Dalitz plot studies of $D^0 \to K_S^0 \pi^+ \pi^-$ decays

Collaboration with :

Robert Kamiński
(Institute of Nuclear Physics PAN, Kraków, Poland),

Jean-Pierre Dedonder and Benoit Loiseau
(LPNHE, Paris, France)

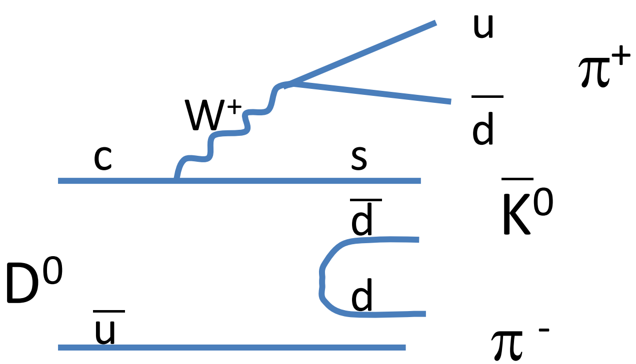
Motivation

Studies of the $D^0 \to K_S^0 \pi^+ \pi^-$ reaction are helpful in:

1. measurements of the **D^0 - $\bar{D}^0$ mixing** parameters,
2. determination of the **CKM angle** γ in the analysis of the decays $B^\pm \to D K^\pm$, $D \to K_S^0 \pi^+ \pi^-$,
3. description of the **final state** interactions between mesons, in particular in the S-waves,
4. testing theoretical models of **form factors**,
5. understanding properties of the meson resonances and their interference effects on the **Dalitz plot**.

Allowed and suppressed transitions

Transition $c \to s u \bar{d}$

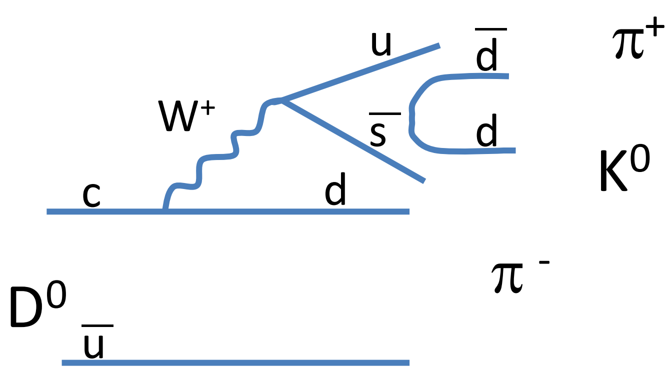


$$O_1 \propto \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A}$$

$$V_{cs} \approx V_{ud} \approx \cos\theta_C$$

allowed

Transition $c \to d u \bar{s}$

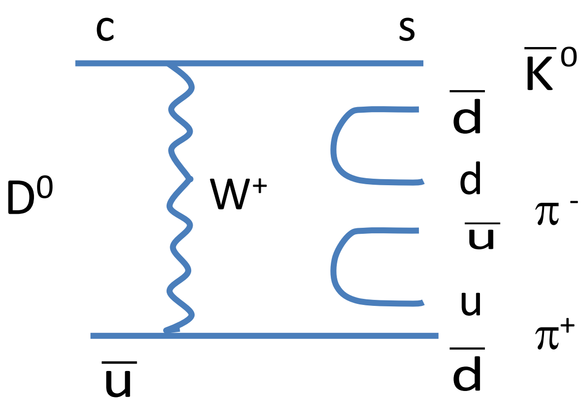


$$O_2 \propto \frac{G_F}{\sqrt{2}} V_{cd}^* V_{us} (\bar{d}c)_{V-A} (\bar{u}s)_{V-A}$$

$$V_{cd} \approx -\lambda, \quad V_{us} \approx \lambda, \quad \lambda = \sin\theta_C \approx 0.225$$

