# Chiral dynamics with vector fields: an application to $\pi\pi$ and $\pi K$ scattering

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The purpose of our work is to apply the novel scheme [A.Gasparyan, M.F.M.Lutz Nucl. Phys. A 848, 126 (2010)] to Goldstone boson scattering, based on the SU(3) chiral Lagrangian with light vector mesons.

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**Resonance saturation mechanism**

The values of the $\mathcal{O}(Q^4)$ parameters in Chiral Lagrangian are basically saturated by vector-meson exchange between Goldstone bosons [G.Ecker, J.Gasser, A.Pich and E.de Rafael, Nucl. Phys. B **321** (1989) 311].

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**Hadrogenesis conjecture**

For instance the leading chiral interaction of Goldstone bosons with light vector mesons generates an axial-vector meson spectrum [M.F.M.Lutz, E.E.Kolomeitsev, Nucl. Phys. A **730** (2004) 392].

# Chiral Lagrangian (No unknown parameters!)

The relevant terms of the chiral Lagrangian for the Goldstone bosons $\Phi$ $(\pi,\, K,\, \bar{K},\eta)$ and vector mesons $\Phi_{\mu\nu}$ $(\rho_{\mu\nu},\, \omega_{\mu\nu},\, K_{\mu\nu},\, \bar{K}_{\mu\nu},\, \phi_{\mu\nu})$

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{4}\,\mathrm{tr}\left\{\partial^\mu \Phi\, \partial_\mu \Phi\right\} - \frac{1}{4}\,\mathrm{tr}\left\{\partial^\mu \Phi_{\mu\alpha}\, \partial_\nu \Phi^{\nu\alpha}\right\} + \frac{1}{8}\, m_V^2\, \mathrm{tr}\left\{\Phi^{\mu\nu}\, \Phi_{\mu\nu}\right\} \\
&- \frac{1}{4}\,\mathrm{tr}\left\{\Phi^2\, \chi_0\right\} + \frac{1}{48 f^2}\,\mathrm{tr}\left\{\Phi^4\, \chi_0\right\} + \frac{1}{8}\, b_D\, \mathrm{tr}\left\{\Phi^{\mu\nu}\, \Phi_{\mu\nu}\, \chi_0\right\} \\
&+ \frac{1}{48 f^2}\,\mathrm{tr}\left\{[\Phi,\, \partial^\mu \Phi]_-\, [\Phi,\, \partial_\mu \Phi]_-\right\} - i\, \frac{f_V\, h_P}{8 f^2}\,\mathrm{tr}\left\{\partial_\mu \Phi\, \Phi^{\mu\nu}\, \partial_\nu \Phi\right\},
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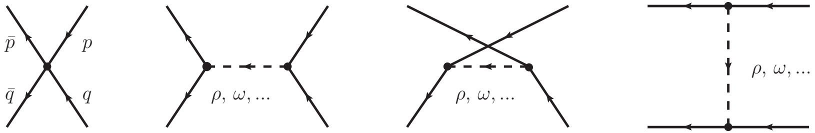
\end{aligned}
$$

All parameters were fixed before, for instance in [M.F.M.Lutz, S.Leupold, Nucl.Phys.A813 (2008), 51-71]

$$
\begin{aligned}
m_V \simeq 0.760\ \mathrm{GeV}, &\quad b_D \simeq 0.95\,, \\
f_V\, h_p \simeq 0.22\ \mathrm{GeV}, &\quad f \simeq 90\,\mathrm{MeV}\,.
\end{aligned}
$$

# Partial-wave projection of the scatting amplitude



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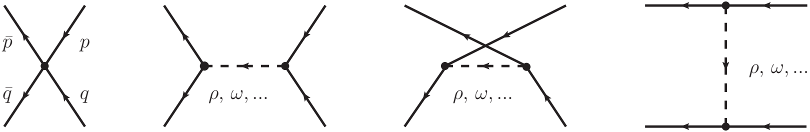


Partial-wave amplitudes are introduced by an average

$$T^J(s) = \int_{-1}^{+1} \frac{d\cos\theta}{2} \left(\frac{\bar{p}_{\rm cm}\, p_{\rm cm}}{s}\right)^J T(s,t,u)\, P_J(\cos\theta)\,,$$

over the center-of mass scattering angle $\theta$.

