To vmd or not to vmd[1]

Vector-meson dominance (vmd) revisited

Stefan Leupold

(collaborators: Carla Terschlüsen, Bruno Strandberg, Matthias Lutz)

UPPSALA UNIVERSITET

MESON 2012, Cracow, Poland, June 2012

[1] title robbed from Rob Pisarski

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Reactions of hadrons with (virtual) photons

Why is it interesting?

- intrinsic structure of proton
  (parton-distribution functions, electromagnetic form factors, GPDs, . . .)
- in-medium changes of properties of hadrons
  (observable in dilepton spectrum)
- $g - 2$ of muon:
  $\hookrightarrow$ background for beyond-standard-model physics

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What do we know?

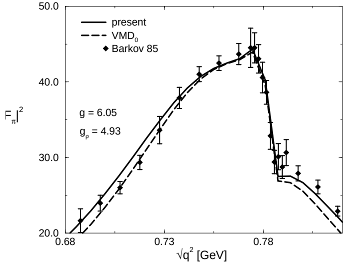
- photons couple directly to vector mesons

$\hookrightarrow$ vector mesons prominently seen in corresponding cross sections

$\hookrightarrow$ “vector-meson dominance” (VMD)

Vector-meson dominance (VMD)

VMD works very well
for pion form factor (FF)
(Sakurai, . . .)
$e^+e^- \to \pi^+\pi^-$:



e.g., Klingl/Kaiser/Weise,
Z. Phys. A356, 193 (1996)

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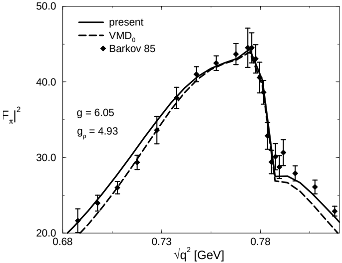
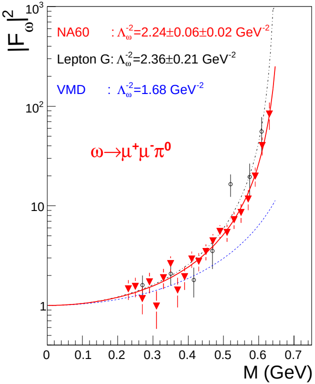


e.g., Klingl/Kaiser/Weise,
Z. Phys. A356, 193 (1996)

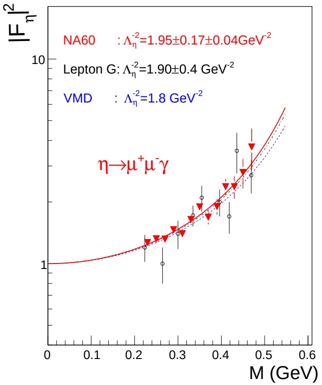
VMD dramatically fails
for omega transition FF
$\omega \to \mu^+\mu^-\pi^0$:



Arnaldi et al. (NA60),
Phys. Lett. B677, 260 (2009)

Vector meson dominance (VMD)

- VMD works very well for eta transition FF $\eta \to \mu^+\mu^-\gamma$:



Arnaldi et al. (NA60), Phys. Lett. B677, 260 (2009)

- VMD works very well for pion transition FF (e.g. $e^-\gamma \to e^-\pi^0$):

$$\Lambda^{-2} := \left.\frac{dF(s)}{ds}\right|_{s=0}$$

$$\Lambda_{\mathrm{PDG}} = (0.76 \pm 0.05)\,\mathrm{GeV},$$
$$\Lambda_{\mathrm{VMD}} = m_V \approx 0.77\,\mathrm{GeV}$$

- does it work for double-virtual transition FF? $\pi^0/\eta \to e^+e^-\,e^+e^-$

⇝ important for hadronic light-by-light scattering

Lagrangian for pseudoscalars and vectors

- want to construct simple Lagrangian for π, ρ, ω (only two flavors for simplicity)
- use lowest-order Lagrangians from chiral perturbation theory for sector with
  - even number of pions: non-linear sigma model
  - $\rightsquigarrow$ two parameters: pion decay constant and pion mass
  - odd number of pions: Wess-Zumino-Witten term
  - $\rightsquigarrow$ parameter free
- use in addition Lagrangian for vector mesons $\rightsquigarrow$ next slides