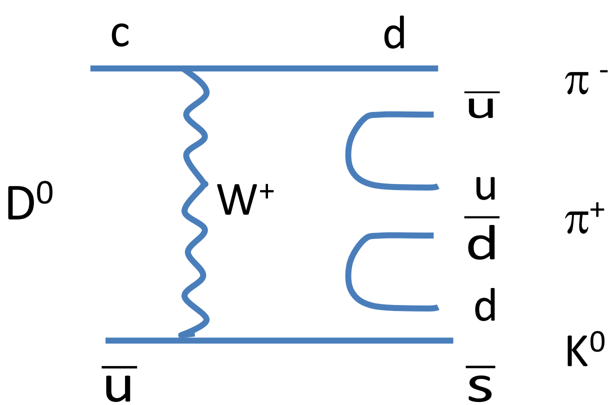
doubly Cabibbo suppressed

Annihilation decay amplitudes



Transition $c \rightarrow su\bar{d}$

allowed



Transition $c \rightarrow du\bar{s}$

doubly Cabibbo **suppressed**

Factorization approach

Quark currents: $j_1 = (\bar{s}c)_{V-A}\,,\; j_2 = (\bar{u}d)_{V-A}\,,\quad j_1' = (\bar{u}c)_{V-A}\,,\; j_2' = (\bar{s}d)_{V-A}$

main part of the effective **Hamiltonian:** $H \propto G_F/\sqrt{2}\; V_{cs}^* V_{ud}\, j_1 \otimes j_2$

Factorization:
$$\langle \bar{K}^0 \pi^- \pi^+ | j_1 \otimes j_2 | D^0 \rangle \approx \langle \bar{K}^0 \pi^- | j_1 | D^0 \rangle \langle \pi^+ | j_2 | 0 \rangle$$
$$+ \langle \pi^- \pi^+ | j_1' | D^0 \rangle \langle \bar{K}^0 | j_2' | 0 \rangle$$
$$+ \langle 0 | j_1' | D^0 \rangle \langle \bar{K}^0 \pi^- \pi^+ | j_2' | 0 \rangle$$

$\langle \pi^+ | j_2^{\mu} | 0 \rangle = i f_\pi p_\pi^{\mu}$ — f_π - pion decay constant

$\langle \bar{K}^0 | j_2'^{\mu} | 0 \rangle = i f_K p_K^{\mu}$ — f_K - kaon decay constant

$\langle 0 | j_1'^{\mu} | D^0 \rangle = -i f_D p_D^{\mu}$ — f_D - D decay constant

Types of decay amplitudes

27 amplitudes for $D^0 \to K_S^0 \pi^+ \pi^-$ decay:

a) **7 allowed** tree amplitudes,

b) **6** doubly Cabibbo **suppressed** tree amplitudes,

c) **14 annihilation** (W-exchange) amplitudes
(7 allowed and 7 doubly Cabibbo suppressed).

Seven partial wave amplitudes include:

1. S-, P- and D- wave amplitudes in the $K\pi$ subsystem,
2. S-, P- and D- wave amplitudes in the $\pi^+ \pi^-$ subsystem, including in addition the $\omega \to \pi^+ \pi^-$ transition .

Resonances in decay amplitudes

Channel:	wave:	name:
$\overline{K}^0\pi^-$	S	$K_0^*(800)^-$ or κ^-, $K_0^*(1430)^-$
	P	$K^*(892)^-, K_1(1410)^-, K^*(1680)^-$
	D	$K_2^*(1430)^-$
$K^0\pi^+$	same list as above but with charge +	
$\pi^+\pi^-$	S	$f_0(600)$ or σ, $f_0(980)$, $f_0(1400)$
	P	$\rho(770), \rho(1450), \omega(782)$
	D	$f_2\,(1270)$

Very rich resonance spectrum $\rightarrow$ **complexity of final state interactions**

Transition matrix elements (1)

Two mesons form a resonance $R=h_2h_3$

$$\langle h_2(p_2)h_3(p_3)|\,j\,|D^0(p_D)\rangle \approx G_{Rh_2h_3}(s_{23})\langle R(p_2+p_3)|\,j\,|D^0(p_D)\rangle$$

