MESON ELECTROMAGNETIC FORM FACTORS

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INTRODUCTION

all **hadrons** - including also **conventional mesons** - $(q\bar{q})$ bound states - are compound of constituent quarks

$\Rightarrow$ in EM interactions manifest **non-point-like EM structure**

- completely described by **scalar functions** $F_i(t)$ (EM FFs), t - squared momentum transferred by the **virtual photon** γ^*

- if $M\gamma^* \to M \Rightarrow F_i(t)$ **elastic FFs**
- if $M\gamma^* \to A'$ or $\gamma \Rightarrow F_i(t)$ **transition FFs**

According to SU(3) classification there are:

scalar mesons 0^+:

$f_0(600), K_0^*(800), f_0(980), a_0(980)$ - the most complete multiplet, however **not necessarily** $(q\bar{q})$ bound states

or $f_0(1370), K_0^*(1430), a_0(1450), f_0(1500)$ - **regular nonet**

pseudoscalar mesons 0^-:

$\pi, K, \bar{K}, \eta, \eta'$

vector mesons 1^-:

$\rho(770), \omega(782), K^*(892), \bar{K}^*(892), \phi(1020)$

tensor mesons 2^+:

$f_2(1270), a_2(1320), f_2'(1525), f_2(1950), f_2(2010), f_2(2300), f_2(2340)$

all **bound states of light quarks - u, d, s**.

Note:

For a **description of the meson EM structure** we use $Unitary\&Analytic$ ($U\&A$) model

- to be consistent **unification of pole and continuum contributions**

- it depends on **effective** t_{in} **thresholds** - free parameters

- it depends on the **coupling constant ratios** (f_{MMV}/f_V) - also free parameters

In order **to determine free parameters of the** $U\&A$ **model** - one needs its **comparison with some exp. data**.

THEREFORE - farther our **attention concentrated only to the nonet of pseudoscalar mesons**:

$\pi^-, \pi^0, \pi^+, K^-, K^0, \bar{K}^0, K^+, \eta, \eta'$

for which **abundant exp. information exists**.

<u>FIRST GENERALLY</u>

Since pseudoscalar mesons M have spin 0^-

$\Rightarrow$ **only one FF** $F_i(t)$ **- describes the meson EM structure completely**, to be defined by the parametrization of the matrix element of the EM current

$$< p_2|J_\mu(0)|p_1 >= eF_M(t)(p_1 + p_2)_\mu \qquad (1)$$

Making use of the transformation:

$J_\mu(x)$ and also the one-particle state vectors $< p_2|$ and $|p_1 > |$

with regard to **all three discrete** C, P, T **transformations** simultaneously

$\Rightarrow F_M(t) = -F_{\bar{M}}(t)$ e.g. $F_{\pi^+}(t) = -F_{\pi^-}(t); F_{K^+}(t) = -F_{K^-}(t); F_{K^0}(t) = -F_{\bar{K}^0}(t)$

From the latter it follows for **true neutral pseudoscalar mesons**: π^0, η, η'

$$F_{\pi^0}(t) = F_\eta(t) = F_{\eta'}(t) \equiv 0 \qquad (2)$$

for **all values** from the interval $-\infty < t < +\infty$.

U&A MODEL OF MESON EM FFs.

General belief - all EM FFs are **analytic in t-plane, besides (branch points) i.e. cuts on the positive real axis**.

U&A model - consistent unification (see Fig.1) of:

- finite number of complex conjugate pairs of poles - reflect an experimental fact of a creation of **unstable neutral vector-meson resonances** with photonic quantum numbers in e^+e^- annihilation processes into hadrons.
- two cut approximation of the analytic properties on

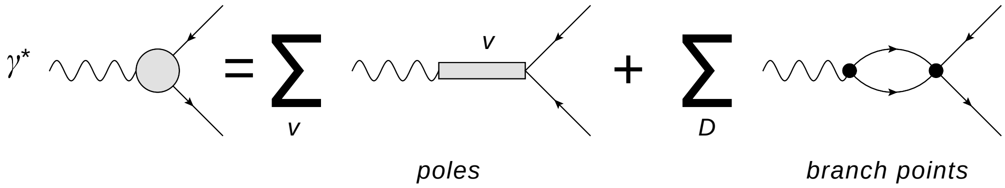

Figure 1: Contributing diagrams to EM FF.

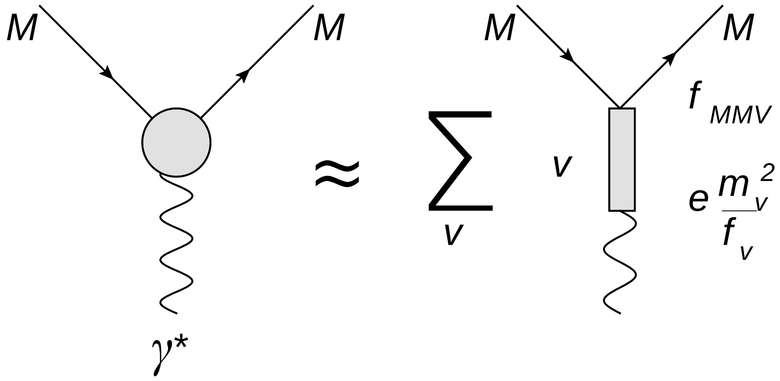

Figure 2: Standard VMD model representation of EM FFs.

the first (called physical) sheet of the Riemann surface, by means of which **just continua contributions** are taken into account.

