On Scalar Mesons from the Combined Analysis of Multi-Channel Pion-Pion Scattering and J/ψ Decays

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Motivation

We present results of the combined coupled-channel analysis of data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on decays $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ for studying f_0-mesons lying below 1.9 GeV. Note that as to parameters of the scalar mesons, obtained from experimental data in various analyses, and even the status of some of the mesons, there is a considerable disagreement (*C.Amsler et al. (PDG), PL B **667 (2008) 1*). As to the first point, we pick out the $f_0(600)/\sigma$-meson, $f_0(980)$ and $f_0(1500)$. As to the second point, one might indicate a situation related to the $f_0(1370)$ when, e.g., D. Bugg (*Eur. Phys.J. C**52** (2007) 55; arXiv: 0710.4452 [hep-ex]*) has indicated a number of data requiring apparently the existence of the $f_0(1370)$. These are the Crystal Barrel data on $\bar{p}p \to \eta\eta\pi^0$ and on $\bar{p}p \to 3\pi^0$ also the BES data on $J/\psi \to \phi\pi^+\pi^-$, and in the GAMS data for $\pi^+\pi^- \to \pi^0\pi^0$ at large $|t|$.**

In works (*W. Ochs, arXiv:1001.4486v1 [hep-ph]; P. Minkowski, W. Ochs, EPJ C9** (1999) 283; hep-ph/0209225*) one did not find evidence for the existence of the $f_0(1370)$. In work (*Yu.S. Surovtsev et al, EPJ A**15** (2002) 409*) also the best description of $\pi\pi \to \pi\pi, K\overline{K}$ was obtained without the $f_0(1370)$, and it was shown that the $K\overline{K}$ scattering length is very sensitive to whether this state exists or not.**

Note a situation with scalar states in the 1500-MeV region. In our previous model-independent analyses of $\pi\pi \to \pi\pi\, K\overline{K}, \eta\eta, \eta\eta'$ (*Yu.S.Surovtsev et al., IJMP A* **24 *(2009) 586*), we saw the wide state $f_0(1500)$ whereas in works of some other authors, analyzing mainly mesons production and decay processes and cited in the PDG tables, the rather narrow $f_0(1500)$ was obtained. We suggested that the wide $f_0(1500)$, observed in the multi-channel $\pi\pi$ scattering, indeed, is a superposition of two states, wide and narrow. The latter is observed just in decays and productions of mesons. Here we verify also this suggestion.**

In view of indicated circumstances, related to parameters and status of scalar mesons, there are the known problems as to determining their QCD nature and assignment to the quark-model configurations in spite of a big amount of work devoted these problems (see, e.g., *V.V.Anisovich, IJMP A* ***21*** *(2006) 3615* **and references therein).**

Here we applied to analyses of experimental data our model-independent method based only on the first principles (analyticity and unitarity) (*D.Krupa, V.Meshcheryakov, Yu.Surovtsev, NC A* ***109*** *(1996) 281 – KMS, 96***). That approach permits us to omit theoretical prejudice in extracting the resonance parameters. Considering the obtained arrangement of resonance poles on the Riemann-surface sheets, obtained coupling constants with channels and resonance masses, we draw definite conclusions about nature of the investigated states.**

The 3-coupled-channel formalism in model-independent approach

Our model-independent method which essentially utilizes an uniformizing variable can be used only for the 2-channel case and under some conditions for the 3-channel one. Only in these cases we obtain a simple symmetric (easily interpreted) picture of the resonance poles and zeros of the S-matrix on the uniformization plane. The 3-channel S-matrix is determined on the 8-sheeted Riemann surface. The matrix elements S_{ij}, where $i, j = 1, 2, 3$ denote channels, have the right-hand cuts along the real axis of the s complex plane (s is the invariant total energy squared), starting with the channel thresholds s_i ($i = 1, 2, 3$), and the left-hand cuts.

The Riemann-surface sheets are numbered according to the signs of analytic continuations of the square roots $\sqrt{s - s_i}$ $(i = 1, 2, 3)$ as follows:

	I	II	III	IV	V	VI	VII	VIII
$\mathrm{Im}\sqrt{s-s_1}$	+	−	−	+	+	−	−	+
$\mathrm{Im}\sqrt{s-s_2}$	+	+	−	−	−	−	+	+
$\mathrm{Im}\sqrt{s-s_3}$	+	+	+	+	−	−	−	−

The resonance representations on the Riemann surface are obtained with the help of formulas from (*KMS, 96*), expressing analytic continuations of the S-matrix elements to all sheets in terms of those on sheet I that have only the resonances zeros (beyond the real axis), at least, around the physical region. Then, starting from the resonance zeros on sheet I, one can obtain an arrangement of poles and zeros of resonance on the whole Riemann surface.