Example: $D^0(p_D)\rightarrow \pi^+(p_1)\overline{K}^0(p_2)\pi^-(p_3)$ $\quad R=K^*(892)^-\rightarrow \overline{K}^0\pi^-$

$p_D=p_1+p_2+p_3,\ \ s_{23}=(p_2+p_3)^2,\ \ p_1^{\ 2}=m_\pi^{\ 2}$ $\quad j=(\bar{s}c)_{V-A}$

$$\langle R(p_2+p_3)|\,j\,|D^0(p_D)\rangle = -i2m_{K^*}\frac{\varepsilon^*\cdot p_D}{p_1^2}p_1^\mu A_0^{DK^*}(m_\pi^2) + 3\ \text{other terms}$$

ε - K^* polarization $\quad A_0^{DK^*}(m_\pi^2)$ **D to K* transition form factor**

Vertex function:

$$G_{K^{*-}\overline{K}^0\pi^-}(s_{23}) = \varepsilon\cdot(p_2-p_2)\frac{1}{m_{K^*}f_{K^*}}F_1^{\overline{K}^0\pi^-}(s_{23})$$

$F_1^{\overline{K}^0\pi^-}(s_{23})$ - **kaon-pion** transition **vector form factor**

Transition matrix elements (2)

$$\langle h_1(p_1)h_2(p_2)h_3(p_3)\,|\, j'\,|0\rangle \approx G_{Rh_2h_3}(s_{23})\langle h_1(p_1)R(p_2+p_3)\,|\, j'\,|0\rangle$$

Example: $h_1 = \overline{K}^0, \quad R = f_0 \to \pi^+\pi^-$

$$p_D = p_1 + p_2 + p_3,\;\; s_{23} = (p_2 + p_3)^2, \qquad j' = (\bar{s}d)_{V-A}$$

$$\langle \overline{K}^0(p_1)\, f_0(p_2+p_3)\,|\, j'^{\mu}\,|0\rangle = -i\,\frac{m_{K^0}^2 - s_{23}}{p_D^2}\, p_D^{\mu} F_0^{\overline{K}^0 f_0}\left(m_D^2\right) +$$ 2nd term

$F_0^{\overline{K}^0 f_0}(m_D^2)$ - **kaon to f_0 transition form factor** (complex number)

$$G_{f_0\pi^+\pi^-}(s_{23}) \approx \chi_2 F_0^{\pi^+\pi^-}(s_{23})$$

$F_0^{\pi^+\pi^-}(s_{23})$ - **pion scalar form factor**, χ_2 - constant

Selected formulae of decay amplitudes (1)

$D^0 \to K_S^0 \pi^+ \pi^-$ $\qquad |K_S^0\rangle \approx \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$

Allowed transitions with $K_S^0 \pi^-$ final state interactions $\qquad \Lambda_1 = V_{cs}^* V_{ud}$

$m_{\mp}$ eff. masses of $K_S^0 \pi^{\mp}$, $m_0 - \pi^+ \pi^-$ eff. mass $\qquad$ a_1 - effective Wilson coefficient

S-wave:

$$A_{1S} = -\frac{G_F}{2} \Lambda_1 a_1 f_\pi (m_D^2 - m_\pi^2)) F_0^{DK_0^{*-}}(m_\pi^2) F_0^{\bar{K}_0 \pi^-}(m_-^2)$$

$F_0^{DK_0^{*-}}(m_\pi^2)$ - D to K_0^* transition scalar form factor

P-wave:

$$A_{1P} = -\frac{G_F}{2} \Lambda_1 a_1 \frac{f_\pi}{f_\rho} [m_0^2 - m_+^2 + \frac{(m_D^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{m_-^2}] A_0^{DK^{*-}}(m_\pi^2) F_1^{\bar{K}_0 \pi^-}(m_-^2)$$

D- wave:

$$A_{1D} = -\frac{G_F}{2} \Lambda_1 a_1 f_\pi F^{DK_2^*}(m_-^2) \frac{G_{K_2^* K_S^0 \pi} D(m_+^2, m_-^2)}{m_{K_2^*}^2 - m_-^2 - i m_{K_2^*} \Gamma_{K_2^*}}$$