Experimental fact of the creation of $\rho, \omega, \phi, \rho', \omega', \phi', etc.$ in $e^+e^- \rightarrow hadrons$ in the **first approximation** can be taken into account by the standard VMD model with stable vector mesons (see Fig.2)

$$F_M(t) = \sum_V \frac{m_V^2}{m_V^2 - t}(f_{MMV}/f_V), \tag{3}$$

which automatically **respects the asymptotic behavior of pseudoscalar meson EM FFs**

$$F_M(t)_{|t|\to\infty} \sim t^{-1} \tag{4}$$

as **predicted by the constituent quark model** of hadrons.

Afterwards the VMD model is **unitarized** by an incorporation of two-cut approximation of the analytic properties of EM FFs with the help of the **non-linear transformation**

$$t = t_0 + \frac{4(t_{in} - t_0)}{[1/W(t) - W(t)]^2}, \tag{5}$$

where:

- t_0 - the square-root branch point corresponding to the **lowest possible threshold**

- t_{in} - an **effective square-root branch point** simulating contributions of all higher relevant thresholds given by the unitarity condition

$$W(t) = i\frac{\sqrt{(\frac{t_{in}-t_0}{t_0})^{1/2} + (\frac{t-t_0}{t_0})^{1/2}} - \sqrt{(\frac{t_{in}-t_0}{t_0})^{1/2} - (\frac{t-t_0}{t_0})^{1/2}}}{\sqrt{(\frac{t_{in}-t_0}{t_0})^{1/2} + (\frac{t-t_0}{t_0})^{1/2}} + \sqrt{(\frac{t_{in}-t_0}{t_0})^{1/2} - (\frac{t-t_0}{t_0})^{1/2}}} \tag{6}$$

is the **conformal mapping** of the four-sheeted Riemann surface into one W-plane, to be just **inverse** to the previous **non-linear transformation**.

As a result - every term $\frac{m_V^2}{m_V^2-t}$ in VMD representation is **factorized**

$$\frac{m_r^2}{m_r^2 - t} = (\frac{1 - W^2}{1 - W_N^2})^2 \times$$
$$\times \frac{(W_N - W_{r0})(W_N + W_{r0})(W_N - 1/W_{r0})(W_N + 1/W_{r0})}{(W - W_{r0})(W + W_{r0})(W - 1/W_{r0})(W + 1/W_{r0})}$$

into:

- **asymptotic term** $(\frac{1-W^2}{1-W_N^2})^2$ completely determining the asymptotic behavior $\sim t^{-1}$ of EM FF

- and into a **resonant term** $\frac{(W_N-W_{r0})(W_N+W_{r0})(W_N-1/W_{r0})(W_N+1/W_{r0})}{(W-W_{r0})(W+W_{r0})(W-1/W_{r0})(W+1/W_{r0})}$, for $\mid t \mid \to \infty$ turning out to **real constant**.

The subindex "0" means that **still stable vector-mesons** are considered.

Generally one can prove

- if $m_r^2 - \Gamma_r^2/4 < t_{in} \Rightarrow W_{r0} = -W_{r0}^*$

- if $m_r^2 - \Gamma_r^2/4 > t_{in} \Rightarrow W_{r0} = 1/W_{r0}^*$

which lead

- in the **first case** to the expression

$$\frac{m_r^2}{m_r^2 - t} = \left(\frac{1 - W^2}{1 - W_N^2}\right)^2 \times$$
$$\times \frac{(W_N - W_{r0})(W_N - W_{r0}^*)(W_N - 1/W_{r0})(W_N - 1/W_{r0}^*)}{(W - W_{r0})(W - W_{r0}^*)(W - 1/W_{r0})(W - 1/W_{r0}^*)}$$

- and in the **second case** to the following expression

$$\frac{m_r^2}{m_r^2 - t} = \left(\frac{1 - W^2}{1 - W_N^2}\right)^2 \times$$
$$\times \frac{(W_N - W_{r0})(W_N - W_{r0}^*)(W_N + W_{r0})(W_N + W_{r0}^*)}{(W - W_{r0})(W - W_{r0}^*)(W + W_{r0})(W + W_{r0}^*)}$$

Finally, introducing the **non-zero widths of resonances** by a formal substitution

$m_r^2 \to (m_r - \Gamma_r/2)^2$

i.e. simply one has **to rid of** 0 **in subindices**, one gets:

- when the **resonance is below** t_{in}

$$\frac{m_r^2}{m_r^2 - t} \to \left(\frac{1-W^2}{1-W_N^2}\right)^2 \times$$
$$\times \frac{(W_N - W_r)(W_N - W_r^*)(W_N - 1/W_r)(W_N - 1/W_r^*)}{(W - W_r)(W - W_r^*)(W - 1/W_r)(W - 1/W_r^*)}$$

- and when the **resonance is beyond** t_{in}

$$\frac{m_r^2}{m_r^2 - t} \to \left(\frac{1-W^2}{1-W_N^2}\right)^2 \times$$
$$\times \frac{(W_N - W_r)(W_N - W_r^*)(W_N + W_r)(W_N + W_r^*)}{(W - W_r)(W - W_r^*)(W + W_r)(W + W_r^*)}$$

where **no more equality** can be used in these relations!