Process	I	II	III	IV	V	VI	VII	VIII
$1 \to 1$	S_{11}	$\frac{1}{S_{11}}$	$\frac{S_{22}}{D_{33}}$	$\frac{D_{33}}{S_{22}}$	$\frac{\det S}{D_{11}}$	$\frac{D_{11}}{\det S}$	$\frac{S_{33}}{D_{22}}$	$\frac{D_{22}}{S_{33}}$
$1 \to 2$	S_{12}	$\frac{iS_{12}}{S_{11}}$	$\frac{-S_{12}}{D_{33}}$	$\frac{iS_{12}}{S_{22}}$	$\frac{iD_{12}}{D_{11}}$	$\frac{-D_{12}}{\det S}$	$\frac{iD_{12}}{D_{22}}$	$\frac{D_{12}}{S_{33}}$
$2 \to 2$	S_{22}	$\frac{D_{33}}{S_{11}}$	$\frac{S_{11}}{D_{33}}$	$\frac{1}{S_{22}}$	$\frac{S_{33}}{D_{11}}$	$\frac{D_{22}}{\det S}$	$\frac{\det S}{D_{22}}$	$\frac{D_{11}}{S_{33}}$
$1 \to 3$	S_{13}	$\frac{iS_{13}}{S_{11}}$	$\frac{-iD_{13}}{D_{33}}$	$\frac{-D_{13}}{S_{22}}$	$\frac{-iD_{13}}{D_{11}}$	$\frac{D_{13}}{\det S}$	$\frac{-S_{13}}{D_{22}}$	$\frac{iS_{13}}{S_{33}}$
$2 \to 3$	S_{23}	$\frac{D_{23}}{S_{11}}$	$\frac{iD_{23}}{D_{33}}$	$\frac{iS_{23}}{S_{22}}$	$\frac{-S_{23}}{D_{11}}$	$\frac{-D_{23}}{\det S}$	$\frac{iD_{23}}{D_{22}}$	$\frac{iS_{23}}{S_{33}}$
$3 \to 3$	S_{33}	$\frac{D_{22}}{S_{11}}$	$\frac{\det S}{D_{33}}$	$\frac{D_{11}}{S_{22}}$	$\frac{S_{22}}{D_{11}}$	$\frac{D_{33}}{\det S}$	$\frac{S_{11}}{D_{22}}$	$\frac{1}{S_{33}}$

In Table, the superscript I is omitted to simplify the notation, $\det S$ is the determinant of the 3×3 S-matrix on sheet I, $D_{\alpha\beta}$ is the minor of the element $S_{\alpha\beta}$, that is,
$D_{11} = S_{22}S_{33} - S_{23}^2$, $D_{22} = S_{11}S_{33} - S_{13}^2$,
$D_{33} = S_{11}S_{22} - S_{12}^2$, $D_{12} = S_{12}S_{33} - S_{13}S_{23}$,
$D_{23} = S_{11}S_{23} - S_{12}S_{13}$, **etc.**

In the 3-channel case, we obtain *7 types* of resonances corresponding to 7 possible situations when there are resonance zeros on sheet I only in S_{11} – (a); S_{22} – (b); S_{33} – (c); S_{11} and S_{22} – (d); S_{22} and S_{33} – (e); S_{11} and S_{33} – (f); S_{11}, S_{22}, and S_{33} – (g).
The resonance of every type is represented by the pair of complex-conjugate clusters (of poles and zeros on the Riemann surface).
A necessary and sufficient condition for existence of the multi-channel resonance is its representation by one of the types of pole clusters.
Whereas cases (a), (b) and (c) can be simply related to the resonance representation by Breit-Wigner forms, cases (d), (e), (f) and (g) practically are lost at the Breit-Wigner description.

The cluster type is related to the nature of state. *E.g.*, **if we consider the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ channels, then a resonance, coupled relatively more strongly to the $\pi\pi$ channel than to the $K\overline{K}$ and $\eta\eta$ ones is described by the cluster of type (a). In the opposite case, it is represented by the cluster of type (e) (say, the state with the dominant $s\bar{s}$ component). The glueball must be represented by the cluster of type (g) as a necessary condition for the ideal case.**

From formulas of the analytic continuations, we conclude that masses and total widths of resonances must be calculated from the pole positions on sheets II, IV and VIII because the analytic continuations only onto these sheets have the forms $\propto 1/S_{11}^{\mathrm{I}}$, $\propto 1/S_{22}^{\mathrm{I}}$ and $\propto 1/S_{33}^{\mathrm{I}}$, respectively, i.e., the pole positions of resonances only on these sheets are at the same points of the complex-energy plane, as the resonance zeros on the physical sheet, and are not shifted due to the coupling of channels.

