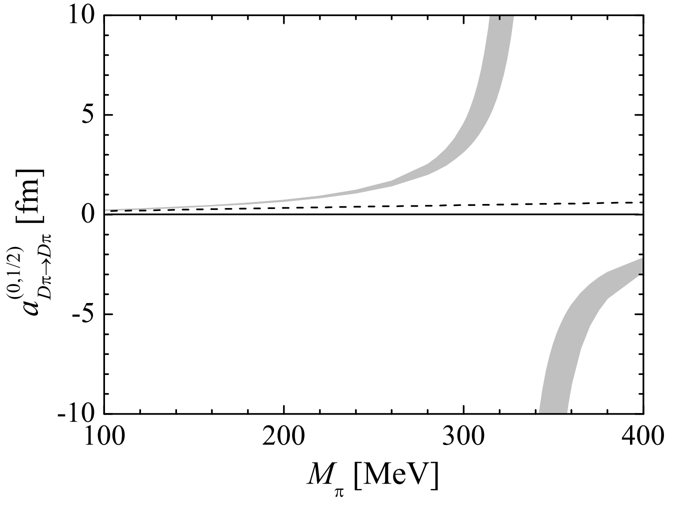
Guo, Hanhart, UGM 2009



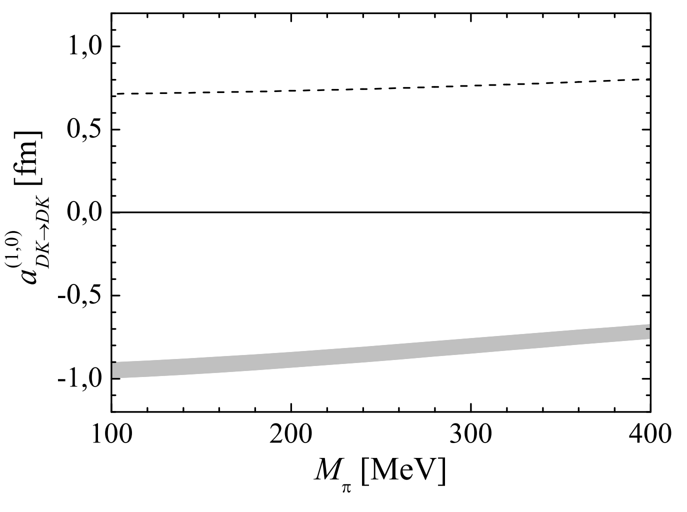
QUARK MASS DEPENDENCE cont'd

- *predictions:* channels with poles $\longrightarrow$ resonances or molecular states

$D\pi \to D\pi\ (0,1/2)$



$DK \to DK\ (1,0)$



a pair of poles above thr.

$$a_{D\pi}^{(0,1/2)} = 0.35(1)\ \text{fm}$$

a bound states below thr. $D_{s0}^*(2317)$

$$a_{DK}^{(1,0)} = -0.93(5)\ \text{fm}$$

$\Rightarrow$ lattice test of the molecular nature

NATURE of the $D_{s1}(2460)$

- Nature of the $D_{s1}(2460)$: $M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)} \simeq M_{D^*} - M_D$

$\Rightarrow$ most likely a $D^\star K$ molecule (if the $D^\star_{s0}(2317)$ is DK)

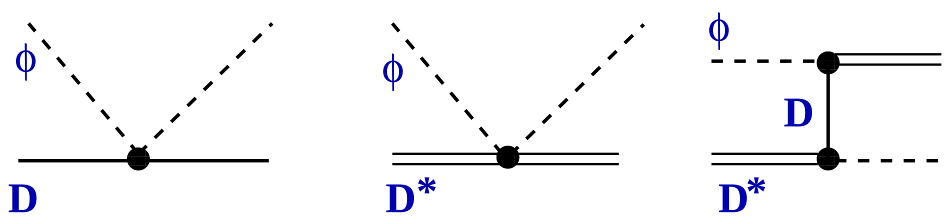
$\Rightarrow$ study Goldstone boson scattering off D- and $D^\star$-mesons

- Use heavy meson chiral perturbation theory   Wise, Donoghue et al., Casalbuoni et al., . . .

$$H_v = \frac{1+\not{v}}{2}\left[\not{V}^*_v + iP_v\gamma_5\right]$$

$$P = (D^0, D^+, D_s^+)\,, \quad V^*_\mu = (D^{*0}_\mu, D^{*+}_\mu, D^{*+}_{s,\mu})$$