Consequently, the $U\&A$ **model of meson EM structure** takes the form

$$F_P[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \times$$
$$\times \left\{ \sum_i \frac{(W_N - W_i)(W_N - W_i^*)(W_N - 1/W_i)(W_N - 1/W_i^*)}{(W - W_i)(W - W_i^*)(W - 1/W_i)(W - 1/W_i^*)} (f_{iPP}/f_i) + \right.$$
$$\left. + \sum_j \frac{(W_N - W_j)(W_N - W_j^*)(W_N + W_j)(W_N + W_j^*)}{(W - W_j)(W - W_j^*)(W + W_j)(W + W_j^*)} (f_{jPP}/f_j) \right\}$$

which is **analytic in the whole complex** t**-plane besides two cuts on the positive real axis**.

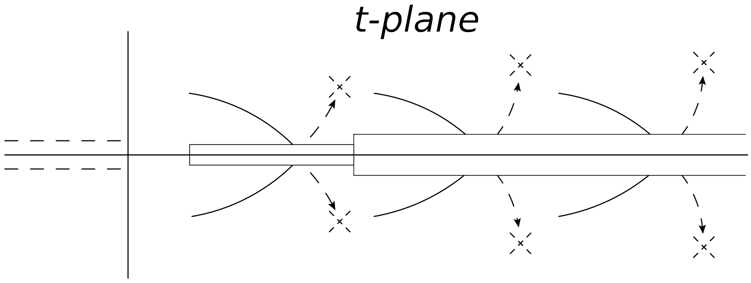

Figure 3: Analytic properties of charged pion EM FF.

NOW ONE BY ONE

$\pi^{\pm}$: The **analytic properties** of $F_\pi(t)$ are in $Fig.3$.

In comparison with expression $F_P[W(t)]$ - there is additional **left-hand cut on the II.Riemann sheet**.

Explanation

Starting from the **elastic unitarity condition** for $F_\pi(t)$

$\frac{1}{2i}\{F_\pi(t+i\varepsilon) - F^*_\pi(t+i\varepsilon)\} = A^{1*}_1(t+i\varepsilon).F_\pi(t+i\varepsilon)$

one can derive the expression for **pion EM FF on the II.Riemann sheet**

$[F_\pi(t)]^{II.} = \frac{F_\pi(t)}{1+2iA^1_1(t)}$

where $A^1_1(t)$ is the P**-wave isovector** $\pi\pi$**-scattering**

amplitude, the analytic properties of which consist

- of **right-hand unitary cut** $4m_\pi^2 < t < \infty$
- and of **left-hand dynamical cut** $-\infty < t < 0$.

<u>**NOTE a)**</u>

The contribution of any cut in Pad'e approximation can be **represented by alternating zeros and poles** on the place of the cut

$\Rightarrow$ we do it in $U\&A$ model of $F_\pi[W(t)]$.

<u>**NOTE b)**</u>

From the same **elastic unitarity condition** and $\delta_1^1(t)_{q\to 0} \sim a_1^1 q^3$ one gets the **threshold behavior** of $ImF_\pi(t)$ to be transformed into **3 threshold conditions**

$ImF_\pi(t)_{q=0} = \frac{dImF_\pi(t)}{dq}_{q=0} = \frac{d^2ImF_\pi(t)}{dq}_{q=0} \equiv 0$, which reduce a number of $(f_{v\pi\pi}/f_v)$ as free parameters.

Taking into account both these **Notes** and also the **normalization** explicitly one gets the $U\&A$ **pion EM FF model**

$$F_\pi[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \frac{(W-W_z)(W_N-W_p)}{(W_N-W_z)(W-W_p)} \times$$
$$\times\Big\{\frac{(W_N-W_\rho)(W_N-W_\rho^*)(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-W_\rho)(W-W_\rho^*)(W-1/W_\rho)(W-1/W_\rho^*)}(f_{\rho\pi\pi}/f_\rho)+$$
$$+\sum_{v=\rho',\rho''}\frac{(W_N-W_v)(W_N-W_v^*)(W_N+W_v)(W_N+W_v^*)}{(W-W_v)(W-W_v^*)(W+W_v)(W+W_v^*)}(f_{v\pi\pi}/f_v)\Big\}$$

with

$$(f_{\rho'\pi\pi}/f_{\rho'}) = \frac{\frac{N_{\rho''}}{|W_{\rho''}|^4}}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}} -$$
$$-\frac{\frac{N_{\rho''}}{|W_{\rho''}|^4}+(1+2\frac{W_z.W_p}{W_z-W_p}.Re[W_\rho(1+|W_\rho|^{-2})])N_\rho}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}}(f_{\rho\pi\pi}/f_\rho)$$

$$(f_{\rho''\pi\pi}/f_{\rho''}) = 1-\frac{\frac{N_{\rho''}}{|W_{\rho''}|^4}}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}}+$$
$$+\Big[\frac{\frac{N_{\rho''}}{|W_{\rho''}|^4}+(1+2\frac{W_z.W_p}{W_z-W_p}.Re[W_\rho(1+|W_\rho|^{-2})])N_\rho}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}}-1\Big](f_{\rho\pi\pi}/f_\rho)$$