Inclusion of vector mesons

relevant Lagrangian with one vector meson

$$-\frac{\mathrm{i}}{4}\, h_P\, m_V\, \mathrm{tr}(V_{\mu\nu}\,[U^\mu, U^\nu]) - \frac{1}{8}\, e_V\, m_V\, \mathrm{tr}(V^{\mu\nu}\, Q)\, F_{\mu\nu}$$

- use anti-symmetric tensor fields, $V_{\mu\nu}$, to represent vector mesons
- Lagrangian saturates low-energy constants of next-to-leading-order chiral perturbation theory (Gasser, Leutwyler, Annals Phys. 158 (1984) 142)
- two parameters: h_P and e_V

$\hookrightarrow$ fix h_P from decay $\rho \to 2\pi$

$\hookrightarrow$ fix e_V from decay $\rho \to e^+ e^-$

Lutz/S.L., Nucl. Phys. A813, 96 (2008)

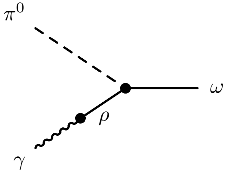
Inclusion of vector mesons

relevant Lagrangian with two vector mesons

$$-\frac{1}{4}\,\mathrm{tr}(D_\mu V^{\mu\alpha}\, D^\nu V_{\nu\alpha}) + \frac{1}{8}\, m_V^2\, \mathrm{tr}(V_{\mu\nu}\, V^{\mu\nu})$$
$$+\,\frac{\mathrm{i}}{8}\, h_A\, \epsilon_{\mu\nu\alpha\beta}\, \mathrm{tr}(\{V^{\mu\nu}, D_\lambda V^{\lambda\alpha}\}\, U^\beta)$$

- one free parameter: h_A
  (corresponds to g_A in pion-nucleon interaction)

$\hookrightarrow$ fix $h_A * e_V$ from decay $\omega \to \pi^0 + \gamma$



Lutz/S.L., Nucl. Phys. A813, 96 (2008)

Electromagnetic form factors

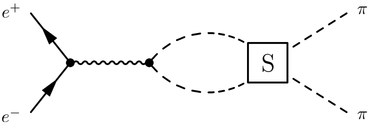
three free parameters fixed from two-body decays

- $\rho \to 2\pi$
- $\rho \to l^+ l^-$ ($l =$ lepton)
- $\omega \to \pi^0 \gamma$

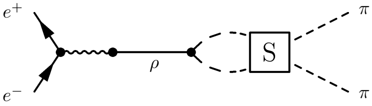
no further parameters needed to determine

- pion form factor, $e^+ e^- \to \pi^+ \pi^-$ (VMD)
- omega transition form factor $\omega \to \pi^0\, l^+ l^-$ (no VMD)
- pion transition form factor
  - single virtual: $\pi^0 \to \gamma\, e^+ e^-$ (VMD)
  - double virtual: $\pi^0 \to e^+ e^-\, e^+ e^-$ (VMD?)
- cross section $e^+ e^- \to \pi^+ \pi^- \pi^0$

Pion form factor

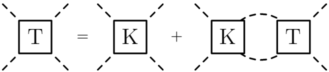


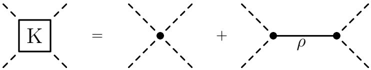
$\sim e$



$\sim e_V * h_P$

- tree level: S $\rightarrow$ 1 (drop S in figure)
- rescattering: S $=$ 1 $+$ 2iT