$F^{DK_2^{*-}}(m_-^2)$ - combination of D to $K_2^{*-}(1430)$ transition form factors

$G_{K_2^* K_S^0 \pi}$ - coupling constant, $D(m_+^2, m_-^2)$ = D-wave angular distribution function

Selected formulae of decay amplitudes (2)

Annihilation (W-exchange) transitions with $\pi^+\pi^-$ **final state interactions**

$m_0 = \pi^+\pi^-$ effective mass a_2 - effective Wilson coefficient

S-wave:

$$An_{2S} = -\frac{G_F}{2}\Lambda_1 a_2 \chi_2 f_D (m_K^2 - m_0^2) F_0^{\bar{K}^0 f_0}(m_D^2) F_0^{\pi^+\pi^-}(m_0^2)$$

$F_0^{\bar{K}^0 f_0}(m_D^2)$ - **$\bar{K}^0$ to f_0 scalar transition form factor**

P-wave:

$$An_{2P} = \frac{G_F}{2}\Lambda_1 a_2 \frac{f_D}{f_\rho}(m_-^2 - m_+^2) A_0^{\rho \bar{K}^0}(m_D^2) F_1^{\pi^+\pi^-}(m_0^2)$$

$A_0^{\rho \bar{K}^0}(m_D^2)$ - **ρ to $\bar{K}^0$ transition form factor**

D-wave:

$$An_{2D} = \frac{G_F}{2}\Lambda_1 a_2 f_D F^{Df_2}(m_0^2) \frac{G_{f_2 \pi\pi} D(m_+^2, m_0^2)}{m_{f_2}^2 - m_0^2 - i m_{f_2} \Gamma_{f_2}(m_0^2)}$$

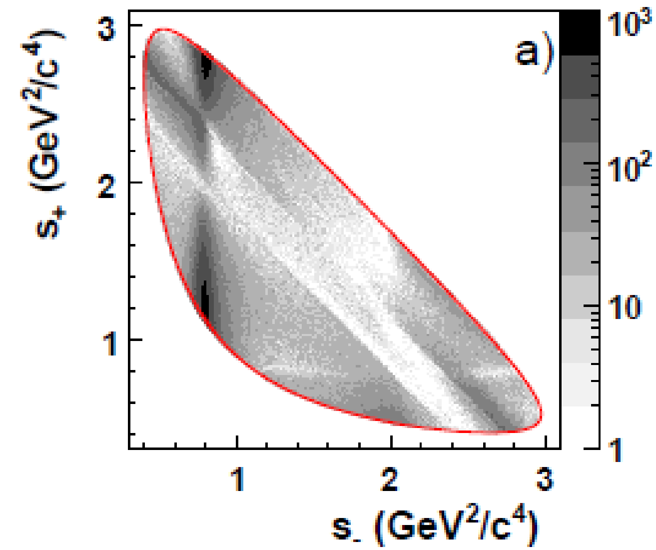
$F^{Df_2}(m_0^2)$ - combination of D to $f_2(1270)$ transition form factors