Due to the $\rho - \omega$ **interference effect** one has to carry out the **fit of existing data** by

$| F_\pi[W(t)] + R.e^{i\phi}\frac{m_\omega^2}{m_\omega^2 - t - im_\omega\Gamma_\omega} |$

with

$\phi = arctg\frac{m_\rho\Gamma_\rho}{m_\rho^2 - m_\omega^2}$.

A **description of existing data** in space-like and time-like regions simultaneously with parameters values

$t_{in} = (1.296 \pm 0.011) GeV^2 \quad R = 0.0123 \pm 0.0032$

$W_z = 0.3722 \pm 0.0008 \quad W_p = 0.5518 \pm 0.0003$

$m_\rho = (759.26 \pm 0.04) MeV \quad \Gamma_\rho = (141.90 \pm 0.13) MeV$

$m_{\rho'} = (1395.9 \pm 54.3) MeV \quad \Gamma_{\rho'} = (490.9 \pm 118.8) MeV$

$m_{\rho''} = (1711.5 \pm 63.6) MeV \quad \Gamma_{\rho''} = (369.5 \pm 112.7) MeV$

$(f_{\rho\pi\pi}/f_\rho) = 1.0063 \pm 0.0024 \quad \chi^2/ndf = 1.58$

is presented in $Fig.4$.

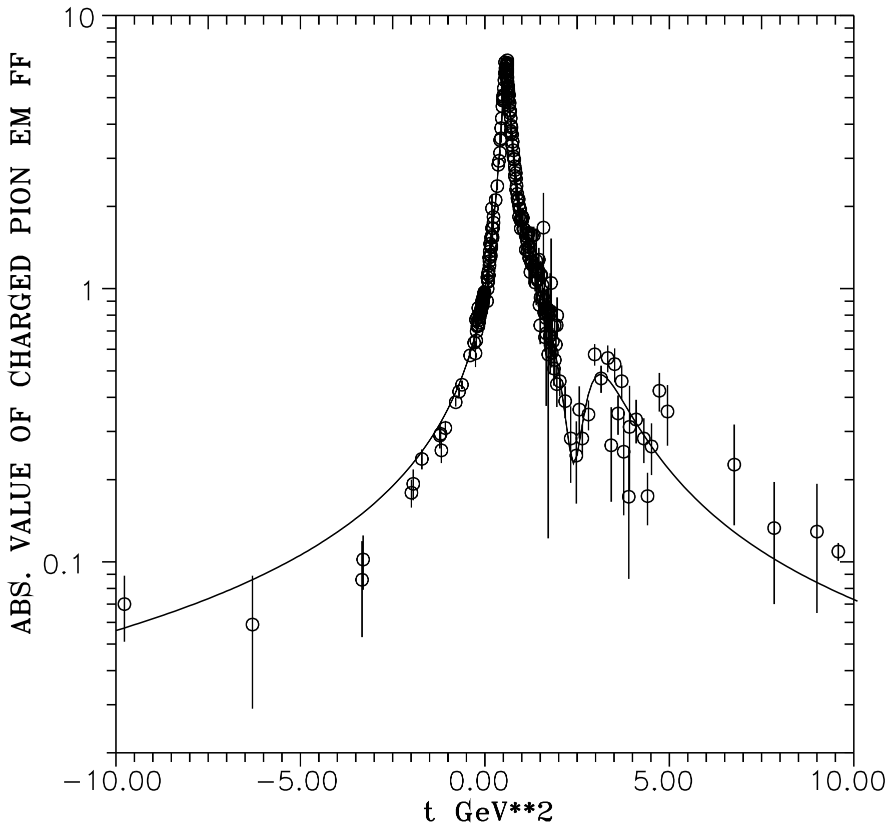

Figure 4: Prediction of pion EM FF behavior by $U\&A$ model.

$\underline{K^\pm, K^0}$:

The K^+ and K^0 belong to the same isomultiplet with $I = 1/2$

$\Rightarrow$ one can introduce, generally, the EM current of K, which splits into sum of **isotopic scalar** and **isotopic vector**.

The corresponding FFs suitable for a construction of the $U\&A$ models are

$$F_K^s(t) = \frac{1}{2}[F_{K^+}(t) + F_{K^0}(t)] \quad F_{K^+}(t) = F_K^s(t) + F_K^v(t)$$

$$F_K^v(t) = \frac{1}{2}[F_{K^+}(t) - F_{K^0}(t)] \quad F_{K^0}(t) = F_K^s(t) - F_K^v(t)$$

from where the normalizations

$$F_K^s(0) = F_K^v(0) = \frac{1}{2}; \quad F_{K^+}(0) = 1; \quad F_{K^0}(0) = 0;$$

follow.

