# Pion-photon Transition Distribution Amplitudes in Nonlocal Chiral Quark Model

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# Outlook

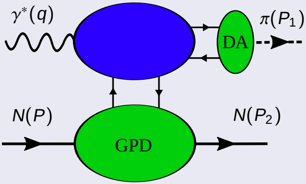
- Transition Distribution Amplitudes (TDAs)
  - Barion-meson TDAs
  - Pion-photon TDAs
- Nonlocal Chiral Quark Model
  - Nonlocal currents
- Results
  - Vector and Axial TDAs
  - Form factors for $\pi^{\pm} \to e^{\pm}\nu\gamma$
  - Transition form factor for $\pi^{0} \to \gamma^{*}\gamma$

# Transition Distribution Amplitudes (TDA)

## Factorization for $\gamma^*(q)\,N(P) \to \pi(P_1)\,N(P_2)$

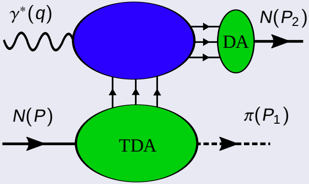
Kinematics: $s = (q+P)^2$, $Q^2 = -q^2$ are large, Bjorken $x_B$ and skewedness $\xi$ are fixed.

*standard approach:*




$$\xi = -\frac{P_2^+ - P^+}{P_2^+ + P^+},\quad t = (P_2 - P)^2,\quad |t| \ll s, Q^2$$

*backward kinematics*[1]*:*




$$\xi = -\frac{P_1^+ - P^+}{P_1^+ + P^+},\quad u = (P_1 - P)^2,\quad |u| \ll s, Q^2$$

$\Rightarrow$ barion-meson TDA $\Leftarrow$

- TDAs are more general objects than GPDs
- they extend factorization for exclusive meson electroproduction to larger kinematic region
- little experimental accessibility so far (potential: JLab, Compass; GSI for $p\bar{p} \to e^+e^-\gamma$)

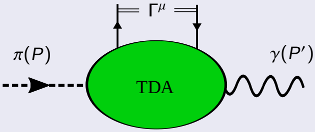
[1] e.g. J. P. Lansberg, B. Pire, K. Semenov-Tian-Shansky, L. Szymanowski

# Pion-Photon TDAs

One can also consider hadron-photon TDAs[1]. They control processes $\gamma^* p \to \gamma p$ in the backward kinematics and $\overline{p}p \to \gamma^*\gamma$ in forward region.

We want to study TDAs in effective model
$\Rightarrow$ choose the simplest hadronic state: pion.

Depending on nature of $\Gamma^\mu$ operator, we have:




$$\Delta^\mu = P'^\mu - P^\mu$$
$$p^\mu = \frac{1}{2}\left(P'^\mu + P^\mu\right)$$

- Vector pion-photon TDA (VTDA)

$$\int \frac{d\lambda}{2\pi} e^{2i\lambda X p^+} \langle \gamma(P',\varepsilon)\big| \overline{d}(-\lambda n)\gamma^\mu u(\lambda n) \left|\pi^+(P)\right\rangle \propto \epsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu P_\alpha P'_\beta\, \boldsymbol{V(X,\xi,t)}$$

- Axial pion-photon TDA (ATDA)

$$\int \frac{d\lambda}{2\pi} e^{2i\lambda X p^+} \langle \gamma(P',\varepsilon)\big| \overline{d}(-\lambda n)\gamma^\mu\gamma_5 u(\lambda n) \left|\pi^+(P)\right\rangle \propto P'^\mu \Delta\cdot\varepsilon^*\, \boldsymbol{A(X,\xi,t)} + \ldots$$

[1] B. Pire, L. Szymanowski

# Pion-Photon TDA (cont.)

## Properties

- polynomiality

$$\int_{-1}^{1} dX\, X^n V(X,\xi,t) = f_0(t) + \xi f_1(t) + \ldots + \xi^n f_n(t)$$

  (and the same for axial TDA).

- normalization of VTDA (but not for ATDA) is fixed by the axial anomaly

$$\int_{-1}^{1} dX\, V(X,\xi,0) = \frac{1}{2\pi^2}$$

- sum rules

$$\int_{-1}^{1} dX\, V(X,\xi,t) \propto F_V(t), \quad \int_{-1}^{1} dX\, A(X,\xi,t) \propto F_A(t)$$

  where $F_V$ and $F_A$ controls $\pi^\pm \to e^\pm \nu\gamma$ decay.