# Partial-wave dispersion relation

The partial-wave dispersion relation

$$T^J_{ab}(s) = U^J_{ab}(s) + \sum_{c,d} \int_{\mu^2_{thr}}^{\infty} \frac{d\bar{s}}{\pi} \frac{s-\mu^2_M}{\bar{s}-\mu^2_M} \frac{T^J_{ac}(\bar{s})\, \rho^J_{cd}(\bar{s})\, T^{J*}_{db}(\bar{s})}{\bar{s}-s-i\epsilon} \,,$$

- separate left and right-hand cuts
- the generalized potential $U^J_{ab}(s)$ contains all left hand cuts

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The phase space fuction

$$\Im\, T^J_{ab}(s) = \sum_{c,d} T^J_{ac}(s)\, \rho^J_{cd}(s)\, T^{J*}_{db}(\bar{s}) , \quad \rho^J_{ab}(s) = \frac{1}{8\pi} \left( \frac{p_{cm}}{\sqrt{s}} \right)^{2\,J+1} \delta_{ab}$$

# Approximation for the generalized potential $U^J_{ab}(s)$

In $\chi$PT one can perform a pert. expansion only in the close-to-threshold region (asymptotically growing potential).

To solve the non-linear integral eq. and restore $T^J_{ab}(s)$ we need $U^J_{ab}(s)$ for energies above threshold.

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To solve the non-linear integral eq. and restore $T^J_{ab}(s)$ we need $U^J_{ab}(s)$ for energies above threshold.

Reliable extrapolation is possible:

Conformal mapping techniques may be used to approximate $U^J_{ab}(s)$ for ($s > \mu^2_{thr}$), based on $U^J_{ab}(s)$ only around threshold $\mu^2_{thr}$.

# Conformal mapping technique

We need $U^J_{ab}(s)$ for energies above threshold $s > \mu^2_{th}$.

$$U^J_{ab}(s) = \sum_{k=0}^{N} C_k \, [\xi(s)]^k \qquad \text{for} \quad s < \Lambda_S$$

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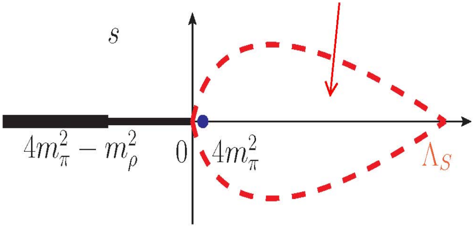
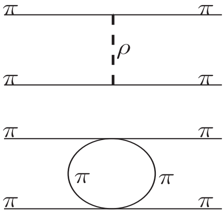
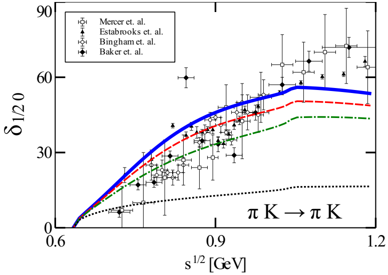
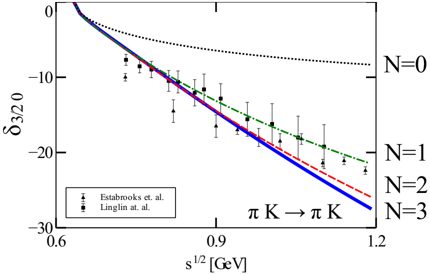
Reliable approximation is within this area

