Theory of Two-Pion Photo- and Electroproduction off the Nucleon

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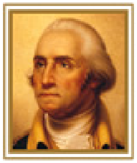
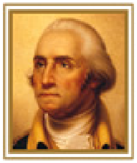


Motivation

- Two-pion production $\gamma N \to \pi\pi N$ is being measured now
- No comprehensive formulation of two-pion photoproduction processes exists that is at the same level of rigor as single-pion production

Procedure

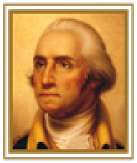
- Use field theory based on hadronic Lagrangians
- Employ LSZ-type mechanisms to couple electromagnetic field to fully dressed hadronic propagators and vertices



NB: The formulation given here can be easily translated to any type of mesons

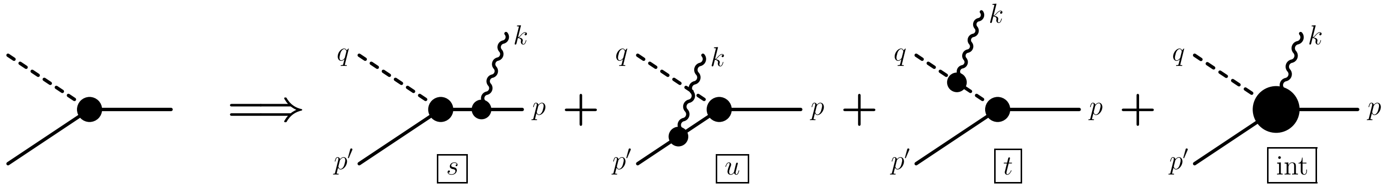
First, something simple. . .

Single-pion Production

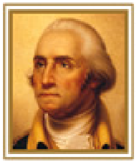


Basic Hadronic Two-pion Production Processes

Attach photon to πNN vertex:



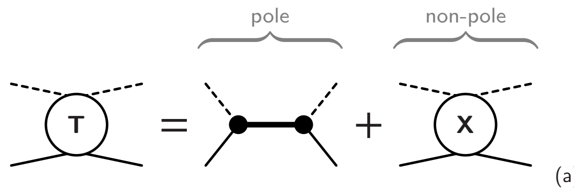
- Simple at tree level
- *Very* complicated for *dressed* vertex

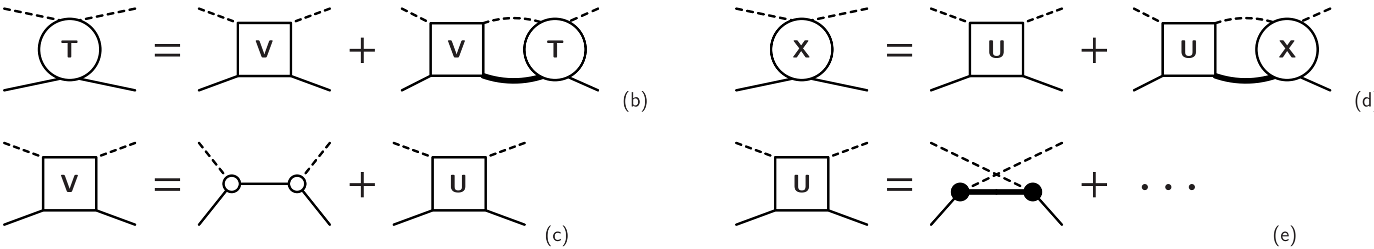


Pions, Nucleons, and Photons

HH, PRC **56**, 2041 (1997)

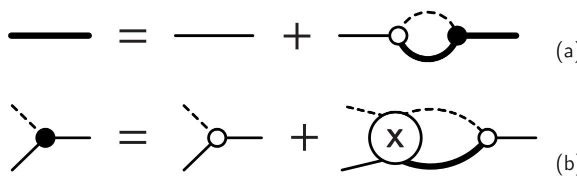
$\boxed{\pi N\ T \text{ matrix}}$



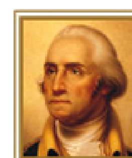


dressed nucleon propagator

dressed πNN vertex



propagator determines current

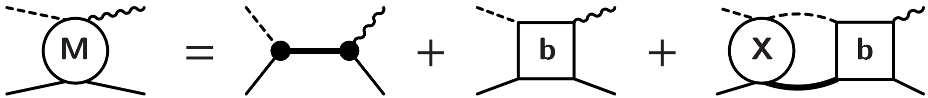


- Tower of *nonlinear* Dyson-Schwinger-type equations

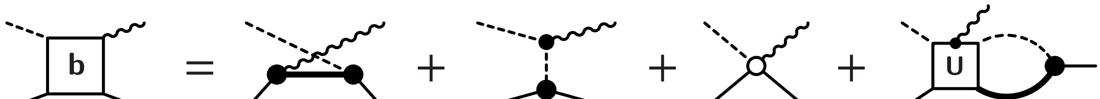
Pion Photoproduction

HH, PRC **56**, 2041 (1997)