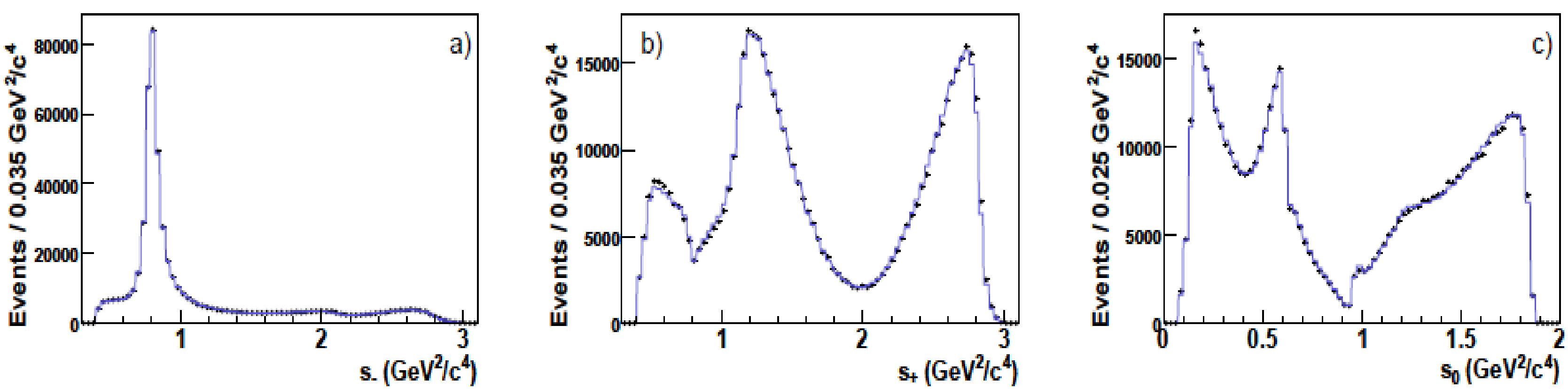
$G_{f_2 \pi\pi}$ - coupling constant, $D(m_+^2, m_0^2)$ - D-wave angular distribution function

Experimental data on $D^0 \to K_S^0 \pi^+ \pi^-$ decay

a) L.M. Zhang et al. (Belle Coll.), Phys. Rev. Lett. 99 (2007) 131803

b) P. del Amo Sanchez et al. (BaBar Coll.), Phys. Rev. Lett. 105 (2010) 081803





$$s_- = (p_{K_S^0} + p_{\pi^-})^2 \qquad s_+ = (p_{K_S^0} + p_{\pi^+})^2 \qquad s_0 = (p_{\pi^+} + p_{\pi^-})^2$$

Isobar model and its problems

1. Amplitudes in the isobar model are **not unitary** neither in three-body decay channels nor in two-body subchannels.
2. It is **difficult** to distinguish the **S-wave** amplitude from the **background** terms. Their interference is often very strong.
3. Some **branching fractions** extracted in such analyses could be unreliable .
4. The isobar model has many free parameters (at least two fitted parameters for each amplitude component).

   Belle used 40 fitted parameters and BaBar 43 parameters.

Why unitarity is important?

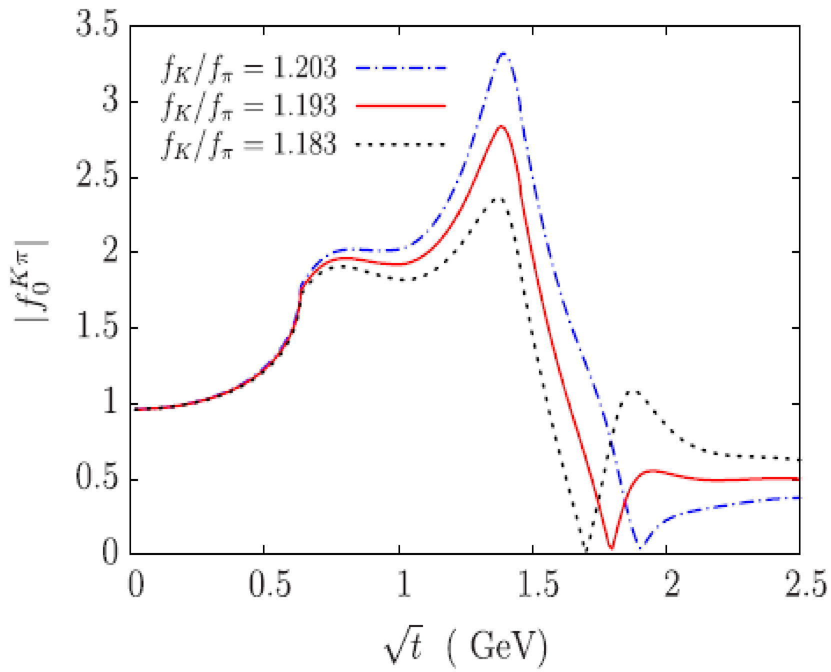
Unitary model allows for:

1. proper construction of D-decay amplitudes,
2. partial wave analyses of final states,
3. explanation of structures seen in Dalitz plots,
4. adequate determination of branching fractions and CP asymmetries for different quasi-two-body decays,
5. extraction of standard model parameters (weak amplitudes) ,
6. application not only in analyses of D decays but also in studies of other reactions.

