Meson resonance spectroscopy, semi-local duality and Weinberg spectral sum rules

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Sketch of my talk

1. Preface
2. Theoretical framework:
   - $U(3)$ χPT at one-loop plus resonance exchanges: meson-meson scattering, form factors and spectral functions
   - N/D approach to implement the unitarization
3. Discussions
   - Fit quality and resonance spectroscopy
   - Semi-local duality at $N_C = 3$ and beyond
   - Weinberg-like spectral sum rules at $N_C = 3$ and beyond
4. Summary

Preface

Thriving studies on the scalar spectroscopy, such as $\sigma, \kappa, f_0(980), a_0(980), \ldots$, emerge recently:

- Roy equation: [Caprini, Colangelo, Leutwyler, PRL'06] [Kaminski, Pelaez, et al., PRD '08 '11] [Descotes-Genon, Moussallam, EPJC '06]
- PKU parametrization: [Zheng et al, NPA '04] [Zhou et al., JHEP '05]
- Bethe-Salpeter Equation: [Nieves, Ruiz Arriola, NPA'00] [Nieves, Pich, Ruiz Arriola, PRD'11]
- Inverse Amplitude Method: [Pelaez, Rios, PRL'06]
- N/D approach: [Oller, Oset, PRD '99] [Albaladejo, Oller, PRL '08] [Guo, Oller, PRD '11]
- ......

Remind: the above approaches are based on the analyses of meson-meson scattering.

Scalar resonances in decay and production processes:

- σ and κ in J/Ψ decays: [BES, PLB'04 '06 '07 '11]
- σ and κ in D decays: [E791, PRL'02]
- κ in photoproduction of $K^*\Sigma$: [Niiyama's talk]
- Scalars in η and η' decays: [Escribano's talk]
- Heavier members in the f_0 scalar family @ BESIII: [Yanping Huang and Yutie Liang's talks]
- σ and $f_0(980)$ in ISR production @ BaBar: [Solodov's talk]

In this talk, we focus more on the theoretical considerations:

- $SS - SS$, $SS - PP$, $PP - PP$ Weinberg sum rules:
  Interplay between Scalar and Pseudoscalar resonances
- Average (or Semi-local) Duality in meson-meson scattering:
  Interplay between Scalar and Vector resonances
- Classification according to large N_C:
  Are they the standard $\bar{q}q$ resonance with a constant mass and width decreasing as $1/N_C$ or something else?

Theoretical Framework :

$U(3)$ χPT and its unitarization

$U(3)$ χPT v.s. $SU(3)$ χPT

Dynamical degrees of freedom

$SU(3)$ χPT: π, K, η_8
$U(3)$ χPT: π, K, η_8, η_1 (massive state, caused by QCD $U_A(1)$ anomaly)

Advantages:

- At large N_C: $U(3)$ χPT contains all the relevant degrees of freedom of QCD at low energy, since η_1 becomes the ninth pseudo-Goldstone boson at large N_C. [Witten, NPB'79]
- In the physical case: $U(3)$ χPT includes both the physical η and η' mesons, while the $SU(3)$ version only explicitly includes the pure octet η_8.

Leading order Lagrangian:

$$\mathcal{L}^{(0)} = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \frac{F^2}{3} M_0^2 \ln^2 \det u \,,$$

[Witten, PRL'80] [Di Vecchia & Veneziano, NPB'80] [Rosenzweig, Schechter & Trahern, PRD'80]

Resonance saturation of the low energy constants is assumed:

$$\mathcal{L}_S = c_d \langle S_8 u_\mu u^\mu \rangle + c_m \langle S_8 \chi_+ \rangle + \widetilde{c}_d S_1 \langle u_\mu u^\mu \rangle + \widetilde{c}_m S_1 \langle \chi_+ \rangle$$

$$\mathcal{L}_V = \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle \,,$$

$$\mathcal{L}_P = i d_m \langle P_8 \chi_- \rangle + i \widetilde{d}_m P_1 \langle \chi_- \rangle \,.$$