- less symmetric than ordinary GPDs

TDAs provide severe test for effective models!

# Nonlocal Chiral Quark Model

## Local Chiral Quark Model

One of the simplest models for pion-quark interctions: semibosonized Nambu-Jona-Lasinio[1]

$$S_{\rm loc} = \int d^4x\, \bar{\psi}(x) \left( i\gamma^\mu D_\mu - M U^{\gamma_5}(x) \right) \psi(x),$$

where $U^{\gamma_5}(x) = \exp\left\{ \frac{i}{F_\pi} \tau^a \pi^a(x) \gamma_5 \right\}$ with $F_\pi = 93\,\mathrm{MeV}$ and $M \sim 350\,\mathrm{MeV}$.

Problem: regularization needed (e.g. cutoff), but its presence violates axial anomaly.

## Nonlocal interactions

Instanton-vacuum[2] motivated interactions:

$$S_{\rm Int} = \int \frac{d^4k\, d^4k'}{(2\pi)^8} \bar{\psi}(k) \sqrt{M(k)}\, U^{\gamma_5}(k-k') \sqrt{M(k')}\, \psi(k'),$$

where $M(k) = M F^2(k)$ and $F(0) = 1, \quad F(k \to \infty) \to 0$.

Problem: naive vector $\bar{\psi}\gamma^\mu\psi$ and axial $\bar{\psi}\gamma^\mu\gamma_5\psi$ currents are not conserved.

[1] e.g. S. P. Klevansky   [2] D. Diakonov, V. Y. Petrov

# Nonlocal Chiral Quark Model (cont.)

## Solution to current non-conservation

Replace naive currents[1]:

- for vector $\bar{\psi}\gamma^\mu\psi \longrightarrow \bar{\psi}\Gamma^\mu\psi$, where

$$\Gamma^\mu(k,k') = \gamma^\mu + g^\mu(k,k'), \quad g^\mu(k,k') = -\frac{k^\mu + k'^\mu}{k^2 - k'^2}\left(M(k) - M(k')\right)$$

- for axial $\bar{\psi}\gamma^\mu\gamma_5\psi \longrightarrow \bar{\psi}\Gamma_5^\mu\psi$, where

$$\Gamma_5^\mu(k,k') = \gamma^\mu\gamma_5 + g_5^\mu(k,k'), \quad g_5^\mu(k,k') = -\frac{k^\mu - k'^\mu}{(k-k')^2}\left(M(k) + M(k')\right)\gamma_5$$

## Mass dependence on momentum

The momentum dependence on mass $M(k) = M F^2(k)$ is chosen as[2]

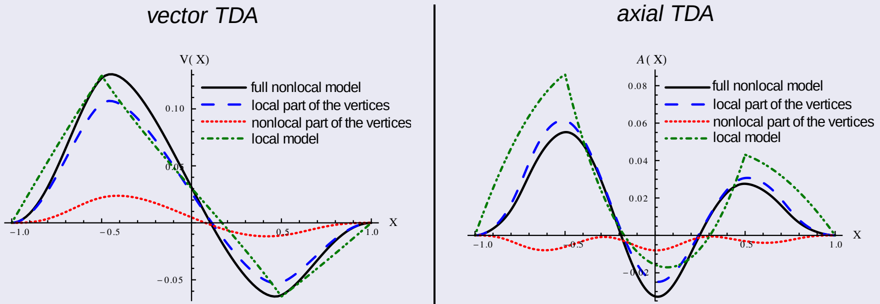
$$F(k) = \left(\frac{-\Lambda_n^2}{k^2 - \Lambda_n^2 + i\epsilon}\right)^n,$$

where $\Lambda_n$ is fixed by the pion decay constant.