- **Pion-production current M^μ:**

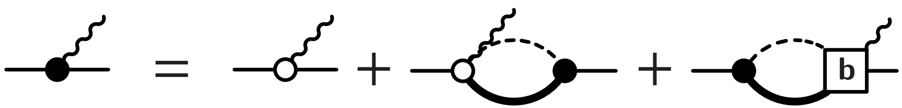


(a)



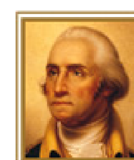
(b)

- **Nucleon current J^μ:**



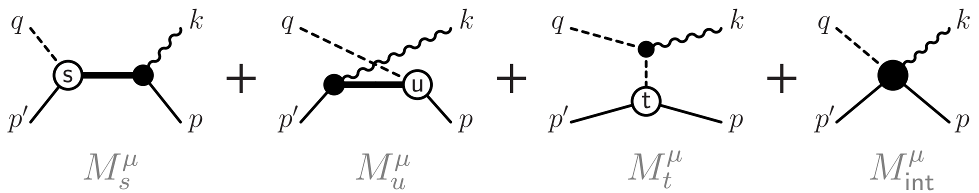
$\Rightarrow$ The internal structures of the dressed nucleon current can be understood by the dynamics of the pion production current.

- Tower of *nonlinear* Dyson-Schwinger-type equations



Gauge Invariance: Generalized Ward–Takahashi Identity

$M^\mu =$

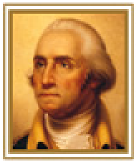


- **Generalized WTI for the full current M^μ:**

$$k_\mu M^\mu = -F_s S(p+k) Q_i S^{-1}(p) + S^{-1}(p') Q_f S(p'-k) F_u + \Delta_\pi^{-1}(q) Q_\pi \Delta_\pi(q-k) F_t$$

Off-shell constraint!

Hadrons on-shell: $k_\mu M^\mu = 0$



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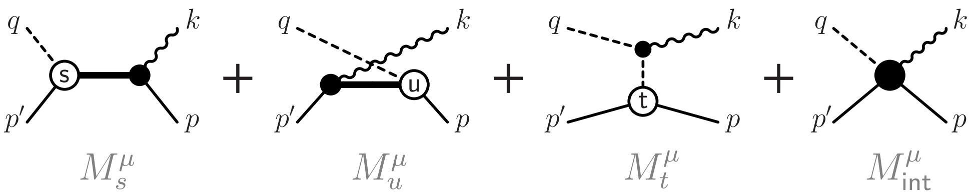

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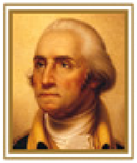
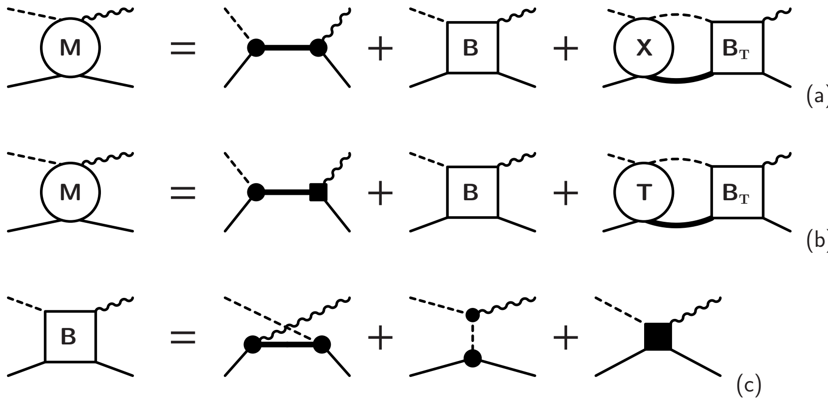
Approximations destroy gauge invariance!