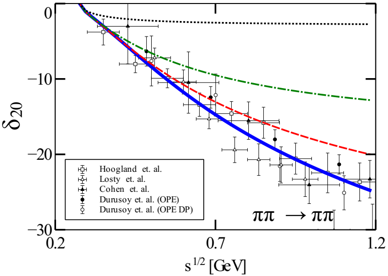


# J=0: The role of the vector meson exchange

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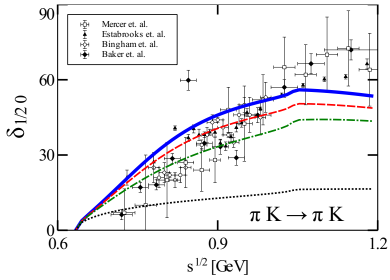


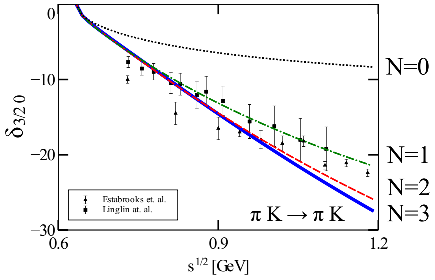


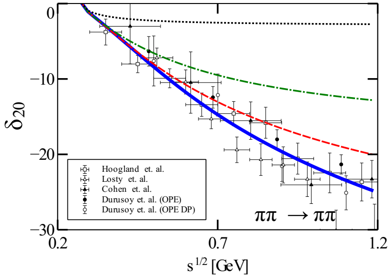


- $U^J_{ab}(s) = \sum_{k=0}^{N} C_k \, [\xi(s)]^k$
- No free parameters adjusted!

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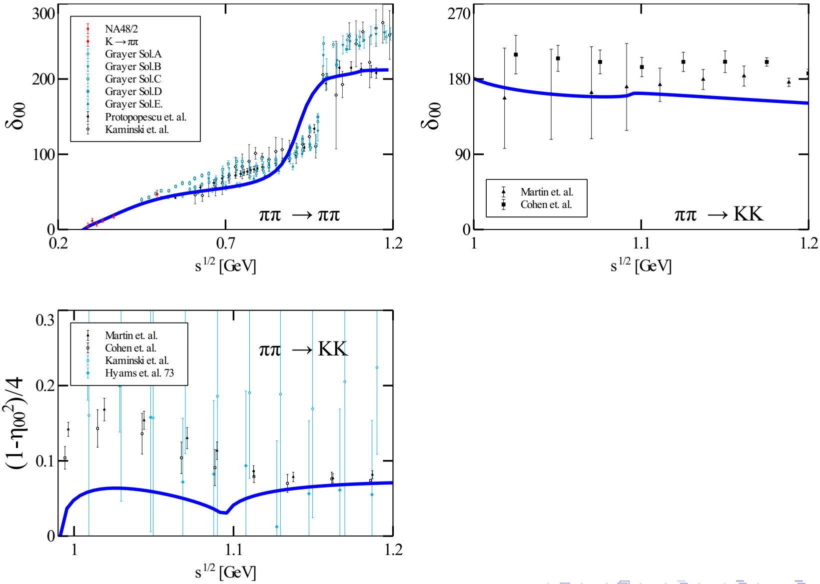

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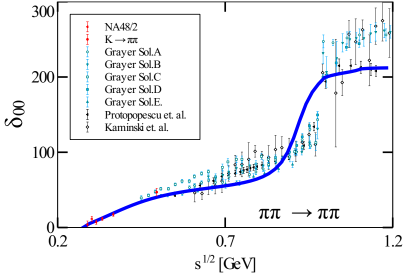


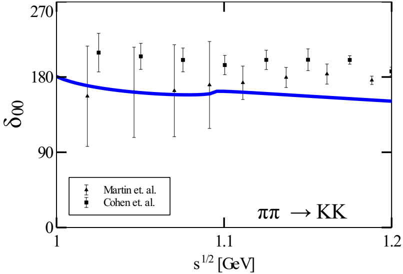


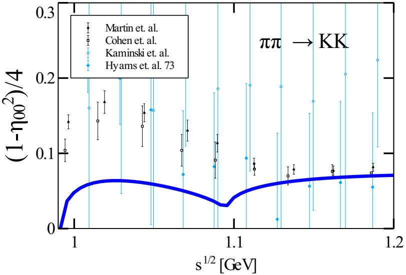


- $f_0(980)$ dynamically generated.