$\sim 1/F^2$ $\sim {h_P}^2$

Pion form factor at tree level

- two contributions: direct ($\sim$ pion charge) and via ρ meson
- form factor: normalize amplitude to direct term

$$F_\pi(s) = 1 + \frac{e_V\, h_P\, m_V^2}{16 e\, f^2}\, \frac{s}{m_V^2 - s}$$

- as compared to VMD

$$F_\pi^{\mathrm{VMD}}(s) = \frac{m_V^2}{m_V^2 - s}$$

- formulae would agree for $e_V\, h_P = 16 e\, f^2 / m_V^2 \approx 0.065$
- here: $e_V \approx 0.22$, $h_P \approx 0.30 \rightsquigarrow e_V\, h_P \approx$

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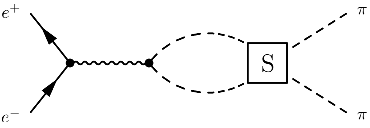
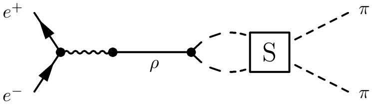
$\hookrightarrow$ very close to VMD

Pion form factor

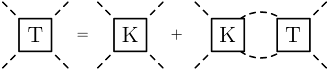


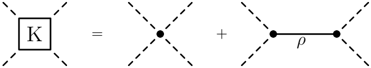
$\sim e$



$\sim e_V * h_P$

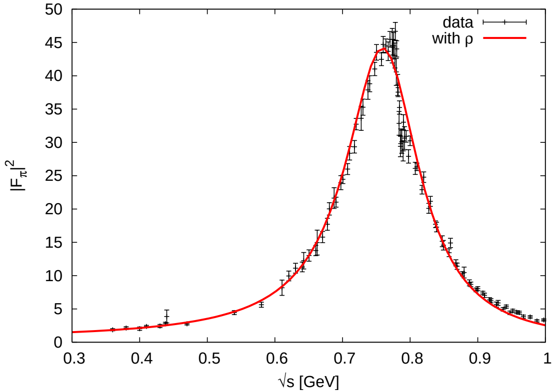
- tree level: $S \rightarrow 1$ (drop S in figure)
- rescattering: $S = 1 + 2iT$





$\sim 1/F^2$ $\sim {h_P}^2$

Pion form factor with rescattering

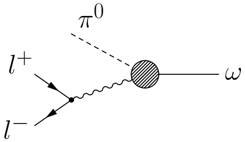


- excellent description
- note: isospin breaking (ρ-ω mixing) not included

S.L., Phys. Rev. D80, 114012 (2009)

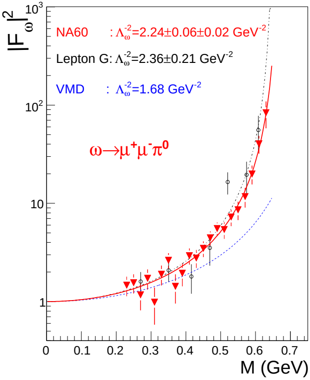
Transition form factor of omega meson — status

- definition: form factor parametrizes deviation from structureless decay; normalized to photon point ($M = 0$)



- experiments show strong deviation from simple vector-meson dominance

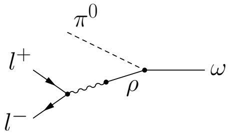
$$F(M^2) = \frac{m_\rho^2}{m_\rho^2 - M^2}$$



Phys. Lett. B 677, 260 (2009)

Transition form factor of omega meson — theory

- our approach:
  decay amplitude $\omega \to \pi^0 + \gamma^{(*)}$
  $\sim h_A\, e_V$ for both real and virtual γ



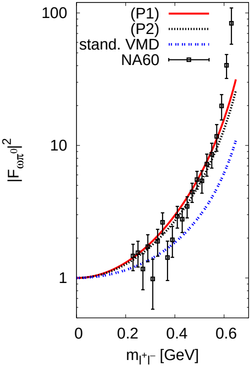
$\hookrightarrow$ $h_A\, e_V$ drops out in form factor

$$F_{\omega\pi}(s) = \frac{m_V^2 + s}{m_V^2 - s} \qquad \leftrightarrow \qquad F_{\omega\pi}^{\mathrm{VMD}}(s) = \frac{m_V^2}{m_V^2 - s}$$