Rewriting the Production Current

- **Pion-production current M^μ:**

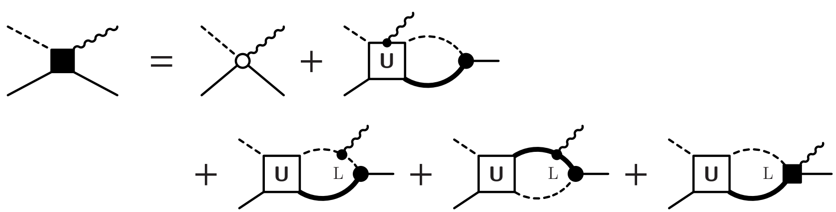


X

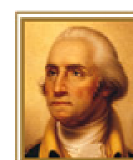
equivalent

T

- **Contact-type current M_c^μ:**



- Tower of *nonlinear* Dyson-Schwinger-type equations

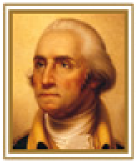


Problems?

- Everything is exact!

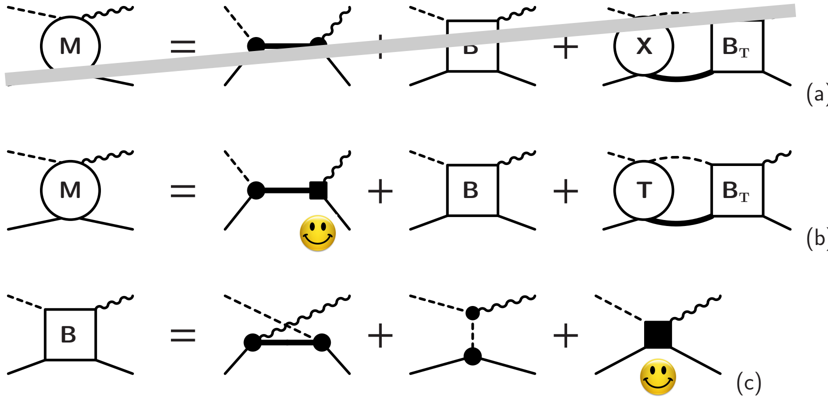
- Everything is nonlinear!

- Everything is hideously complicated!

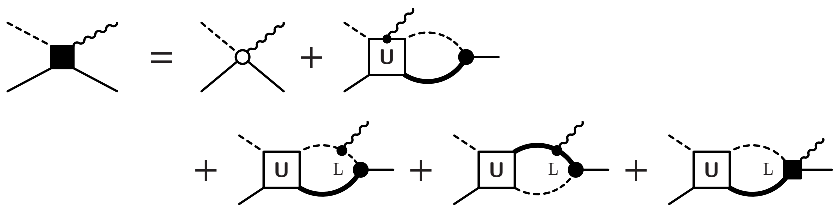


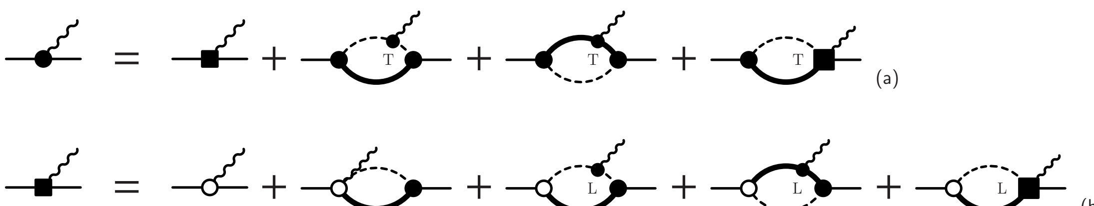
Let’s cut the Gordian knot!

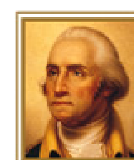
HH, F. Huang, K. Nakayama, PRC **83**, 065502 (2011)



Do not use X.
Work with full T.