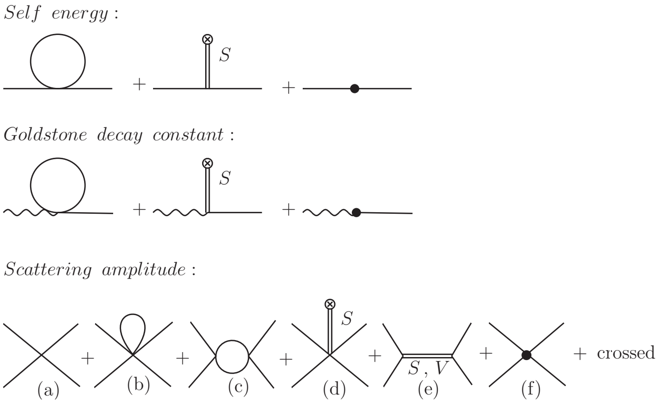
[Ecker, Gasser, Pich, de Rafael, NPB'89]

Another two local operators are also considered:

$$\frac{\delta L_8}{2} \langle \chi_+ \chi_+ + \chi_- \chi_- \rangle \,, \quad -\Lambda_2 \frac{F^2}{12} \langle U^+ \chi - \chi^+ U \rangle \ln \det u^2$$

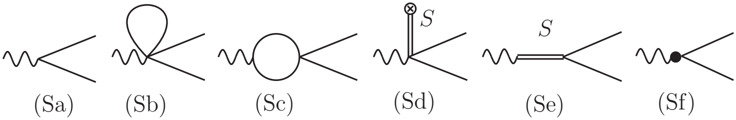
[Guo, Oller, PRD'11] [Guo, Oller, Ruiz de Elvira, PLB'12]

Perturbative calculations

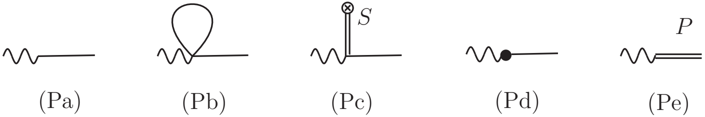


[Guo, Oller, PRD'11]
http://www.um.es/oller/u3FullAmp16.nb

Scalar form factor :



Pseudoscalar form factor :



[Guo, Oller, Ruiz de Elvira, PLB'12]

Unitarization: to extend the applicable energy region of perturbative results

The unitarized scattering amplitudes and form factors are constructed using a simplified N/D approach [Oller, Oset, PRD'99], [Meißner, Oller, NPA'01]

$$T^{IJ}(s) = \left[1 + N^{IJ}(s)\, g^{IJ}(s)\right]^{-1} N^{IJ}(s)\,, \tag{1}$$

$$F^{I}(s) = \left[1 + N^{IJ}(s) g^{IJ}(s)\right]^{-1} R^{I}(s)\,, \tag{2}$$

where $N^{IJ}(s)$ only contains the crossed channel cuts, $R^I(s)$ is real and $g^{IJ}(s)$ only includes the right hand cuts required by unitarity:

$$N^{IJ}(s) = T^{IJ}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}} + T^{IJ}(s)^{(2)}\, g^{IJ}(s)\, T^{IJ}(s)^{(2)}\,, \tag{3}$$

$$R^{I}(s) = F^{I}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}} + N^{IJ}(s)^{(2)}\, g^{IJ}(s)\, F^{I}(s)^{(2)}\,. \tag{4}$$

$$\begin{aligned} 16\pi^2 g^{IJ}(s) &= a_{SL}(\mu) + \log\frac{m_B^2}{\mu^2} - x_+ \log\frac{x_+ - 1}{x_+} - x_- \log\frac{x_- - 1}{x_-}\,, \qquad (5) \\ x_\pm &= \frac{s + m_A^2 - m_B^2}{2s} \pm \frac{1}{-2s}\sqrt{-4s(m_A^2 - i0^+) + (s + m_A^2 - m_B^2)^2}\,. \end{aligned}$$

Relevant channels considered in our work

- IJ=00: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$

  $T^{00}(s)$, $N^{00}(s)$ and $g^{00}(s)$ are 5×5 matrices.
  $R^0(s)$ is a column vector with 5 rows.