We can distinguish, in a model-independent way, a bound state of colourless particles (*e.g.*, $K\overline{K}$ molecule) and a $q\bar{q}$ bound state. Just as in the 1-channel case, the existence of the particle bound-state means the presence of a pole on the real axis under the threshold on the physical sheet, so in the 2-channel case, the existence of the bound-state in channel 2 ($K\overline{K}$ molecule) that, however, can decay into channel 1 ($\pi\pi$ decay), would imply the presence of the pair of complex conjugate poles on sheet II under the second-channel threshold without the corresponding shifted pair of poles on sheet III.

In the 3-channel case, the bound state in channel 3 ($\eta\eta$) that, however, can decay into channels 1 ($\pi\pi$ decay) and 2 ($K\overline{K}$ decay), is represented by the pair of complex conjugate poles on sheet II and by the pair of shifted poles on sheet III under the $\eta\eta$ threshold without the corresponding poles on sheets VI and VII.

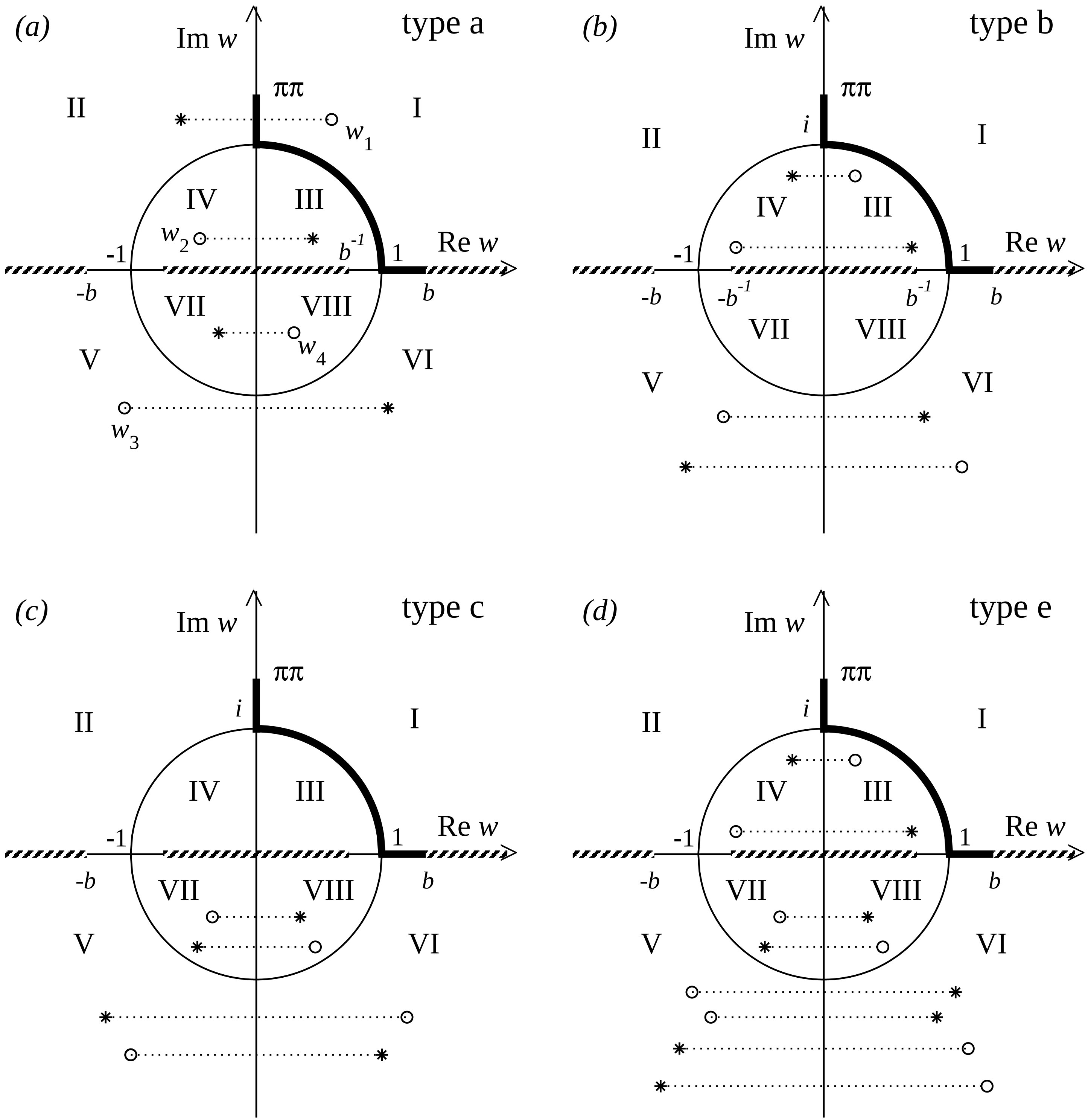
According to this test (*D.Morgan, M.R.Pennington, PR D* ***48* *(1993) 1185; KMS, 96*), earlier in (*KMS, 96*), we rejected interpretation of the $f_0(980)$ as the $K\overline{K}$ molecule because this state is represented by the cluster of type (a) in the 2-channel analysis of processes $\pi\pi \to \pi\pi, K\overline{K}$ and, therefore, does not satisfy the necessary condition to be the $K\overline{K}$ molecule.**