The specific **6 - resonance** $(\rho, \omega, \phi, \rho', \phi', \rho'')$ $U\&A$ **model of the kaon EM structure** has the form

$$F_K^s[V(t)] = \left(\frac{1-V^2}{1-V_N^2}\right)^2 \Bigg[\frac{1}{2}\frac{(V_N-V_\omega)(V_N-V_\omega^*)(V_N-1/V_\omega)(V_N-1/V_\omega^*)}{(V-V_\omega)(V-V_\omega^*)(V-1/V_\omega)(V-1/V_\omega^*)} + \\ + \Bigg\{\frac{(V_N-V_\phi)(V_N-V_\phi^*)(V_N-1/V_\phi)(V_N-1/V_\phi^*)}{(V-V_\phi)(V-V_\phi^*)(V-1/V_\phi)(V-1/V_\phi^*)} - \\ - \frac{(V_N-V_\omega)(V_N-V_\omega^*)(V_N-1/V_\omega)(V_N-1/V_\omega^*)}{(V-V_\omega)(V-V_\omega^*)(V-1/V_\omega)(V-1/V_\omega^*)}\Bigg\}(f_{\phi KK}/f_\phi) + \\ + \Bigg\{\frac{(V_N-V_{\phi'})(V_N-V_{\phi'}^*)(V_N-1/V_{\phi'})(V_N-1/V_{\phi'}^*)}{(V-V_{\phi'})(V-V_{\phi'}^*)(V-1/V_{\phi'})(V-1/V_{\phi'}^*)} - \\ - \frac{(V_N-V_\omega)(V_N-V_\omega^*)(V_N-1/V_\omega)(V_N-1/V_\omega^*)}{(V-V_\omega)(V-V_\omega^*)(V-1/V_\omega)(V-1/V_\omega^*)}\Bigg\}(f_{\phi' KK}/f_{\phi'})\Bigg] \tag{7}$$

$$F_K^v[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \Bigg[\frac{1}{2}\frac{(W_N-W_\rho)(W_N-W_\rho^*)(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-W_\rho)(W-W_\rho^*)(W-1/W_\rho)(W-1/W_\rho^*)} + \\ + \Bigg\{\frac{(W_N-W_{\rho'})(W_N-W_{\rho'}^*)(W_N-1/W_{\rho'})(W_N-1/W_{\rho'}^*)}{(W-W_{\rho'})(W-W_{\rho'}^*)(W-1/W_{\rho'})(W-1/W_{\rho'}^*)} - \\ - \frac{(W_N-W_\rho)(W_N-W_\rho^*)(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-W_\rho)(W-W_\rho^*)(W-1/W_\rho)(W-1/W_\rho^*)}\Bigg\}(f_{\rho' KK}/f_{\rho'}) + \\ + \Bigg\{\frac{(W_N-W_{\rho''})(W_N-W_{\rho''}^*)(W_N-1/W_{\rho''})(W_N-1/W_{\rho''}^*)}{(W-W_{\rho''})(W-W_{\rho''}^*)(W-1/W_{\rho''})(W-1/W_{\rho''}^*)} - \\ - \frac{(W_N-W_\rho)(W_N-W_\rho^*)(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-W_\rho)(W-W_\rho^*)(W-1/W_\rho)(W-1/W_\rho^*)}\Bigg\}(f_{\rho'' KK}/f_{\rho''})\Bigg] \tag{8}$$

Both functions are **analytic in the whole com-**

plex t-planes besides two cuts on the positive real axis, generated by $t_0^s = 9m_\pi^2$ and t_{in}^s in $F_K^s[V(t)]$ and by $t_0^v = 4m_\pi^2$ and t_{in} in $F_K^v[W(t)]$.

- they are **real on the whole real negative axis up to positive values** $t_0^s = 9m_\pi^2$ **and** $t_0^v = 4m_\pi^2$, respectively

- automatically **normalized** to 1/2 with $ImF_K^s(t) \neq 0$ and $ImF_K^v(t) \neq 0$, starting from $9m_\pi^2$ and $4m_\pi^2$, respectively, as it is **required by the unitarity conditions**.

- they possess **complex conjugate pairs of poles** on unphysical sheets of the Riemann surface, corresponding to considered vector-mesons with quantum numbers of the photon.

A simultaneous **reproduction of all existing kaon EM FF data** by the $U\&A$ models is presented in $Fig.5$ and $Fig.6$

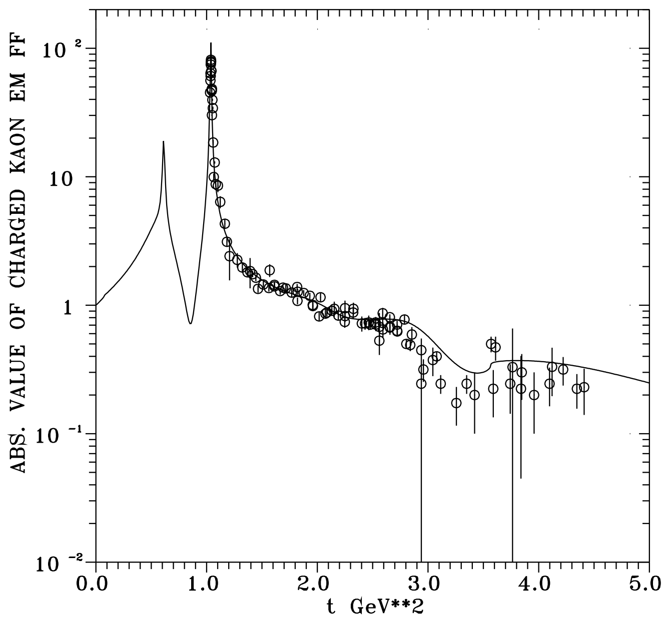

Figure 5: Prediction of charge kaon EM FF behavior by $U\&A$ model.