- IJ=$\frac{1}{2}$ 0 or $\frac{1}{2}$ 1: $K\pi$, $K\eta$, $K\eta'$

- IJ=10: $\pi\eta$, $K\bar{K}$, $\pi\eta'$

- IJ=11: $\pi\pi$, $K\bar{K}$

- IJ=20: $\pi\pi$

- IJ=$\frac{3}{2}$ 0: $K\pi$

Spectral functions and Weinberg sum rules

The scalar spectral function or the imaginary part of the scalar two-point correlator can be calculated through

$$\mathrm{Im}\Pi_{S^a}(s) = \sum_i \rho_i(s) \left|F_i^a(s)\right|^2 \theta(s - s_i^{\mathrm{th}}), \quad (6)$$

$$F_i^a(s) = \frac{1}{B}\langle 0 | \bar{q}\lambda_a q \, | (PQ)_i \rangle, \quad (7)$$

$$\rho_i(s) = \frac{\sqrt{[s-(m_A+m_B)^2][s-(m_A-m_B)^2]}}{16\pi s}, \quad (8)$$

with λ_a $(a = 1, 2, ..., 8)$ the Gell-Mann matrix and $\lambda_0 = \sqrt{2/3}\, I_{3\times 3}$.

We consider the strangeness conserving cases: $a = 0, 8, 3$.

Strangeness changing cases: [Jamin, Oller, Pich, NPB'02]

The pseudoscalar spectral function is calculated through

$$\mathrm{Im}\,\Pi_{P^a}(s) = \sum_j \pi\,\delta(s-m_{P_j}^2)\,|H_j^a(s)|^2\,, \quad (9)$$

$$H_j^a(s) = \frac{1}{B}\langle\, 0|\, i\,\bar{q}\gamma_5\lambda_a q\,|\,(P)_j\,\rangle\,. \quad (10)$$

Scalar and Pseudoscalar types of Weinberg sum rules:
[Gasser, Leutwyler, NPB'85] [Mousllam, EPJC'00]
[Golterman, Peris, PRD'00] [Bijnens, Gamiz, Prades, JHEP'01]

$$\int_0^{s_0}\left[\mathrm{Im}\,\Pi_R(s)-\mathrm{Im}\,\Pi_{R'}(s)\right]ds+\int_{s_0}^{\infty}\left[\mathrm{Im}\,\Pi_R(s)-\mathrm{Im}\,\Pi_{R'}(s)\right]ds=0\,, \quad (11)$$

with R and $R' = S^{a=0,8,3}$ or $P^{a=0,8,3}$.

We know, with the OPE results at $\mathcal{O}(\alpha_s)$ with demension 5 operators, that the second integral in the above equation vanishes in the chiral limit. [Jamin, Munz, ZPC'93]

Semi-local duality: Regge theory and hadronic degrees of freedom

In $\pi\pi$ scattering, the relations between the t- and s-channel amplitudes with definite isospin numbers can be deduced from crossing symmetry

$$T_{\rm t}^{(0)}(s,t) = \frac{1}{3}T_{\rm s}^{(0)}(s,t) + T_{\rm s}^{(1)}(s,t) + \frac{5}{3}T_{\rm s}^{(2)}(s,t)\,, \tag{12}$$

$$T_{\rm t}^{(1)}(s,t) = \frac{1}{3}T_{\rm s}^{(0)}(s,t) + \frac{1}{2}T_{\rm s}^{(1)}(s,t) - \frac{5}{6}T_{\rm s}^{(2)}(s,t)\,, \tag{13}$$

$$T_{\rm t}^{(2)}(s,t) = \frac{1}{3}T_{\rm s}^{(0)}(s,t) - \frac{1}{2}T_{\rm s}^{(1)}(s,t) + \frac{1}{6}T_{\rm s}^{(2)}(s,t)\,. \tag{14}$$