We use the Le Couteur-Newton relations (*K.J.LeCouteur, Proc.Roy.Soc. A* ***256* *(1960) 115; R.G.Newton, J.Math.Phys.* ***2*** *(1961) 188; M.Kato, Ann.Phys.* ***31*** *(1965) 130*). They express the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(\sqrt{s-s_1},\cdots,\sqrt{s-s_n})$ that is a real analytic function with the only square-root branch-points at $\sqrt{s-s_i}=0$.**

The important branch points, corresponding to the thresholds of the coupled channels and to the crossing ones, are taken into account in the proper uniformizing variable. Here we used a new uniformizing variable, in which we neglect the lowest $\pi\pi$-threshold branch-point and take into account the threshold branch-points related to two remaining channels and the left-hand branch-point at $s=0$:

$$w=\frac{\sqrt{(s-s_2)s_3}+\sqrt{(s-s_3)s_2}}{\sqrt{s(s_3-s_2)}}.$$

$s_2=m_K^2, \quad s_3=4m_\eta^2.$


(a)
type a
Im w
ππ
II
I
w1
IV
III
w2
-1
b-1
1
Re w
-b
b
VII
VIII
V
VI
w4
w3
(b)
type b
Im w
ππ
i
II
I
IV
III
-1
1
Re w
-b
-b-1
b-1
b
VII
VIII
V
VI
(c)
type c
Im w
ππ
i
II
I
IV
III
-1
1
Re w
-b
b
VII
VIII
V
VI
(d)
type e
Im w
ππ
i
II
I
IV
III
-1
1
Re w
-b
b
VII
VIII
V
VI

On the w-plane, the Le Couteur-Newton relations are

$$S_{11} = \frac{d^*(-w^*)}{d(w)}, \qquad S_{22} = \frac{d(-w^{-1})}{d(w)}, \qquad S_{33} = \frac{d(w^{-1})}{d(w)},$$

$$S_{11}S_{22} - S_{12}^2 = \frac{d^*(w^{*-1})}{d(w)}, \qquad S_{11}S_{33} - S_{13}^2 = \frac{d^*(-w^{*-1})}{d(w)}.$$

$$d = d_B d_{res}, \qquad d_{res}(w) = w^{-\frac{M}{2}} \prod_{r=1}^{M} (w + w_r^*)$$

M is the number of resonance zeros.

$$d_B = \exp[-i(a + \sum_{n=1}^{3} \frac{\sqrt{s - s_n}}{2m_n}(\alpha_n + i\beta_n))],$$

$$\alpha_n = a_{n1} + a_{n\sigma}\frac{s - s_\sigma}{s_\sigma}\theta(s - s_\sigma) + a_{nv}\frac{s - s_v}{s_v}\theta(s - s_v),$$

$$\beta_n = b_{n1} + b_{n\sigma}\frac{s - s_\sigma}{s_\sigma}\theta(s - s_\sigma) + b_{nv}\frac{s - s_v}{s_v}\theta(s - s_v).$$

s_σ is the $\sigma\sigma$ threshold; s_v is the combined threshold of the $\eta\eta'$, $\rho\rho$, $\omega\omega$ channels.