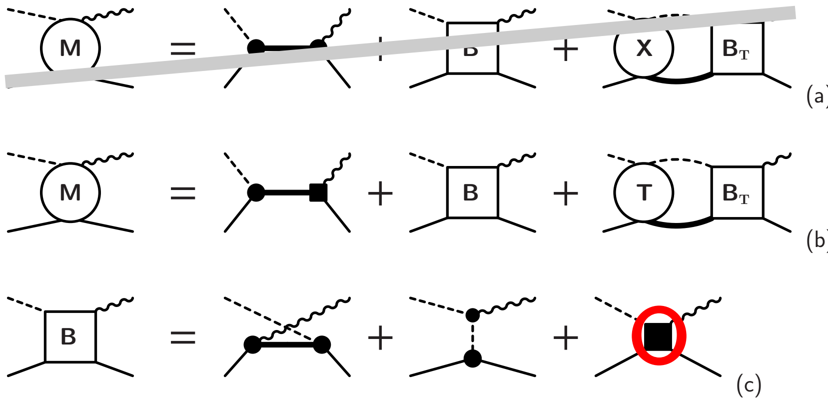






Approximating M_c^μ

HH, F. Huang, K. Nakayama, PRC **83**, 065502 (2011)

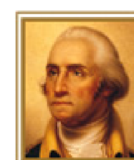


M_c^μ

- **Lowest-order approximation in terms of phenomenological form factors:**

$$M_c^\mu = ge\gamma_5 \frac{i\sigma^{\mu\nu}k_\nu}{4m^2}\tilde{\kappa}_N - (1-\lambda)g\frac{\gamma_5\gamma^\mu}{2m}\tilde{F}_t e_\pi - G_\lambda \left[e_i \frac{(2p+k)^\mu}{s-p^2}\left(\tilde{F}_s - \hat{F}\right) \right.$$
$$+ e_f \frac{(2p'-k)^\mu}{u-p'^2}\left(\tilde{F}_u - \hat{F}\right)$$
$$\left. + e_\pi \frac{(2q-k)^\mu}{t-q^2}\left(\tilde{F}_t - \hat{F}\right) \right]$$

Don't try to read the details. What is important is that this is a simple expression, easy to evaluate, and that it helps preserve gauge invariance of the entire production current.



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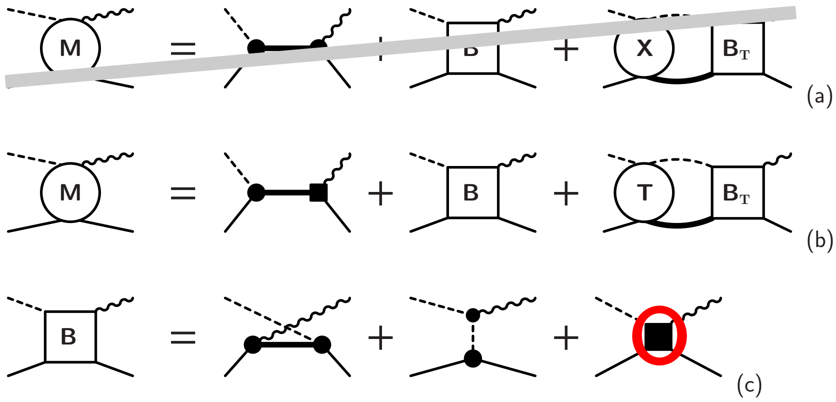

M_c^μ

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$$+ e_f\frac{(2p'-k)^\mu}{u-p'^2}\left(\tilde{F}_u-\hat{F}\right)$$

$$\left.+ e_\pi\frac{(2q-k)^\mu}{t-q^2}\left(\tilde{F}_t-\hat{F}\right)\right]$$

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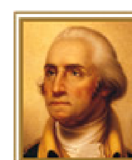


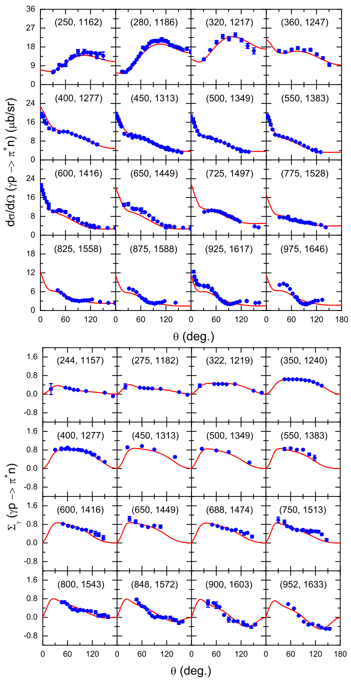
Approximation can be made more sophisticated if necessary. . .

Does it work? — Yes!