A useful quantity to measure the semi-local (average) duality is [Pelaez, Pennington, Ruiz de Elvira and Wilson, PRD'11]

$$F_n^{I=2,\,I'=1} = \frac{\int_{\nu_{\rm threhold}}^{\nu_{\rm max}} \nu^{-n}\,{\rm Im}\,T_{\rm t}^{(2)}(\nu,t)}{\int_{\nu_{\rm threhold}}^{\nu_{\rm max}} \nu^{-n}\,{\rm Im}\,T_{\rm t}^{(1)}(\nu,t)}\,, \tag{15}$$

$${\rm Im}\,T_{\rm s}^{(I)}(\nu,t) = \sum_J (2J+1)\,{\rm Im}\,T^{IJ}(s)\,P_J(z_s)\,, \tag{16}$$

with $\nu = \frac{s-u}{2}$, $z_s = 1 + 2t/(s-4m_\pi^2)$ and $P_J(z_s)$ the Legendre polynomials.

Discussions

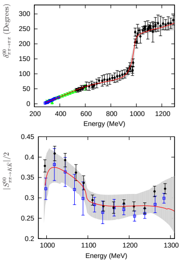


Figure: S-wave: I=0

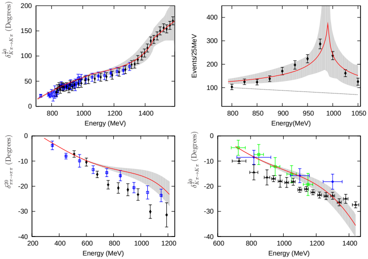


Figure: S-wave: I=$\frac{1}{2}$, 1, 2, $\frac{3}{2}$

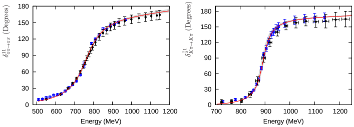


Figure: P-wave: I=1, $\frac{1}{2}$

See [Guo, Oller, PRD'11] [Guo, Oller, Ruiz de Elvira, PLB'12] for details.

Resonance contents in our study

R	M (MeV)	$\Gamma/2$ (MeV)	$\lvert\text{Residues}\rvert^{1/2}$ (GeV)	Ratios
$f_0(600)$ or σ	442^{+4}_{-4}	246^{+7}_{-5}	$3.02^{+0.03}_{-0.04}\ (\pi\pi)$	$0.50^{+0.04}_{-0.08}\ (K\bar{K}/\pi\pi)$ $0.17^{+0.09}_{-0.09}\ (\eta\eta/\pi\pi)$ $0.33^{+0.06}_{-0.10}(\eta\eta'/\pi\pi)$ $0.11^{+0.05}_{-0.06}(\eta'\eta'/\pi\pi)$
$f_0(980)$	978^{+17}_{-11}	29^{+9}_{-11}	$1.8^{+0.2}_{-0.3}(\pi\pi)$	$2.6^{+0.2}_{-0.3}(K\bar{K}/\pi\pi)$ $1.6^{+0.4}_{-0.2}(\eta\eta/\pi\pi)$ $1.0^{+0.3}_{-0.2}(\eta\eta'/\pi\pi)$ $0.7^{+0.2}_{-0.3}(\eta'\eta'/\pi\pi)$
$f_0(1370)$	1360^{+80}_{-60}	170^{+55}_{-55}	$3.2^{+0.6}_{-0.5}(\pi\pi)$	$1.0^{+0.7}_{-0.3}(K\bar{K}/\pi\pi)$ $1.2^{+0.7}_{-0.3}(\eta\eta/\pi\pi)$ $1.5^{+0.4}_{-0.5}(\eta\eta'/\pi\pi)$ $0.7^{+0.2}_{-0.3}(\eta'\eta'/\pi\pi)$
$K_0^*(800)$	643^{+75}_{-30}	303^{+25}_{-75}	$4.8^{+0.5}_{-1.0}(K\pi)$	$0.9^{+0.2}_{-0.3}(K\eta/K\pi)$ $0.7^{+0.2}_{-0.3}(K\eta'/K\pi)$
$K_0^*(1430)$	1482^{+55}_{-110}	132^{+40}_{-90}	$4.4^{+0.2}_{-1.1}(K\pi)$	$0.3^{+0.3}_{-0.3}\ (K\eta/K\pi)$ $1.2^{+0.2}_{-0.2}(K\eta'/K\pi)$
$a_0(980)$	1007^{+75}_{-10}	22^{+90}_{-10}	$2.4^{+3.2}_{-0.4}(\pi\eta)$	$1.9^{+0.2}_{-0.5}\ (K\bar{K}/\pi\eta)$ $0.03^{+0.10}_{-0.03}(\pi\eta'/\pi\eta)$
$a_0(1450)$	1459^{+70}_{-95}	174^{+110}_{-100}	$4.5^{+0.6}_{-1.7}(\pi\eta)$	$0.4^{+1.2}_{-0.2}(K\bar{K}/\pi\eta)$ $1.0^{+0.8}_{-0.3}(\pi\eta'/\pi\eta)$
$\rho(770)$	760^{+7}_{-5}	71^{+4}_{-5}	$2.4^{+0.1}_{-0.1}(\pi\pi)$	$0.64^{+0.01}_{-0.02}(K\bar{K}/\pi\pi)$
$K^*(892)$	892^{+5}_{-7}	25^{+2}_{-2}	$1.85^{+0.07}_{-0.07}(K\pi)$	$0.91^{+0.03}_{-0.02}(K\eta/K\pi)$ $0.41^{+0.07}_{-0.06}(K\eta'/K\pi)$
$\phi(1020)$	$1019.1^{+0.5}_{-0.6}$	$1.9^{+0.1}_{-0.1}$	$0.85^{+0.01}_{-0.02}(K\bar{K})$	