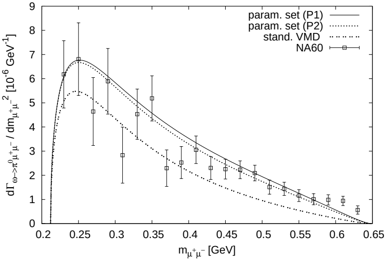
$\hookrightarrow$ no free parameters and significant deviation from VMD

- note: tree level sufficient for $\omega \to \pi^0 +$ dilepton
  (would not be sufficient for $e^+e^- \to \omega + \pi^0$)

Transition form factor of omega meson — results



corresponding differential decay rate:

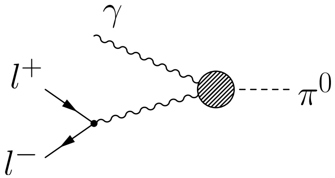


C. Terschlüsen, S.L., Phys. Lett. B691, 191 (2010)

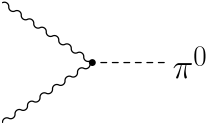
- satisfactory description, much better than VMD
- NA60: dimuons, planned: WASA with dielectrons

Pion transition form factor

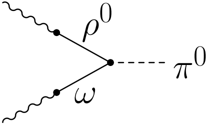
- in general: form factor parametrizes deviation from structureless decay; normalized to photon point ($\pi^0 \to 2\gamma$)



- experiments: agreement with VMD (slope $\approx m_V$)
- our approach: two contributions
  (leptons not displayed any more)



WZW



$\sim h_A * e_V^2$

Pion transition form factor — results

- two contributions:
  - direct: Wess-Zumino-Witten term $\sim 1/f$
  - via vector mesons: π-ρ-ω vertex $\sim h_A$
- form factor: normalized to direct term

$$F_{\pi\gamma}(s) = 1 + \frac{\pi^2\, h_A\, e_V^2}{12 e^2} \frac{s}{m_V^2 - s}$$

- as compared to VMD

$$F_{\pi\gamma}^{\mathrm{VMD}}(s) = \frac{m_V^2}{m_V^2 - s}$$

- formulae would agree for $h_A\, e_V^2 = 12 e^2/\pi^2 \approx 0.11$
- here: $e_V \approx 0.22$, $h_A \approx 2.1 \rightsquigarrow h_A\, e_V^2 \approx$

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- here: $e_V \approx 0.22$, $h_A \approx 2.1 \rightsquigarrow h_A e_V^2 \approx 0.10$

$\hookrightarrow$ again close to VMD

Double-virtual transition form factor

- single-virtual $\pi^0 \to \gamma\,\gamma^*$ agrees with VMD
- double-virtual $\pi^0 \to \gamma^*\gamma^*$ might not
- in our approach:

$$F(s_1, s_2) = 1 + \frac{\pi^2 h_A e_V^2}{12 e^2} \frac{m_V^2 (s_1 + s_2)}{(m_V^2 - s_1)(m_V^2 - s_2)}$$

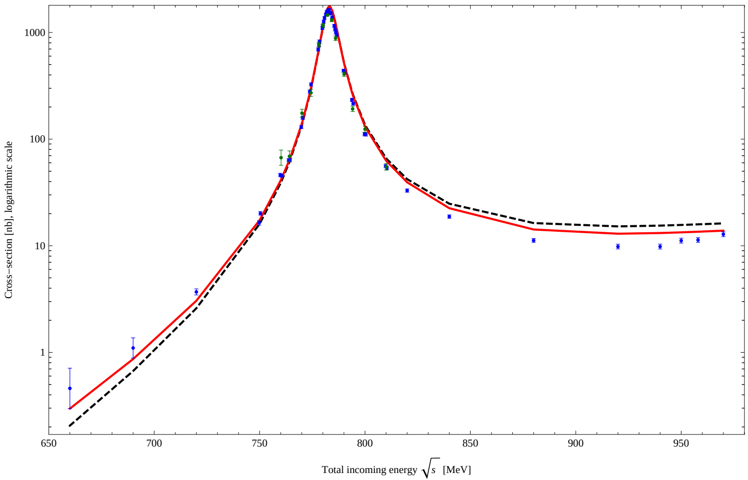
$$\approx 1 + \frac{m_V^2 (s_1 + s_2)}{(m_V^2 - s_1)(m_V^2 - s_2)}$$