Analysis of the data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on decays $J/\psi \to \phi\pi\pi, \phi K\overline{K}$

To the combined analysis of data on processes

$$\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta',$$

we added also data from Mark III (*W.Lockman, Hadron'89, Proceedings, p.109*) and DM2 (*A.Falvard et al., PR D38** (1988) 2706*) on decays $J/\psi \to \phi\pi\pi, \phi K\overline{K}$. Formalism for calculating di-meson mass distributions of these decays can be found in Refs. (*D.Morgan, M.R.Pennington, PR D **48** (1993) 1185; PR D **48** (1993) 5422; B.S.Zou, D.V.Bugg, PR D **50** (1994) 591*). There is assumed that pairs of pseudo-scalar mesons of final states have $I = J = 0$ and only they undergo strong interactions, and the ϕ meson acts as a spectator.**

For the $\pi\pi$ scattering, the data from the threshold to 1.89 GeV are taken from (*B.Hyams et al., NP B **64** (1973) 134; **100** (1975) 205 (1975); A.Zylbersztejn et al., PL B **38** (1972) 457; P.Sonderegger, P.Bonamy, in Proc. 5th Intern. Conf. on Elem. Part., Lund, 1969, paper 372; J.R.Bensinger et al., PL B **36** (1971) 134; J.P.Baton et al., PL B **33** (1970) 525, 528; P.Baillon et al., PL B **38** (1972) 555; L.Rosselet et al., PR D **15** (1977) 574; A.A.Kartamyshev et al., Pis'ma v ZhETF **25** (1977) 68; A.A. Bel'kov et al., Pis'ma v ZhETF **29** (1979) 652*). **For $\pi\pi \to K\overline{K}$, practically all the accessible data are used (***W.Wetzel et al., NP B **115** (1976) 208; V.A.Polychronakos et al., PR D **19** (1979) 1317; P.Estabrooks, PR D **19** (1979) 2678 ; D.Cohen et al., PR D **22** (1980) 2595; G.Costa et al., NP B **175** (1980) 402; A.Etkin et al., PR D **25** (1982) 1786*)**.**

For $\pi\pi \to \eta\eta$, we used data for $|S_{13}|^2$ from the threshold to 1.72 GeV (*F.Binon et al., NC A **78** (1983) 313*)**.**

The amplitudes for $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ decays are related with the scattering amplitudes T_{ij} $i, j = 1 - \pi\pi, 2 - K\overline{K}$ as follows

$$F(J/\psi \to \phi\pi\pi) = \sqrt{\frac{2}{3}}[c_1(s)T_{11} + c_2(s)T_{21}],$$

$$F(J/\psi \to \phi K\overline{K}) = \sqrt{\frac{1}{2}}[c_1(s)T_{12} + c_2(s)T_{22}]$$

where $c_i(s)$ are functions of couplings of the J/ψ to channel i.

$$c_i = \gamma_{i0} + \gamma_{i1}s$$

with γ_{i0} and γ_{i1} free parameters.

$$\frac{N|F|^2\sqrt{s - s_i}}{\sqrt{(m_\psi^2 - (\sqrt{s} - m_\phi)^2)}(m_\psi^2 - (\sqrt{s} + m_\phi)^2}$$

gives the di-meson mass distributions. N (normalization to experiment) is 0.73 for Mark III and 0.28 for DM2.

$\gamma_{10}, \gamma_{11}, \gamma_{20}, \gamma_{21} = 1.6148, 1.3169, -1.0962, -1.64.$

We supposed that in the 1500-MeV region there are two resonances. The $f_0(600)$ is described by the cluster of type (a); $f_0(1370)$, type (b); $f_0(1500)$, type (c); $f_0'(1500)$, type (g); $f_0(1710)$, type (b); the $f_0(980)$ is represented only by the pole on sheet II and shifted pole on sheet III in both variants.

Satisfactory combined description of all analyzed processes the total $\chi^2/\mathrm{NDF} = 408.166/(364-51) \approx 1.30$.

One can see (*R. Kamiński et al., Z.Phys. C74 (1997) 79*) that the data for $\pi\pi$ scattering below 1 GeV admit two solutions for the phase shift: "up" and "down". The above solution is "down".

The "up" solution gives practically the same result: The total $\chi^2/\mathrm{NDF} = 400.249/(364-51) \approx 1.27$.