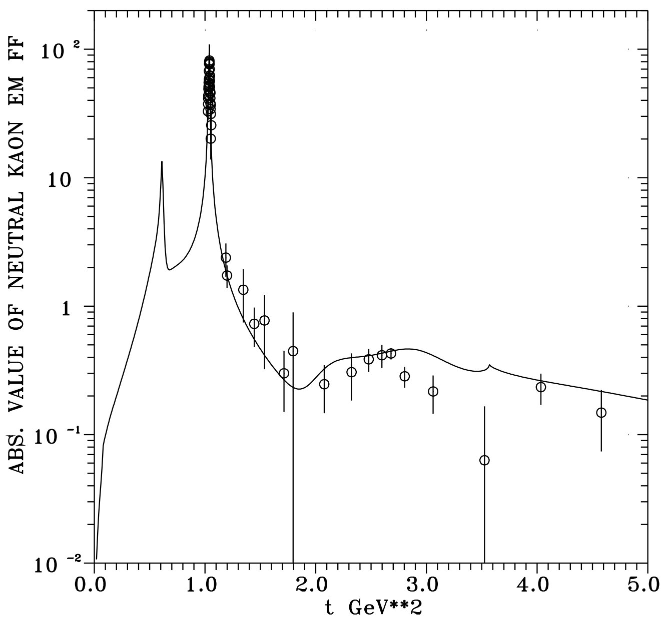

Figure 6: Prediction of neutral kaon EM FF behavior by $U\&A$ model.

and the following **values of free parameters of the model** have been determined - $m_\rho, \Gamma_\rho, m_\omega, \Gamma_\omega$ are **fixed at the TABLE values**.

$q^s_{in} = \sqrt{(t^s_{in} - 9)/9} = 2.2326[m_\pi]$; $q^v_{in} = \sqrt{(t^v_{in} - 4)/4} = 6.6721[m_\pi]$

$(f_{\omega KK}/f_\omega) = 0.14194 \quad (f_{\rho KK}/f_\rho) = 0.5615$

$m_\phi = 7.2815[m_\pi] \quad m_{\rho'} = 10.3940[m_\pi]$

$\Gamma_\phi = 0.03733[m_\pi] \quad \Gamma_{\rho'} = 1.6284[m_\pi]$

$(f_{\phi KK}/f_\phi) = 0.4002 \quad (f_{\rho' KK}/f_{\rho'}) = -.3262$

$m_{\phi'} = 11.8700[m_\pi] \quad m_{\rho''} = 13.5650[m_\pi]$

$\Gamma_{\phi'} = 1.3834[m_\pi] \quad \Gamma_{\rho''} = 3.3313[m_\pi]$

$(f_{\phi' KK}/f_{\phi'}) = -.04214 \quad (f_{\rho'' KK}/f_{\rho''}) = -.02888$

From the obtained results one observes that **the contribution of $\rho'''(2150)$ resonance to**
$e^+e^- \to K\bar{K}$ **processes**
is favored prior to the $\rho''(1700)$ one by existing data in the charge and neutral kaon EM FFs.

What about π^0, η, η':

They are **true neutral particles**

$\Rightarrow$ their **elastic EM FFs**

$F_{\pi^0}(t) = 0$

$F_\eta(t) = 0$

$F_{\eta'}(t) = 0$

i.e. these particles are **point-like** according to EM interactions !

However, one can define **nonzero single FF for each $\gamma^* \to \gamma P$ transition** by a parametrization of the matrix element of the EM current

$J_\mu^{EM} = 2/3\bar{u}\gamma_\mu u - 1/3\bar{\gamma}_\mu d - 1/3\bar{s}\gamma_\mu s$

$< P(p) \mid J_\mu^{EM} \mid 0 >= \varepsilon_{\mu\nu\alpha\beta} p^\nu \epsilon^\alpha k^\beta F_{\gamma P}(q^2)$

ϵ^α - the polarization vector of γ

$\varepsilon_{\mu\nu\alpha\beta}$ - antisymmetric tensor

The **transition FF** is related with total cross sections

$\sigma_{tot}(e^+e^- \to P\gamma) = \frac{\pi\alpha^2}{6}(1 - \frac{m_P^2}{t})^3 \mid F_{P\gamma}(t) \mid^2$

giving experimental data on $F_{\pi^0\gamma}(t)$, $F_{\eta\gamma}(t)$ and $F_{\eta'\gamma}(t)$ in $t > 0$ region.