$$= \underbrace{1 - \frac{m_V^2}{m_V^2 - s_1} - \frac{m_V^2}{m_V^2 - s_2}}_{\text{“half” VMD}} + 2 \underbrace{\frac{m_V^4}{(m_V^2 - s_1)(m_V^2 - s_2)}}_{\text{VMD type}}$$

$\hookrightarrow$ different from VMD for $s_1, s_2 \neq 0$

cross section $e^+e^- \longrightarrow \pi^+\pi^-\pi^0$

vector-meson contribution $e^+e^- \to \omega^* \to \rho^*\pi \to 3\pi$ (black)
plus Wess-Zumino-Witten term (sum in red)



Summary

- have utilized respective lowest order Lagrangians of
  - χPT in normal sector (non-linear sigma model)
  - χPT in anomalous sector (Wess-Zumino-Witten term)
  - vector mesons represented by antisymmetric tensor fields
- pion form factor and pion transition form factor:
  - direct term and term with vector mesons
  - parameters allow cancellations such that VMD emerges
- omega transition form factor:
  - only one term $\rightsquigarrow$ no cancellation
  - no VMD
  - but also parameter-free description $(m_V^2 + s)/(m_V^2 - s)$
- all results in good agreement with experiments
- prediction: double-virtual transition form factor of pion deviates from VMD

Summary II

three coupling constants of vector-meson Lagrangian sufficient to describe

- $\rho \to \pi^+\pi^-$, $\rho/\omega \to l^+l^-$ (l = lepton)
- $\rho/\omega \to \pi\gamma$ Lutz/S.L., Nucl. Phys. A813, 96 (2008)
- $e^+e^- \to \pi^+\pi^-$, pion form factor
- $\pi^+\pi^- \to \pi^+\pi^-$ in p-wave (not shown here)
  S.L., Phys. Rev. D80, 114012 (2009)
- $\omega \to \pi^0 l^+l^-$, transition form factor of omega to pion
  Terschlüsen/S.L., Phys. Lett. B691, 191 (2010)
- $\pi^0 \to \gamma e^+e^-$, transition form factor of pion to photon
  C. Terschlüsen, Diploma thesis, Giessen (2010)
- $\omega \to \pi^+\pi^-\pi^0$ (not shown here)
  S.L./Lutz, E. P. J. A 39, 205 (2009)
- $e^+e^- \to \pi^+\pi^-\pi^0$
  B. Strandberg, Master thesis, Uppsala (2012)

Outlook

- microscopic understanding of cancellations

- extension to three flavors

$\hookrightarrow$ inclusion of terms which break flavor and/or nonet symmetry

$\hookrightarrow$ especially inclusion of η' is interesting and challenging

$\hookrightarrow$ stay tuned for talk by Carla Terschlüsen on Monday, session A4

(C. Terschlüsen, SL, M.F.M. Lutz, arXiv:1204.4125 [hep-ph])

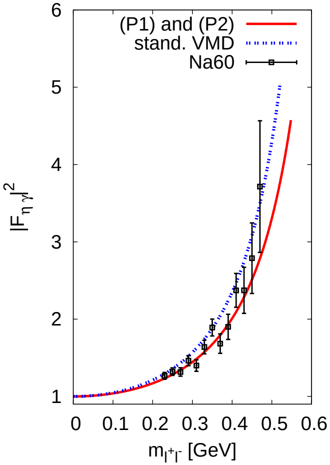
- useful as starting point for an effective field theory?