We considered also a possibility of description without the $f_0(1370)$: The total $\chi^2/\mathrm{NDF} = 397.609/(364-47) \approx 1.25$.

$$T^{res} = \sqrt{s}\,\Gamma_{el}/(m_{res}^2 - s - i\sqrt{s}\,\Gamma_{tot})$$

	"down"		"up"		"up" without $f_0(1370)$	
State	m_{res}[MeV]	Γ_{tot}[MeV]	m_{res}[MeV]	Γ_{tot}[MeV]	m_{res}[MeV]	Γ_{tot}[MeV]
$f_0(600)$	771.1±15	1032.0±26	768.5±9.8	521.6±20	768.5±9.2	521.5±20
$f_0(980)$	1008.2±4	75.4±12	1008.0±3	75.0±7	1008.3±3	75.0±7
$f_0(1370)$	1365.1±16	339.6 ±30	1383.7±17	344.6±27	–	–
$f_0(1500)$	1500.2±14	114.2±22	1499.6±13	115.8±21	1499.5±11	115.7±20
$f_0'(1500)$	1538.3±14	662.9±26	1538.0±12.5	662.2±25	1538.4±12	663.0± 25
$f_0(1710)$	1740.3±15	215.0±32	1741.5±14	257.2±28	1741.6±12	225.2±22

As to a description, it is impossible to prefer any of these solutions. However, we select the "up" solution with the $f_0(1370)$, mainly because here the parameters of the $f_0(600)$ remarkably accord with prediction ($m_\sigma \approx m_\rho$ and $\Gamma_{tot} \approx 600$ MeV) on the basis of mended symmetry by Weinberg (*S.Weinberg, PRL **65 (1990) 1177*). Existence of the $f_0(1370)$ is for now a standard point of view.**

The pole clusters for resonances on the complex energy plane $\sqrt{s}$. The poles on sheets IV, VI, VIII and V, corresponding to the $f_0'(1500)$, are of the 2nd and 3rd order, respectively (this is an approximation). $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$.

Sheet		II	III	IV	V	VI	VII	VIII
$f_0(600)$	E_r	722.9±15	766.4±15			772.7±15	729.2±15	
	$\Gamma_r/2$	260.8±19	260.8±19			260.8±19	260.8±19	
$f_0(980)$	E_r	1007.3±4	969.5±9					
	$\Gamma_r/2$	37.5±6.6	53.6±10.7					
$f_0(1370)$	E_r		1372.9±19	1372.9±19	1372.9±19	1372.9±19		
	$\Gamma_r/2$		171.7±17	172.3±17	253.1±17	252.5±17		
$f_0(1500)$	E_r				1498.5±14	1498.5±14	1498.5±14	1498.5±14
	$\Gamma_r/2$				70.1±13	57.9±13	45.7±13	57.9±13
$f_0'(1500)$	E_r	1502.4±18	1502.8±16	1502.4±18	1497.5±14	1512.5±15	1502.8±20	1502.4±18
	$\Gamma_r/2$	331.8±16	136.3±10	224.4±16	139.9±14	184.5±17	98.8±11	331.4±16
$f_0(1710)$	E_r		1736.7±17	1736.7±17	1736.7±17	1736.7±17		
	$\Gamma_r/2$		110.6±23	128.6±23	288.0±23	270.0±23		

$a = 0.0$, $a_{11} = 0.2806$, $a_{1\sigma} = -0.0131$, $a_{1v} = 0$, $b_{11} = b_{1\sigma} = 0$, $b_{1v} = 0.0504$, $a_{21} = -0.9792$, $a_{2\sigma} = -0.416$, $a_{2v} = -6.644$, $b_{21} = 0.0289$, $b_{2\sigma} = 0$, $b_{2v} = 6.955$, $b_{31} = 0.6417$, $b_{3\sigma} = 0.6104$, $b_{2v} = 0$; $s_\sigma = 1.638\ \mathrm{GeV}^2$, $s_v = 2.085\ \mathrm{GeV}^2$.

The result for the $f_0(980)$: This state lies slightly above the $K\overline{K}$ threshold and is described by the pole on sheet II and by the shifted pole on sheet III under the $\eta\eta$ threshold without the corresponding (for standard clusters) poles on sheets VI and VII. This corresponds, e.g., to the description of the $\eta\eta$ bound state.

The $f_0(1370)$ and $f_0(1710)$ are described by the clusters pointing up to the dominant $s\bar{s}$ component of these states.