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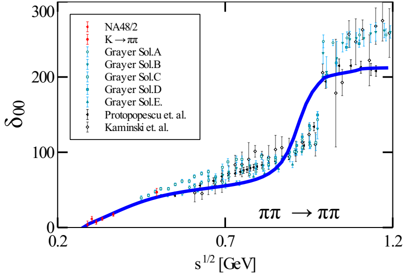


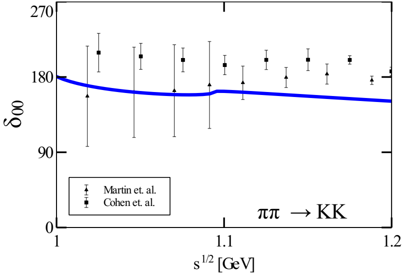


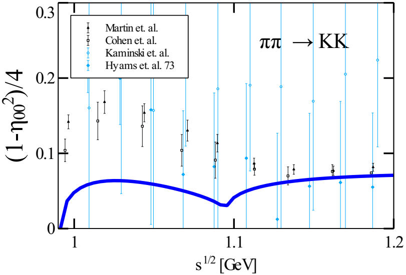


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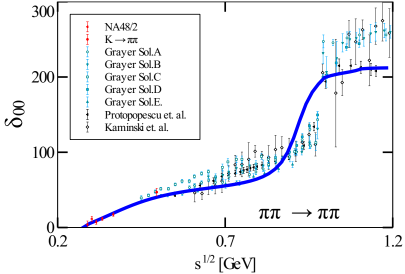


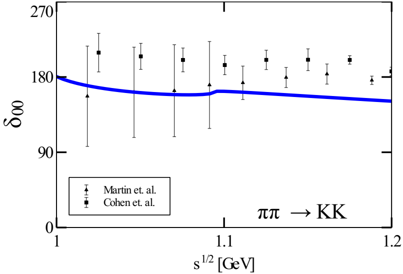


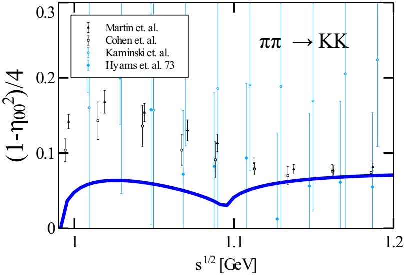


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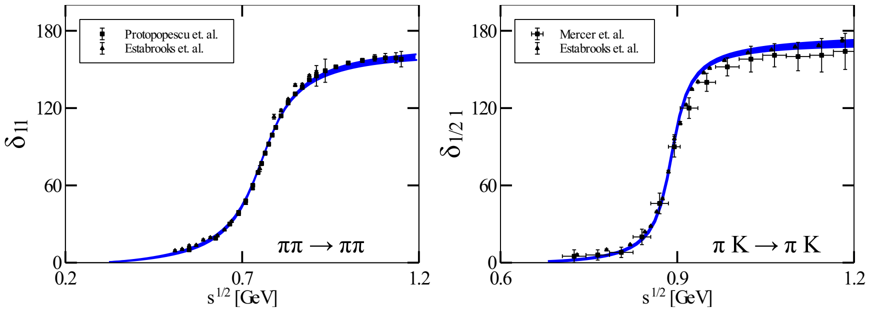
# J=1: The role of the vector meson exchange



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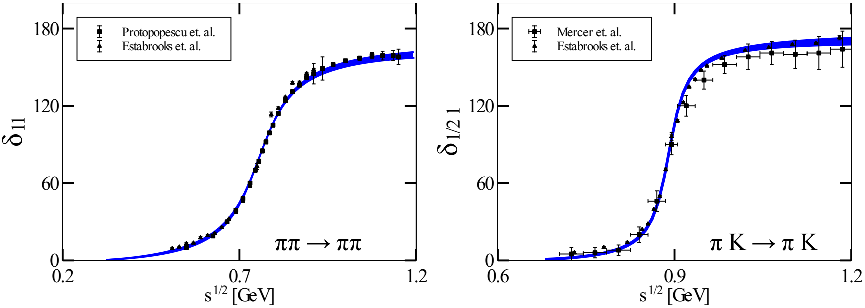