$\hookrightarrow$ calculations beyond leading order $\rightsquigarrow$ loops backup slides

Transition form factor of eta meson

PRELIMINARY

our approach extended to three flavors:
leading-order term from vector-meson Lagrangian plus leading-order term from pure χPT (Wess-Zumino-Witten)



PRELIMINARY

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Why is it interesting?

- intrinsic structure of proton

Intrinsic structure of proton

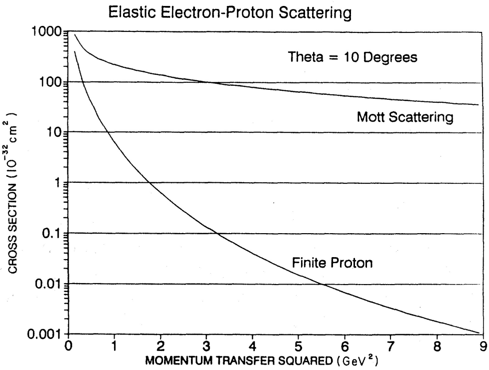

FIG. 4. Elastic scattering cross sections for electrons from a "point" proton and for the actual proton. The differences are attributable to the finite size of the proton.

H.W. Kendall, Rev. Mod. Phys. 63, 597 (1991) (Nobel prize lecture 1990)

Reactions of hadrons with (virtual) photons

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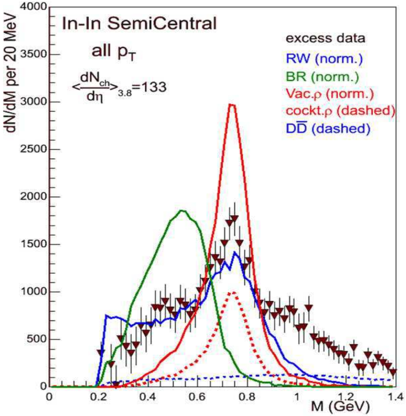
In-medium changes of properties of hadrons



dilepton spectrum from a nucleus-nucleus collision

models:

- dropping mass
- broad spectral function

data: NA60

S. Damjanovic et al. (NA60), Nucl. Phys. A 774, 715 (2006)

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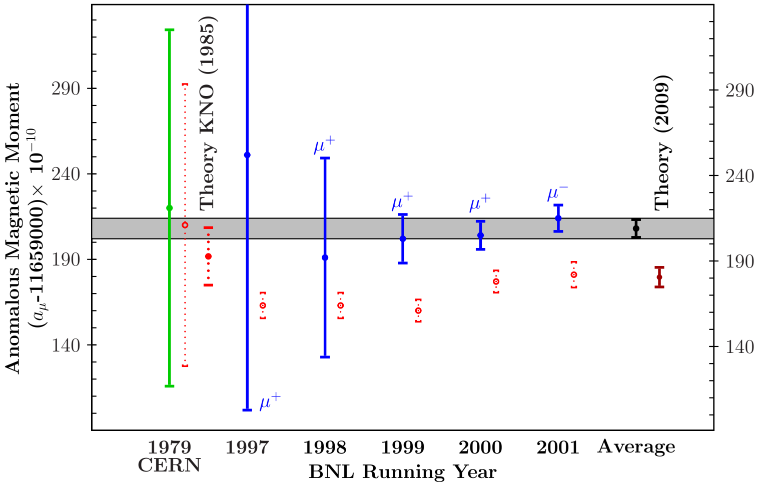
  (parton distribution functions, electromagnetic form factors, GPDs, ...)
- in-medium changes of properties of hadrons
- $g-2$ of muon:
  $\hookrightarrow$ background for beyond-standard-model physics