Weinberg-like sum rules

	W_{S^0}			W_{S^8}			W_{S^3}			W_{P^0}	W_{P^8}	W_{P^3}	$\overline{W}$	σ_W	$\sigma_W/\overline{W}$
Physical masses	8.6	9.0	9.6	7.4	7.5	7.7	7.0	7.2	7.4	8.9	11.3	10.1	9.0	1.5	0.16
$m_q = 0, M_0 \neq 0$	6.9	7.0	7.1	6.8	7.0	7.3	6.6	6.8	7.0	5.5	7.4	7.4	6.9	0.7	0.10
$m_q = 0, M_0 = 0$	8.4	8.8	9.3	8.1	8.8	9.3	7.8	8.4	8.7	6.1	8.4	8.4	8.1	1.0	0.12

Table: Three different values of s_0 are used: 2.5, 3.0, 3.5 GeV^2. W_i is given in GeV^2. We set a_{SL} to be equal for the $m_q = 0$ cases as required by $SU(3)$ symmetry. [Jido,Oller,Oset,Ramos,Meissner, NPA'03]

$$W_i = 16\pi \int_0^{s_0} \mathrm{Im}\,\Pi_i(s)\,ds\,, \quad i = S^8, S^0, S^3, P^0, P^8, P^3\,,$$

$$\overline{W} = \frac{\sum_{i=(S^8,S^0,S^3,P^0,P^8,P^3)} W_i}{3\times 6}\,,$$

$$\sigma_W^2 = \sum_{i=(S^8,S^0,S^3,P^0,P^8,P^3)} \frac{(W_i - \overline{W})^2}{17}\,.$$

Different strategies to extrapolate N_C

N_C scaling at Leading order:

$$\left\{c_{d,m}(N_C), G_V(N_C), d_m(N_C), g_T(N_C)\right\} = \left\{c_{d,m}(3), G_V(3), d_m(3), g_T(N_C)\right\} \times \sqrt{\frac{N_C}{3}} ,$$

$$\left\{M_R(N_C),\ a_{SL}(N_C),\ \delta_{L_8}(N_C)\right\} = \left\{M_R(3),\ a_{SL}(3),\ \delta_{L_8}(3)\right\},\ \left\{M_0^2(N_C),\ \Lambda_2(N_C)\right\} = \left\{M_0^2(3),\ \Lambda_2(3)\right\} \times \frac{3}{N_C} .$$