The cluster of type (g) of $f_0'(1500)$ tells us on the approximately equal coupling constants of this state with the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ systems. This points up to its glueball nature (*C.Amsler, F.E.Close, PR D* ***53*** *(1996) 295***).**

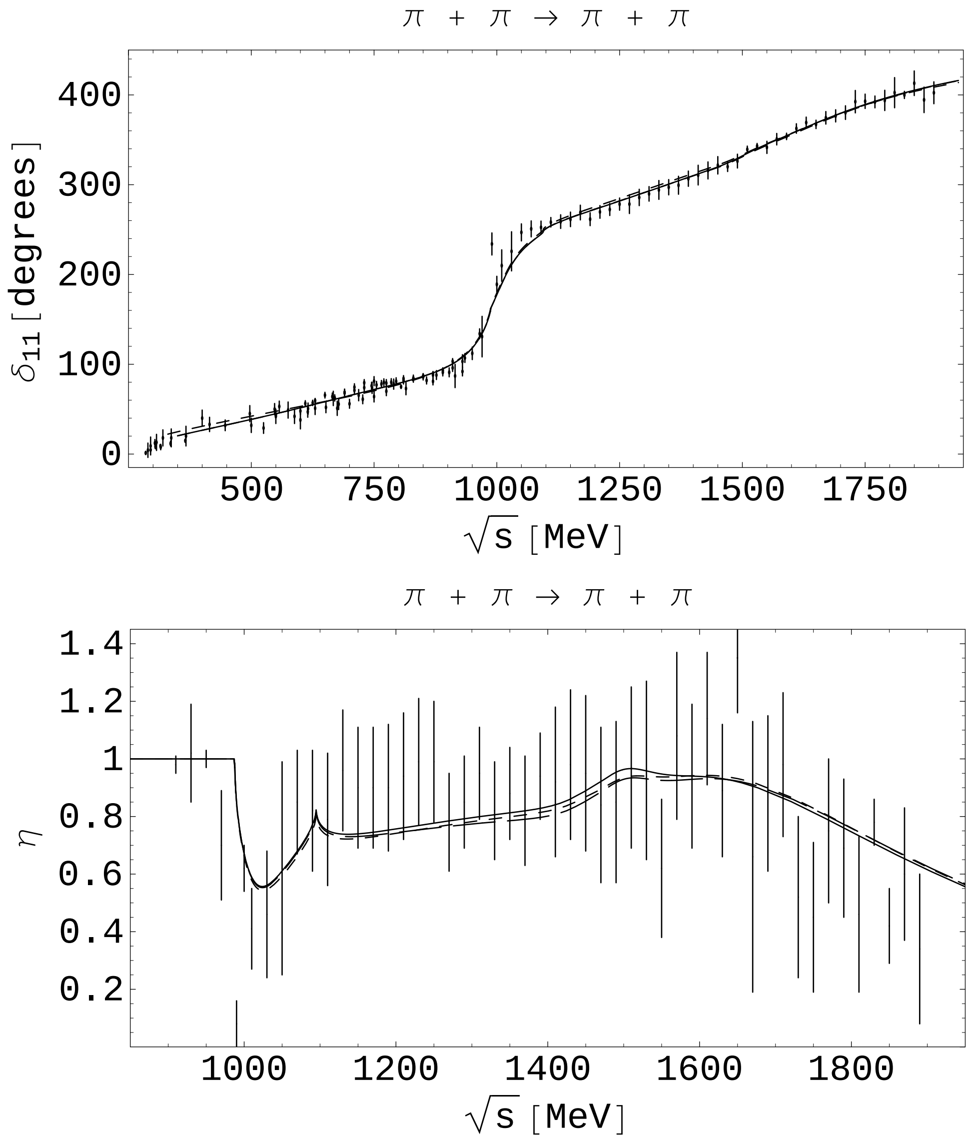


Figure 2: The phase shift and module of the $\pi\pi$-scattering S-wave matrix element.