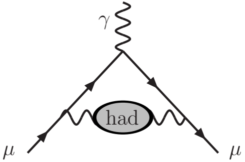
Muon's $g - 2$



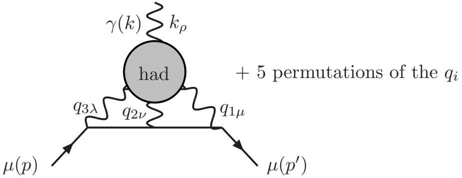
Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009)

Muon's $g-2$

Largest uncertainty of standard model: hadronic contributions



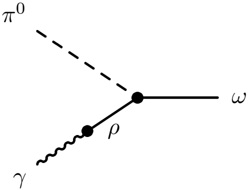
vacuum polarization

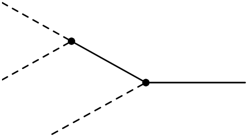


light-by-light scattering

Decay of omega to three pions

decays of ω meson



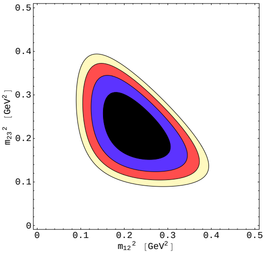


- same vertex appears in both processes $\omega \to \gamma\pi$ and $\omega \to 3\pi$ in leading order
- use first process to fix coupling of second one

⇝ prediction: $\Gamma_{\omega\to 3\pi} = 7.3\,\text{MeV}$

$\Gamma^{\text{exp}}_{\omega\to 3\pi} = (7.57 \pm 0.13)\,\text{MeV}$

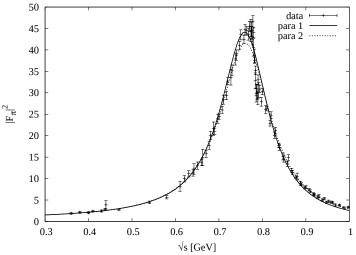
S.L./Lutz, E. P. J. A 39, 205 (2009)

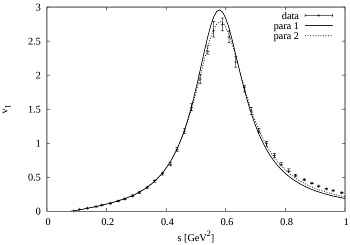


presently also investigated by WASAatCOSY and Kubis et al.

Pion form factor

from e^+e^- and from τ decay





$e_V = 0.230$, $h_P = 0.300$-0.308

Representations for vector mesons

- need three active degrees of freedom for spin-1 state
- vector representation:
  use four-component field, V_μ, together with one constraint
  $\partial^\mu V_\mu = 0$
- anti-symmetric tensor representation:
  use six-component field, $V_{\mu\nu}$, together with constraints
  $\epsilon^{\mu\nu\alpha\beta} \partial_\nu V_{\alpha\beta} = 0$
- connection: $V_\mu \sim \frac{1}{m_V} \partial^\nu V_{\mu\nu}$
- note: rewriting Lagrangian with tensor fields into Lagrangian with vector fields one finds vector-meson exchange and point interactions — but with fixed relative weight; example:

$$F_{\omega\pi}(s) = \frac{m_V^2 + s}{m_V^2 - s} = \frac{2\, m_V^2}{m_V^2 - s} - 1$$

Cancellations

- pion form factor

$\hookrightarrow$ VMD for $e_V\, h_P = 16 e\, f^2/m_V^2 \approx 0.065$

- pion transition form factor

$\hookrightarrow$ VMD for $h_A\, e_V^2 = 12 e^2/\pi^2 \approx 0.11$

- our parameters

$\hookrightarrow$ $e_V \approx 0.22$, $h_P \approx 0.30$, $h_A \approx 2.1$

- KSFR relations

$\hookrightarrow$ $e_V = 4\sqrt{2}\, e\, f/m_V \approx 0.199$, $h_P = 2\sqrt{2}\, f/m_V \approx 0.328$,
no statement about h_A!

$\hookrightarrow$ if one turns logic around and demands VMD:

$\rightsquigarrow$ $h_A = 3\, m_V^2/(8\pi^2 f^2) \approx 2.82$