Sub-leading order N_C scaling (taking G_V as an example): $G_V(3)$, $G_V(\infty)$

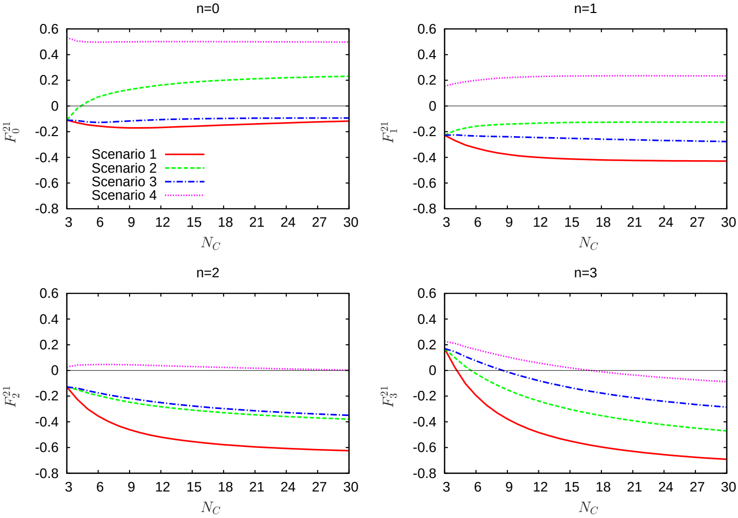
$$G_V(\infty) = \frac{F(\infty)}{3} ,$$ [Guo, Sanz Cillero, Zheng, JHEP′07], [Pich, Rosell, Sanz Cillero, JHEP′11],

$$G_V(N_C) = G_V(3)\sqrt{\frac{N_C}{3}} \times \left[1 + \frac{G_V(3) - G_V^{\rm Nor}(\infty)}{G_V(3)}\left(\frac{3}{N_C} - 1\right)\right] , \text{ with } G_V^{\rm Nor}(\infty) = G_V(\infty)\sqrt{\frac{3}{N_C}} .$$

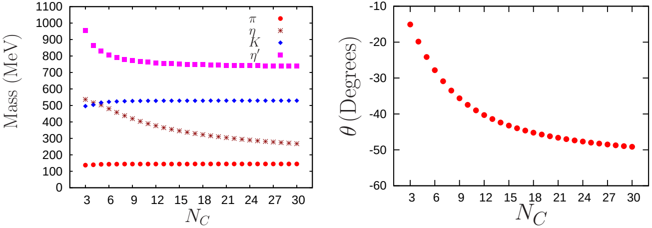
	G_V	M_ρ, M_{S_1}	D-wave
Scenario 1	LO	LO	NO
Scenario 2	LO+NLO	LO	NO
Scenario 3	LO+NLO	LO+NLO	NO
Scenario 4	LO+NLO	LO+NLO	YES

Tensor resonances (crucial to D-wave): [Ecker, Zauner, EPJC'07]

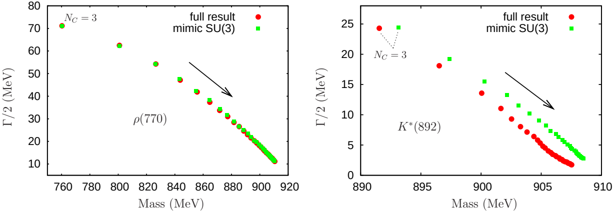
Semi-local duality at $N_C = 3$ and beyond