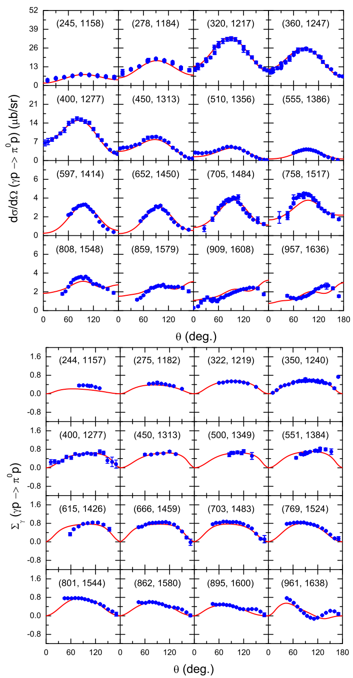
- **Results for $\gamma N \to \pi N$**

$$\frac{d\sigma}{d\Omega}$$

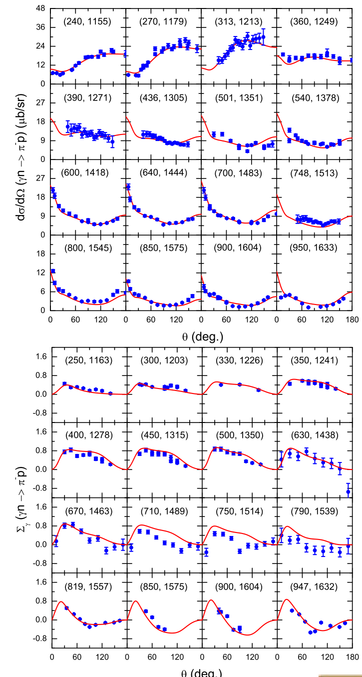
$$\Sigma$$



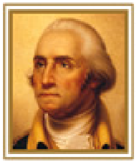
$$\gamma p \to \pi^+ n$$



$$\gamma p \to \pi^0 p$$



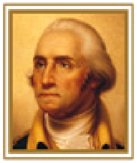
$$\gamma n \to \pi^- p$$



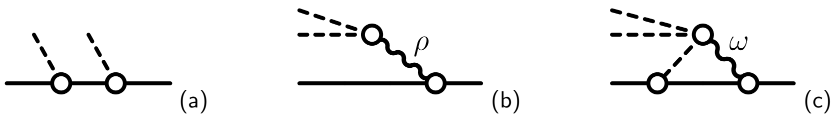
F. Huang, M. Döring, H. Haberzettl, J. Haidenbaum, C. Hanhart, S. Krewald, U.-G. Meißner, and K. Nakayama, Phys. Rev. C **85**, 054003 (2012)

Now, the real thing...

Two-pion Production



Basic Hadronic Two-pion Production Processes

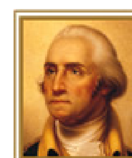


etc.

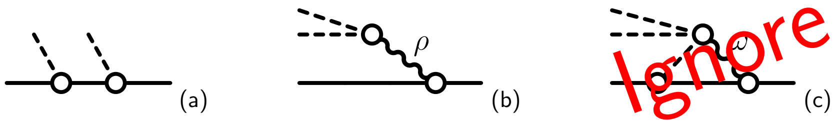
(a) sequential production off nucleon
(b) production off intermediate vector meson
(c) production off intermediate three- or more-pion vertex

Procedure

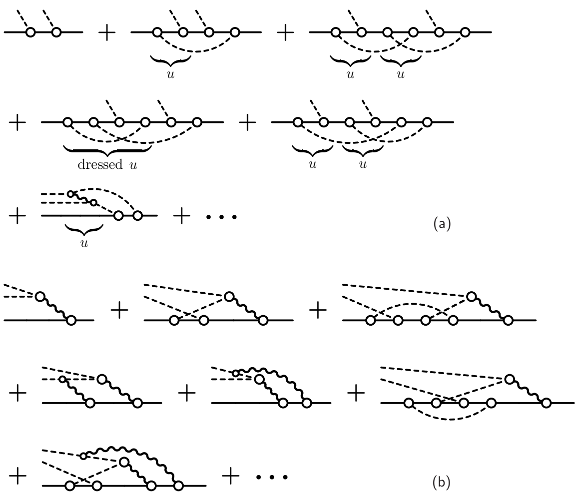
(1) Iterate bare hadronic processes and sum up to obtain dressed mechanisms
(2) Attach photon — employ (gauge-invariant) single-pion amplitudes