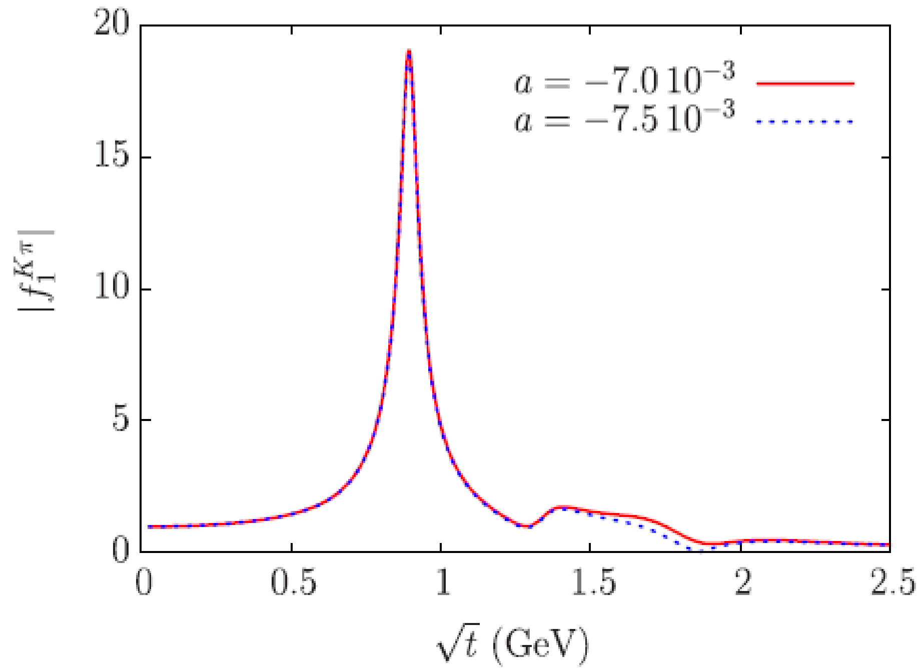
Towards a unitary approach

1. Construction of unitary three-body strong interaction amplitudes in a wide range of effective masses is difficult.
2. As **a first step** we attempt to incorpotate **in our model two-body unitarity** into the D-decay amplitudes with final state interactions in the following channels:
   a) K^0 π S-wave amplitude,
   b) K^0 π P-wave amplitude,
   b) π π S-wave amplitude.
3. The branching fraction corresponding to a sum of these amplitudes can exceed 80% of the total braching fraction for the $D^0 \rightarrow K_S{}^0 \pi^+ \pi^-$ decay.

kaon-pion form factors



scalar Kπ form factor
[K_0*(800) + K_0*(1430)]



vector Kπ form factor
[K*(892) + K_1(1410)+ K*(1680)]