Pseudo-Goldstone masses and leading order η-η' mixing angle with varying N_C

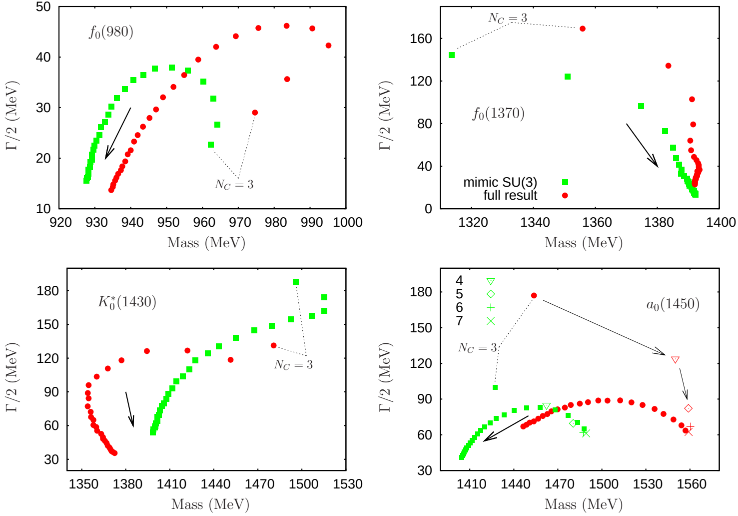


N_C trajectories of $\rho(770)$ and $K^*(892)$

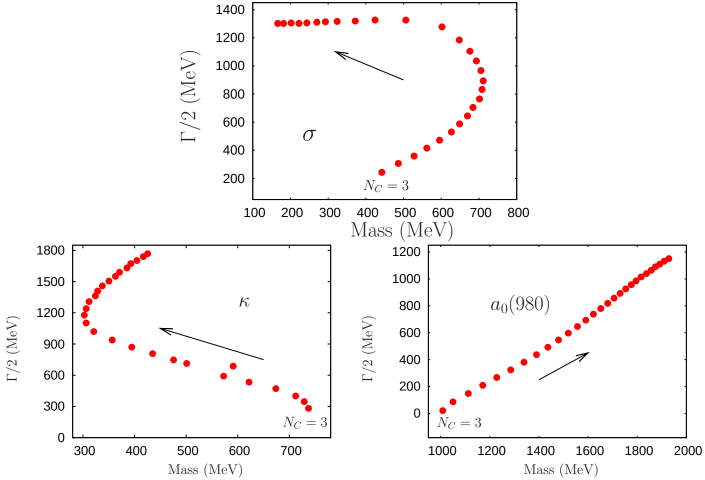


mimic SU(3): fix the π, K, η, η' masses and $\theta = 0$ when varying N_C

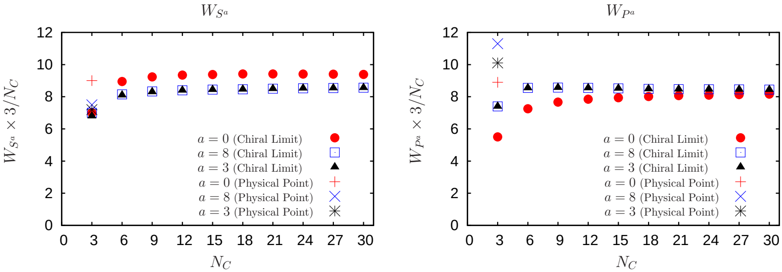
N_C trajectories of $f_0(980)$, $f_0(1370)$, $K_0^*(1430)$ and $a_0(1450)$



N_C trajectories of $f_0(600)$ (or σ), $K_0^*(800)$ (or κ) and $a_0(980)$



Weinberg sum rules with varying N_C: $W_i \times \frac{3}{N_C}$



$$W_i = 16\pi \int_0^{s_0} \mathrm{Im}\, \Pi_i(s)\, ds\,, \quad i = S^8, S^0, S^3, P^0, P^8, P^3$$

Summary

- The one-loop calculations of all meson-meson scattering amplitudes, scalar and pseudoscalar form-factors within $U(3)$ χPT plus tree level exchanges of resonances, which are also unitarized through the N/D approach, have been worked out.
- Resonance pole positions at $N_C \geqslant 3$ and their coupling strengths to the pseudo-Goldstone boson pairs are discussed: $f_0(600)$, $a_0(980)$, $K_0^*(800)$, $f_0(980)$, $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$, $\rho(770)$, $K^*(892)$ and $\phi(1020)$.
- Studies of semi-local duality and Weinberg-like sum rules for $N_C \geqslant 3$ pose strong constraints on the spectra and the evolution of N_C. This shows a clear support about the emerging pictures for the scalar dynamics proposed by us.

Dziekuje !