A straightforward calculation of $F_{P\gamma}(t)$ in QCD **impossible !**

One has to construct sophisticated **phenomenological models**.

In a construction of the $U\&A$ model - it is again suitable to split $F_{P\gamma}(t)$ into two terms depending on the isotopic character of the photon

$F_{P\gamma}(t) = F_{P\gamma}^{I=0}(t) + F_{P\gamma}^{I=1}(t)$

$F_{P\gamma}^{I=0}(t)$ - saturated by **isoscalar vector-mesons** $\omega, \phi, \omega', \phi'$ etc.

$F_{P\gamma}^{I=1}(t)$ - saturated by **isovector vector-mesons** ρ, ρ', ρ'' etc.

QUESTION - how many vector-meson resonances have to be taken into account?

It is **prescribed by the existing data interval** on the corresponding FF in $t > 0$ region.

The data on $\pi^0\gamma$ transition FF allow to **consider all** 3 **ground state vector mesons:** $\rho(770), \omega(782), \phi(1020)$ and also $\omega'(1420)$ and $\rho'(1450)$, in order to construct automatically normalized $U\&A$ models.

NOTE:

With the aim of obtaining **comparable results**, the same number of resonances is considered also for η and η'.

Further:

- the **resonance parameters are fixed** at the TABLE values

- the normalization of FFs are

$F_{P\gamma}(0) = \frac{2}{\alpha m_P}\sqrt{\frac{\Gamma(P\to\gamma\gamma)}{\pi m_P}}$

where $\Gamma(P \to \gamma\gamma)$ are fixed at the **world averaged values** from TABLE

- the **analytic properties** of $F_{P\gamma}(t)$ - FFs are analytic in t - plane besides the cut from $t = m^2_{\pi^0}$ up to $+\infty$.

$\Rightarrow$ the $U\&A$ model of $F_{P\gamma}(t)$ takes the form

$$F^{I=0}_{P\gamma}[V(t)] = \left(\frac{1-V^2}{1-V_N^2}\right)^2 \times \tag{9}$$

$$\times\{\frac{1}{2}F_{P\gamma}(0)H(\omega') + [L(\omega) - H(\omega')]a_\omega + [H(\phi) - H(\omega')]a_\phi\}$$

$$F^{I=1}_{P\gamma}[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \times \tag{10}$$

$$\times\{\frac{1}{2}F_{P\gamma}(0)H(\rho') + [L(\rho) - H(\rho')]a_\rho\}$$

with

$L(\omega) = \frac{(V_N-V_\omega)(V_N-V_\omega^*)(V_N-1/V_\omega)(V_N-1/V_\omega^*)}{(V-V_\omega)(V-V_\omega^*)(V-1/V_\omega)(V-1/V_\omega^*)}$

$H(i) = \frac{(V_N-V_i)(V_N-V_i^*)(V_N+V_i)(V_N+V_i^*)}{(V-V_i)(V-V_i^*)(V+V_i)(V+V_i^*)}, \quad i = \phi, \omega'$

$$L(\rho) = \frac{(W_N - W_\rho)(W_N - W_\rho^*)(W_N - 1/W_\rho)(W_N - 1/W_\rho^*)}{(W - W_\rho)(W - W_\rho^*)(W - 1/W_\rho)(W - 1/W_\rho^*)}$$

$$H(\rho') = \frac{(W_N - W_{\rho'})(W_N - W_{\rho'}^*)(W_N + W_{\rho'})(W_N + W_{\rho'}^*)}{(W - W_{\rho'})(W - W_{\rho'}^*)(W + W_{\rho'})(W + W_{\rho'}^*)}$$

and the normalization points

$$V(0) = V_N, \quad W(0) = W_N.$$

The model depends on 5 **free parameters**

$t_{in}^s, t_{in}^v, a_j = (f_{\gamma P j}/f_j), \quad j = \rho, \omega, \phi$

determined in an optimal description of existing data.

for π^0: see $Fig.7$

$q_{in}^s = 5.5210 \pm 0.0084 \quad q_{in}^v = 5.61220 \pm 0.1414$

$a_\omega = 0.0063 \pm 0.0013 \quad a_\rho = 0.0212 \pm 0.0006$

$a_\phi = -.0004 \pm 0.0001 \quad \chi^2/ndf = 121/75 = 1.61$

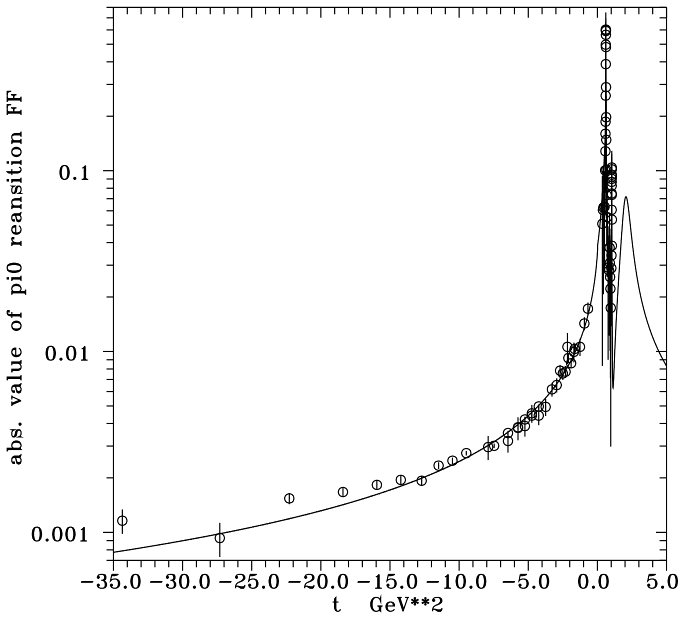

Figure 7: Prediction of $\pi^0\gamma$ transition EM FF behavior by $U\&A$ model.