[1] R. D. Bowler, M. C. Birse; B. Holdom, R. Lewis [2] M. Praszalowicz, A. Rostworowski

# Results: pion-photon TDAs

## TDAs in nonlocal model




$\xi = 0.5,\ t = -0.1\,\mathrm{GeV}^2$ $\xi = 0.5,\ t = -0.1\,\mathrm{GeV}^2$

- nonlocality makes TDAs smooth along the support
- in local models the normalization of ATDA and VTDA is the same
- in nonlocal model the normalization of ATDA is much smaller

# Results: vector and axial formfactors

## Vector formfactor

- model independent value (if anomaly requirement satisfied)

$$F_V(0) \approx 0.0272$$

- experimental value (PDG)

$$F_V^{\mathrm{exp}}(0) = 0.0254 \pm 0.0017$$

## Axial formfactor

- local quark models

$$F_A(0) = F_V(0) \approx 0.0272$$

- experimental value (PDG)

$$F_A^{\mathrm{exp}}(0) = 0.0119 \pm 0.0001$$

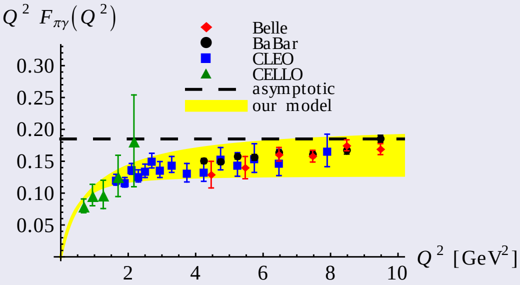
- our model

| $M$ [MeV] | 225 | 350 | 350 | 400 | 400 |
|---|---|---|---|---|---|
| $n$ | 1 | 1 | 5 | 1 | 5 |
| $F_A(0)$ | 0.0217 | 0.0168 | 0.0163 | 0.0161 | 0.0152 |

$\Rightarrow$ nonlocality shifts $F_A$ towards experimental values.

# Results: $\pi$-$\gamma$ form factor

The form factor $F_V(t)$ is directly related to $\pi^0 \to \gamma^*\gamma$ formfactor $F_{\pi\gamma}\left(Q^2\right)$.



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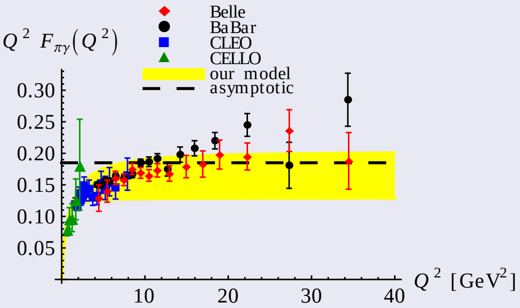


be carefull: this is not the domain of applicability of our model

# Summary

- Transition Distribution Amplitudes further generalize Generalized Parton Distributions to transitions between different states
- TDAs for pion transitions can be studied within chiral quark model and provide severe test for its symmetries

- We have obtained pion-photon TDAs in our version of nonlocal chiral quark model
- The normalization of vector TDA originating in axial anomaly is recovered due to non-local currents
- The normalization of axial TDA is not fixed and is shifted towards experimental values
- For pion-photon transition form factor we find agreement in the domain of applicability of our model

# BACKUP

# TDAs precise definitions

## VTDA definition

$$\int \frac{d\lambda}{2\pi} e^{2i\lambda X p^+} \langle \gamma(P',\varepsilon) \big| \, \overline{d}(-\lambda n)\gamma^\mu u(\lambda n) \, \big| \pi^+(P) \big\rangle = \frac{-e}{4\sqrt{2} F_\pi p^+} \, \epsilon^{\mu\nu\alpha\beta} \varepsilon^*_\nu P_\alpha P'_\beta \, \boldsymbol{V\,(X,\xi,t)}$$

## ATDA definition

$$\int \frac{d\lambda}{2\pi} e^{2i\lambda X p^+} \langle \gamma(P',\varepsilon) \big| \, \overline{d}(-\lambda n)\gamma^\mu u(\lambda n) \, \big| \pi^+(P) \big\rangle = \frac{ie}{4\sqrt{2} F_\pi p^+} \, P'^\mu \Delta \cdot \varepsilon^* \, \boldsymbol{A\,(X,\xi,t)}$$

$$+ \Delta^\mu \Delta \cdot \varepsilon^* \, \frac{i\sqrt{2} e F_\pi \, \mathrm{sign}\,(\xi)}{t} \, \phi_\pi \left( \frac{X+\xi}{2\xi} \right)$$

# Formfactors

$$\int_{-1}^{1} dX\, V(X,\xi,t) = \frac{2\sqrt{2}F_\pi}{m_\pi} F_V(t)$$

$$\int_{-1}^{1} dX\, A(X,\xi,t) = \frac{2\sqrt{2}F_\pi}{m_\pi} F_A(t)$$

$$F_{\pi\gamma}(t) = \frac{\sqrt{2}}{m_\pi} F_V(t)$$