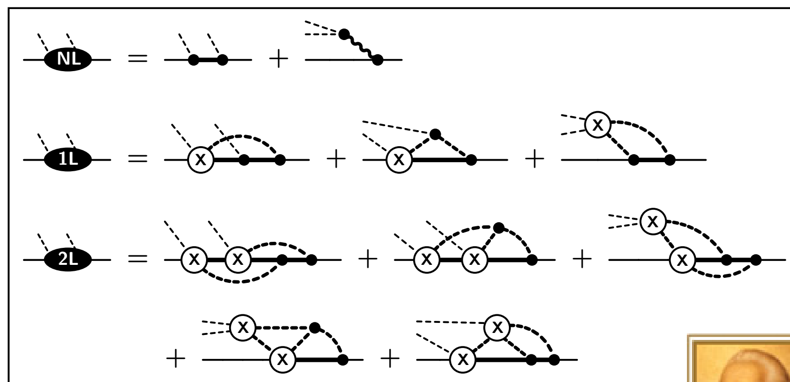


Iterated Hadronic Two-pion Production Processes

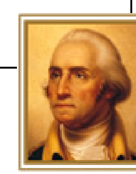


etc.



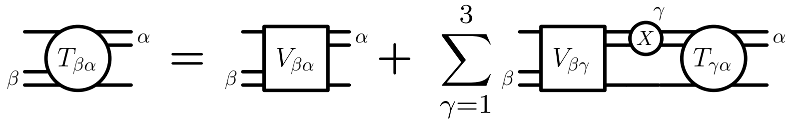


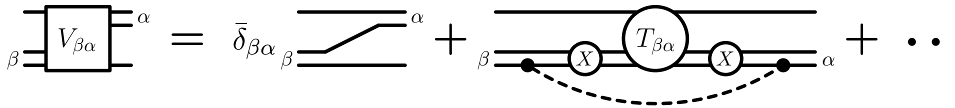
Lowest orders of
3-body multiple scattering series

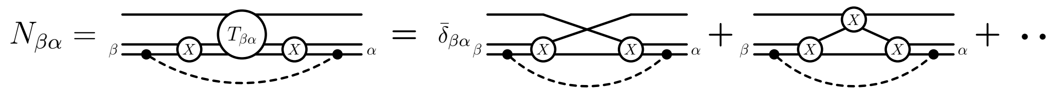


Faddeev-type Alt-Grassberger-Sandhas Equations

NP B2, 167 (1967)

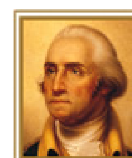






$$\boxed{T_{\beta\alpha} = V_{\beta\alpha} + \sum_{\gamma=1}^{3} V_{\beta\gamma} G_0 X_\gamma G_0 T_{\gamma\alpha}}$$

$$\alpha,\ \beta,\ \gamma: \quad \begin{aligned} \text{“1”} &= (\pi_1 N, \pi_2) \\ \text{“2”} &= (\pi_2 N, \pi_1) \\ \text{“3”} &= (\pi_1 \pi_2, N) \end{aligned}$$



Closed-form Expression for $N \to \pi\pi N$ 'Vertex'

$$M_\beta = \sum_\alpha \left(\delta_{\beta\alpha} + \sum_\gamma T_{\beta\gamma} G_0 X_\gamma \bar{\delta}_{\gamma\alpha} \right) f_\alpha$$

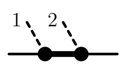
$$+ \sum_\gamma \left(\delta_{\beta\gamma} + T_{\beta\gamma} G_0 X_\gamma G_0 \right) \sum_\alpha N_{\gamma\alpha} G_0 f_\alpha$$

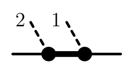
$$= f_\beta \qquad \Leftarrow \text{ no loop}$$

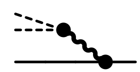
$$+ \sum_{\gamma,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\alpha} X_\gamma G_0 f_\alpha \qquad \Leftarrow \text{ one loop}$$

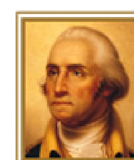
$$+ \sum_{\gamma,\kappa,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\kappa} \bar{\delta}_{\kappa\alpha} X_\gamma G_0 X_\kappa G_0 f_\alpha \qquad \Leftarrow \text{ two loops}$$

$$+ \sum_\alpha N_{\beta\alpha} G_0 f_\alpha \cdots$$

where $f_1 =$ 

$f_2 =$ 

$f_3 =$ 



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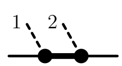
$$= f_\beta \qquad \Leftarrow \text{ no loop}$$

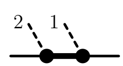
$$+ \sum_{\gamma,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\alpha} X_\gamma G_0 f_\alpha \qquad \Leftarrow \text{ one loop}$$