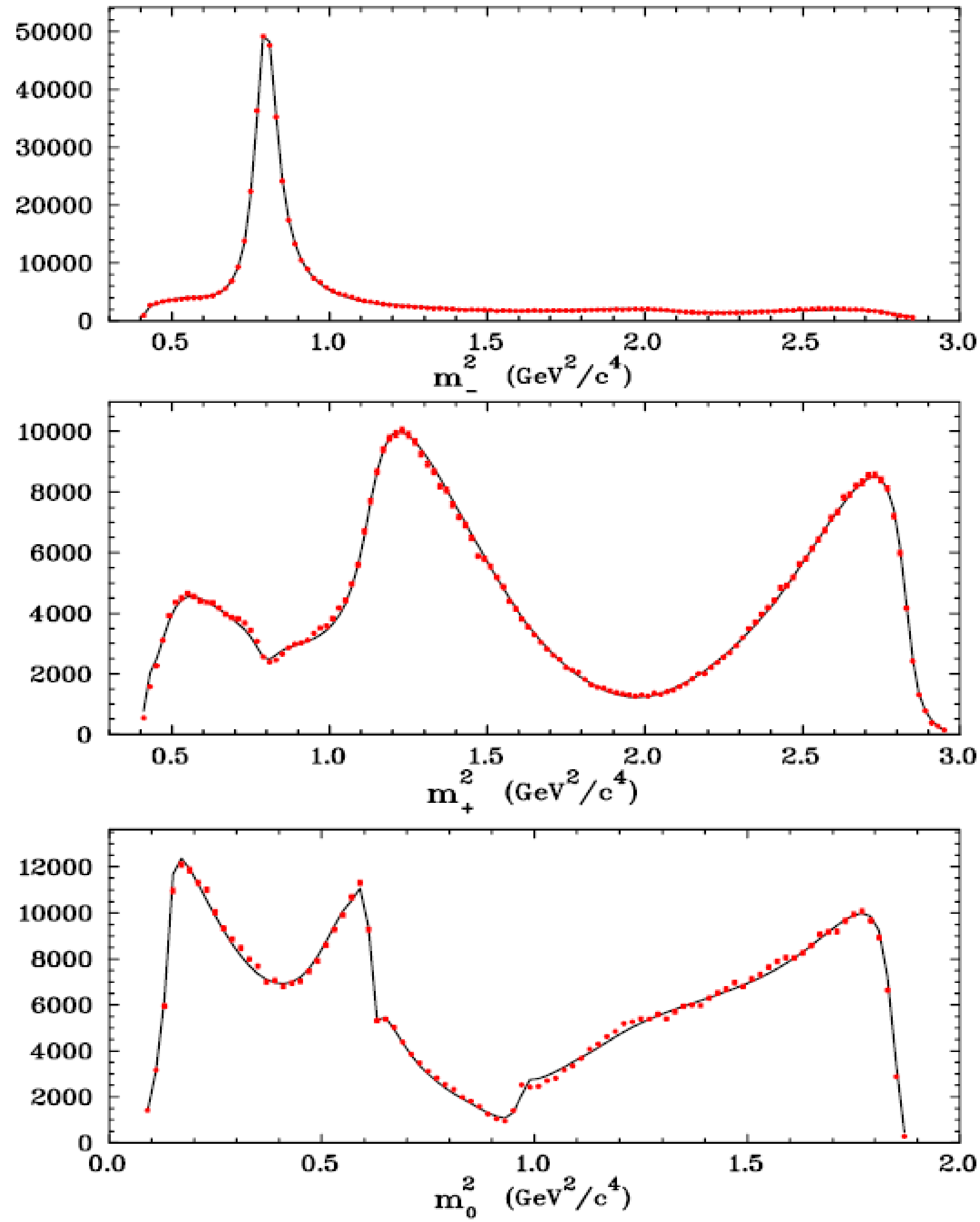
Figures are taken from Physical Review D 79, 094005 (2009)
Authors: B. El-Bennich, A. Furman, R. Kamiński, L. Leśniak, B.Loiseau, B. Moussallam

Comparison with the Belle data

Model results are preliminary.

There are 28 free parameters (many unknown transition form factors).

Number of events per 0.02 GeV^2



Summary

1. The **$D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays** are analysed using the QCD factorization approximation.
2. The annihilation (via W-echange) amplitudes are added to the weak-decay tree amplitudes.
3. The strong interactions between kaon-pion and pion-pion pairs in the S-, P- and D-final states are described in terms of the corresponding form factors.
4. The kaon-pion or pion-pion scalar and vector form factors are constrained using unitarity, analyticity and chiral symmetry.
5. Preliminary results of the theoretical model compare fairly well with the effective mass distributions obtained by the Belle Collaboration .