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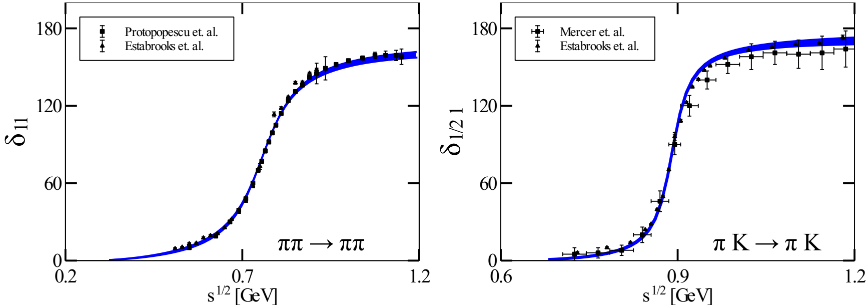


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- $\Gamma_\phi \simeq 3.31$ MeV    Exp: $\Gamma_\phi(K\bar{K}) = 3.54 \pm 0.04$ MeV

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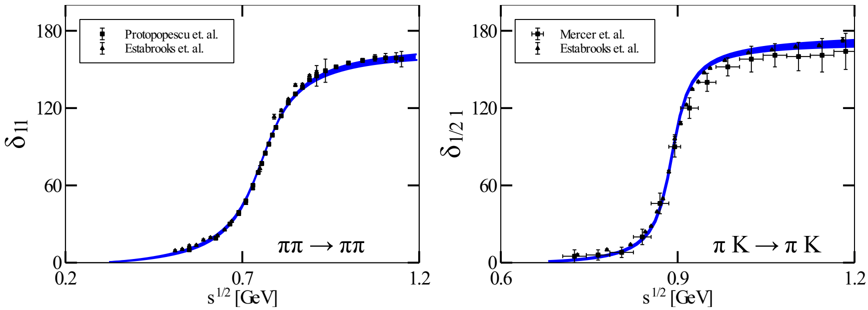


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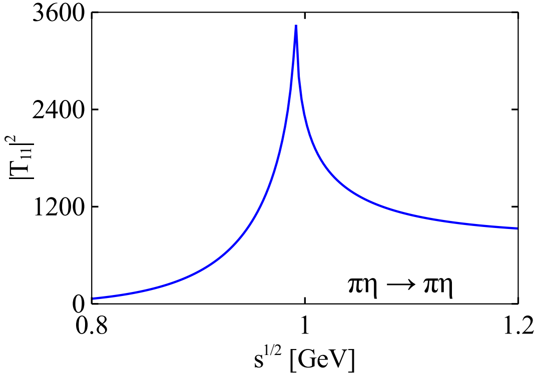
We conclude that the dynamics depends sensitively on the details of the vector meson exchange.

## $a_0(980)$

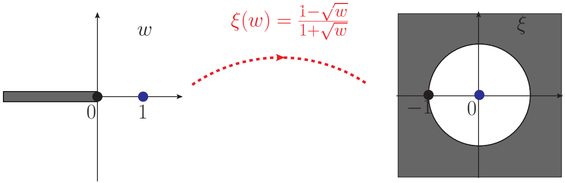



$J = 0$ and $(I^G, S) = (1^-, 0)$

# Conformal mapping technique

Typical example: $U(\omega) = ln\,(\omega) \quad = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}\,[\omega - 1]^k$

$$= \sum_{k=0}^{\infty} C_k\,[\xi(\omega)]^k$$



$$ln\,(\omega): \qquad \sum_{k=1}^{N} \frac{(-1)^{k+1}}{k!}\,[\omega - 1]^k \quad vs. \quad \sum_{k=0}^{N} C_k\,[\xi(\omega)]^k$$