for η**:** see $Fig.8$

$$q^s_{in} = 6.7104 \pm 0.0190 \quad q^v_{in} = 5.5006 \pm 0.0632$$

$$a_\omega = 0.0002 \pm 0.0014 \quad a_\rho = 0.0250 \pm 0.0013$$

$$a_\phi = -.0020 \pm 0.0003 \quad \chi^2/ndf = 52/52 = 1.00$$

for η'**:** see $Fig.9$

$$q^s_{in} = 5.5366 \pm 0.0891 \quad q^v_{in} = 7.7554 \pm 0.0158$$

$$a_\omega = -.1134 \pm 0.0078 \quad a_\rho = 0.1241 \pm 0.0026$$

$$a_\phi = 0.0098 \pm 0.0091 \quad \chi^2/ndf = 59/50 = 1.18$$

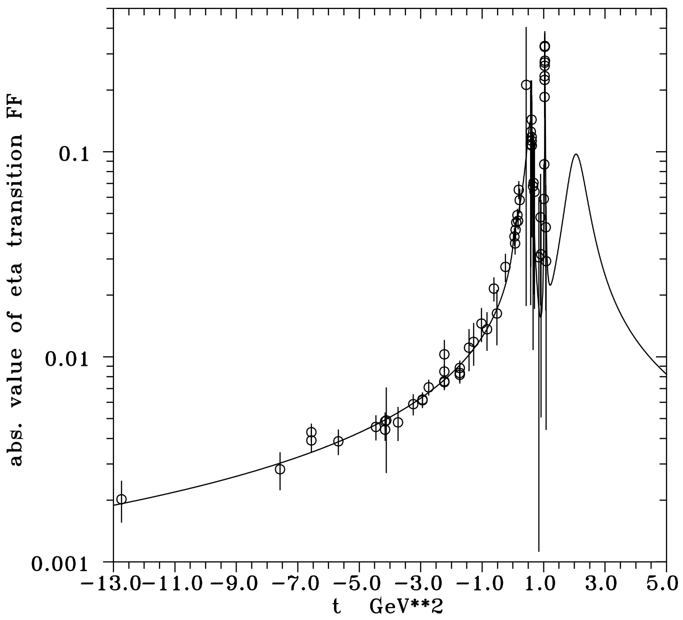

Figure 8: Prediction of $\eta\gamma$ transition EM FF behavior by $U\&A$ model.

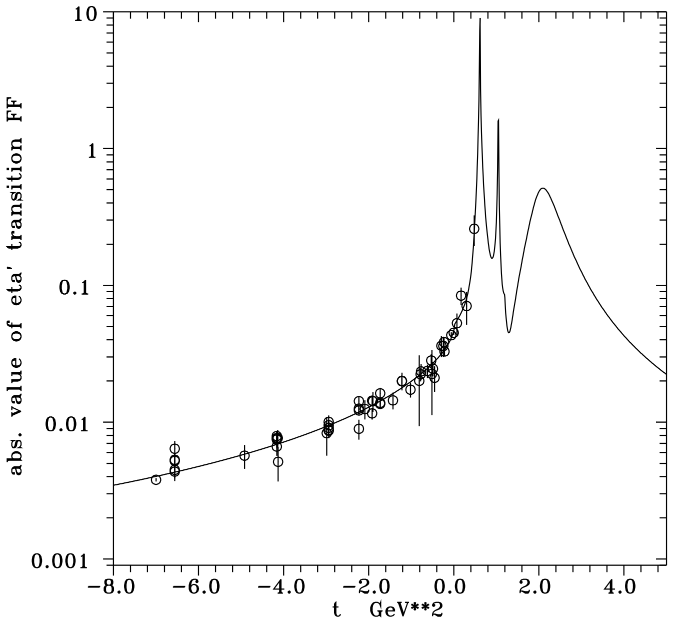

Figure 9: Prediction of $\eta'\gamma$ transition EM FF behavior by $U\&A$ model.

CONCLUSIONS

- We have investigated **EM structure of pseudoscalar mesons** to be described by corresponding EM FFs.
- Since there is **no possibility to describe the latter in the framework of** QCD, the universal $U\&A$ **models have been elaborated.**
- More or less **successful description of all existing data** on the whole complete nonet $\pi^-, \pi^0, \pi^+, K^-, K^0, \bar{K}^0, K^+, \eta, \eta'$ **has been achieved** in space-like and time-like regions simultaneously.