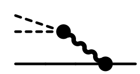
$$+ \sum_{\gamma,\kappa,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\kappa} \bar{\delta}_{\kappa\alpha} X_\gamma G_0 X_\kappa G_0 f_\alpha \qquad \Leftarrow \text{ two loops}$$

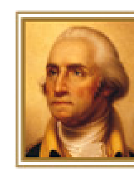
$$+ \sum_\alpha N_{\beta\alpha} G_0 f_\alpha \cdots$$

attach photon order by order

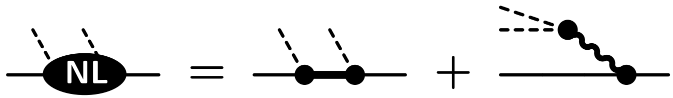
where $f_1 =$ 

$f_2 =$ 

$f_3 =$ 

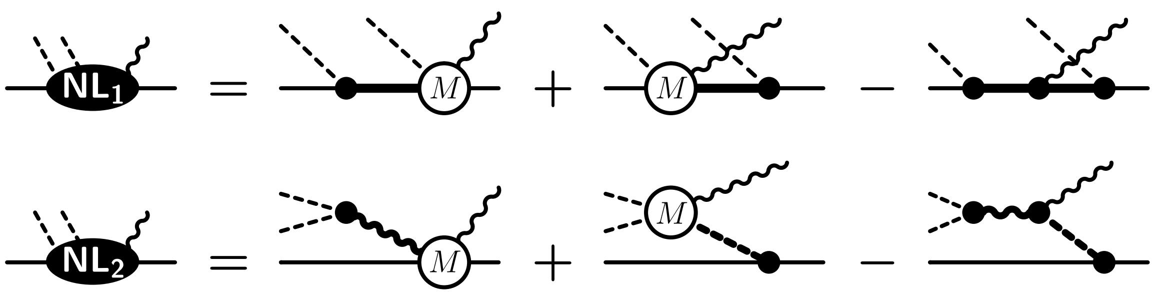


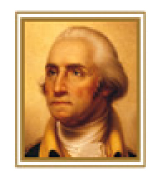
Attach Photon — No-loop Graphs



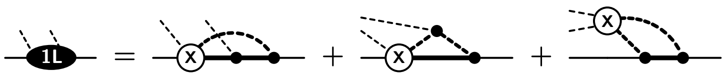
$\Downarrow$ attach photon

separately gauge invariant!