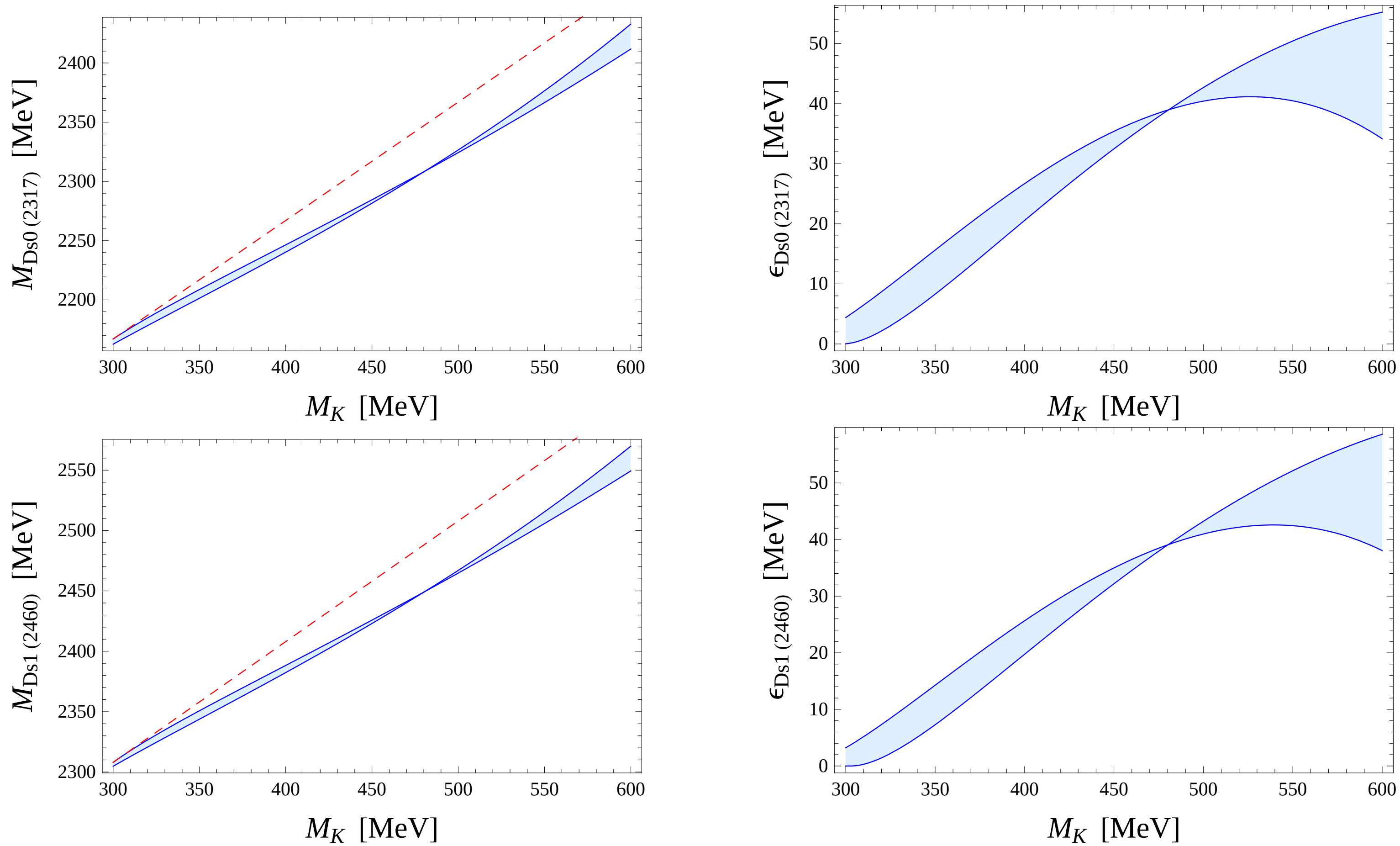
- T-matrix:



- Unitarization (as before) $\longrightarrow$ find poles in the complex plane

KAON MASS DEPENDENCE

- Mass and binding energy: $M_{\rm mol} = M_K + M_H - \epsilon$ Guo, Hanhart, UGM 2009



$\Rightarrow$ typical for a molecule $\rightarrow$ test in LQCD

SUMMARY & OUTLOOK

- Hadron-hadron scattering: important role of chiral symmetry (CHPT)
  $\longrightarrow$ combine with dispersion relations, unitarization, lattice
- Pion-pion scattering
  $\longrightarrow$ a fine test of the Standard Model
- Pion-kaon scattering
  $\longrightarrow$ tension between lattice and Roy-Steiner solution
- Pion-nucleon scattering
  $\longrightarrow$ superbe accuracy from EFTs for pionic hydrogen/deuterium
- Antikaon-nucleon scattering
  $\longrightarrow$ consistent determination of the scattering lengths possible
- Goldstone-boson scattering off $D, D^\star$–mesons
  $\longrightarrow$ lattice test of molecular states possible

$\Rightarrow$ **exciting times ahead of us**

SPARES