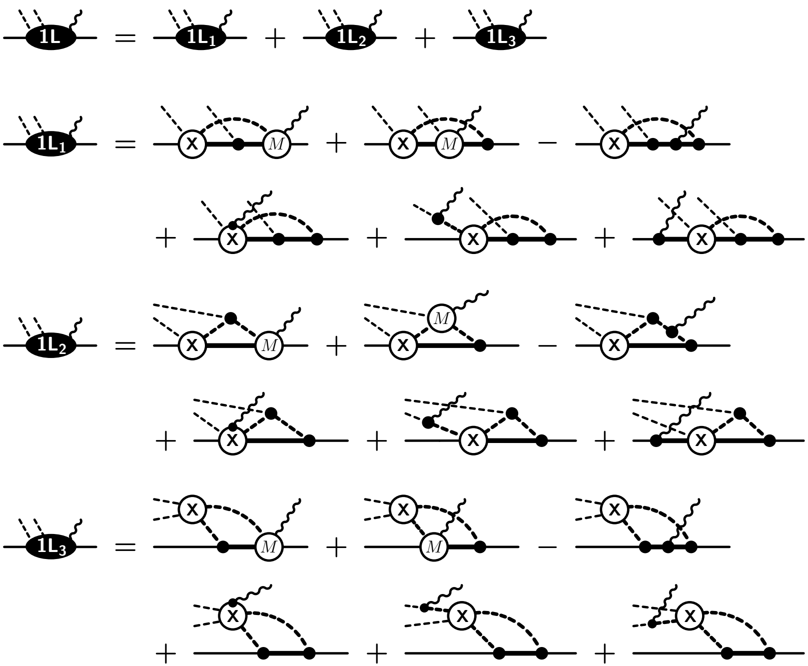




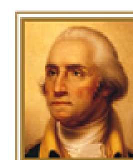
Attach Photon — One-loop Graphs

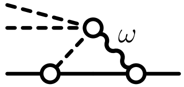


$\Downarrow$ attach photon

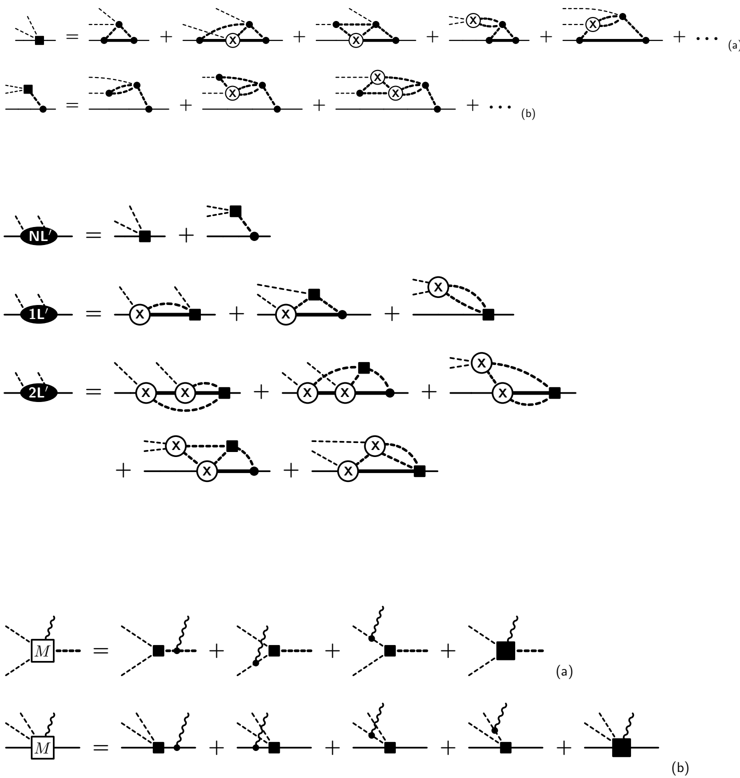


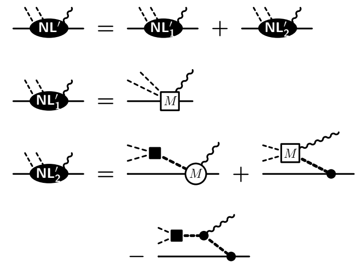
separately gauge invariant!



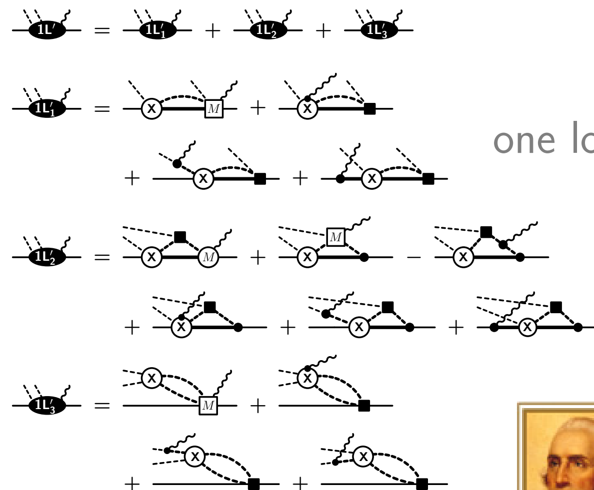
What about  ?

(Don't try to read this page!)

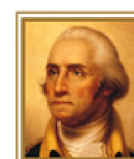




no loop

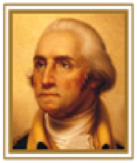


one loop



Summary

- Theory presented provides a complete description of the $\pi\pi$ production process based on field theory. *(This is not a model!* — In principle, the formalism could be implemented to an arbitrary degree of sophistication for any given set of interaction Lagrangians.)
- Consistent expansion of the two-pion production current in terms of the $\pi\pi N$ Faddeev ordering structure.
- Full implementation of gauge invariance order by order in terms of *Generalized Ward–Takahashi Identities* at all levels of the reaction dynamics.
  $\Longrightarrow$ Essential for the microscopic consistency of all reaction mechanisms.
- Valid for hadronic two- and three-point functions dressed by arbitrary internal mechanisms — even nonlinear ones.
- Resulting $\pi\pi$ production current can also be written in closed form accounting for full three-body dynamics.
- Extension beyond one-photon approximation straightforward.
- Translation to other two-meson production processes straightforward.



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Thank you!