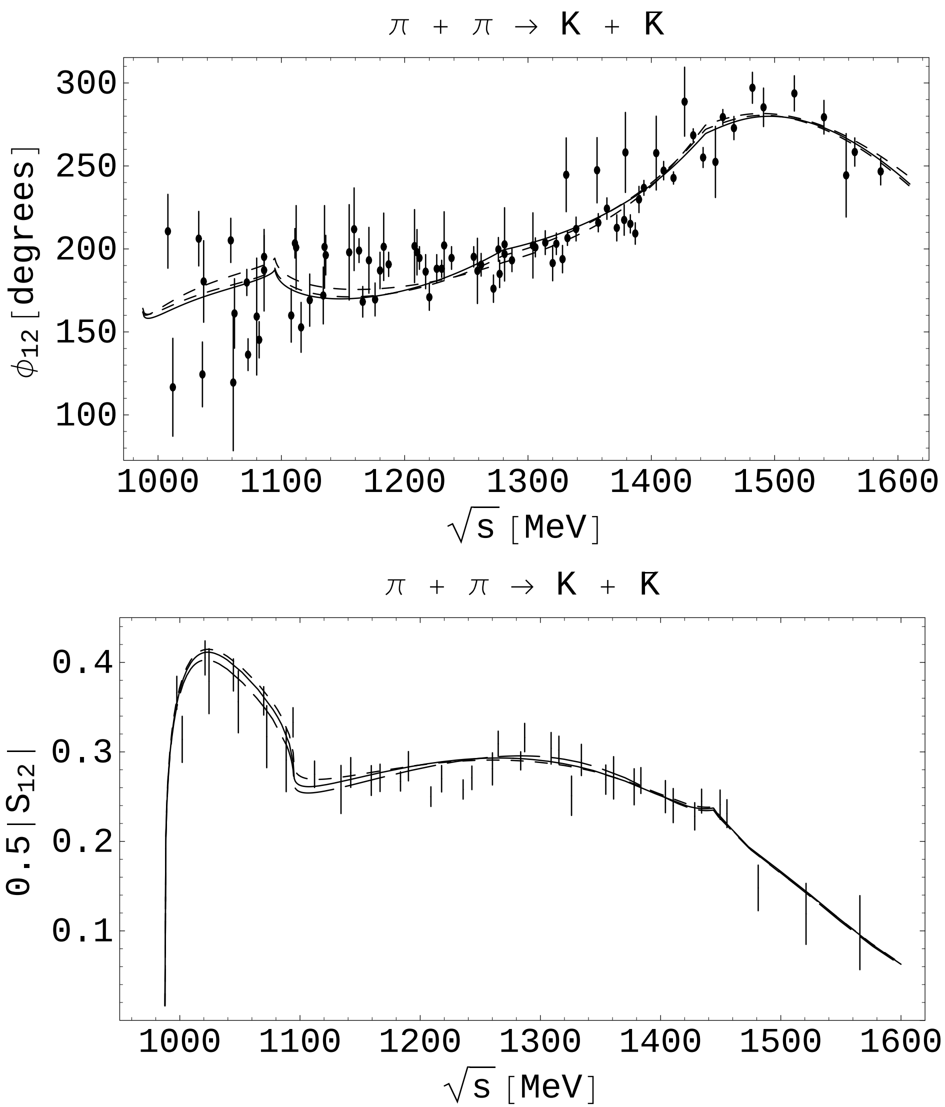


Figure 3: The phase shift and module of the $\pi\pi \to K\overline{K}$ S-wave matrix element.

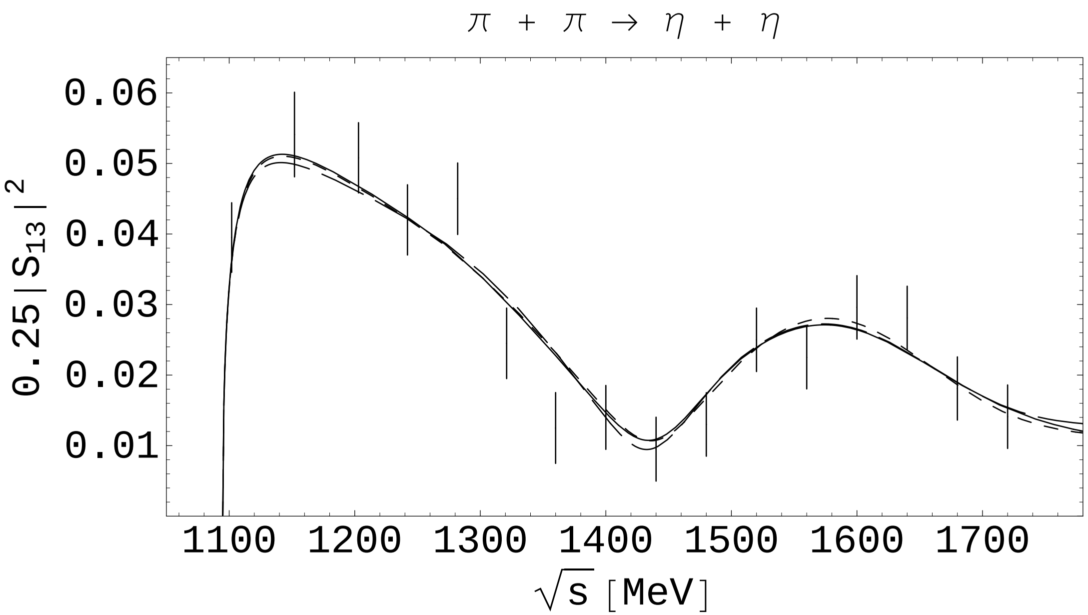


Figure 4: The squared modules of the $\pi\pi \longrightarrow \eta\eta$ S-wave matrix element.

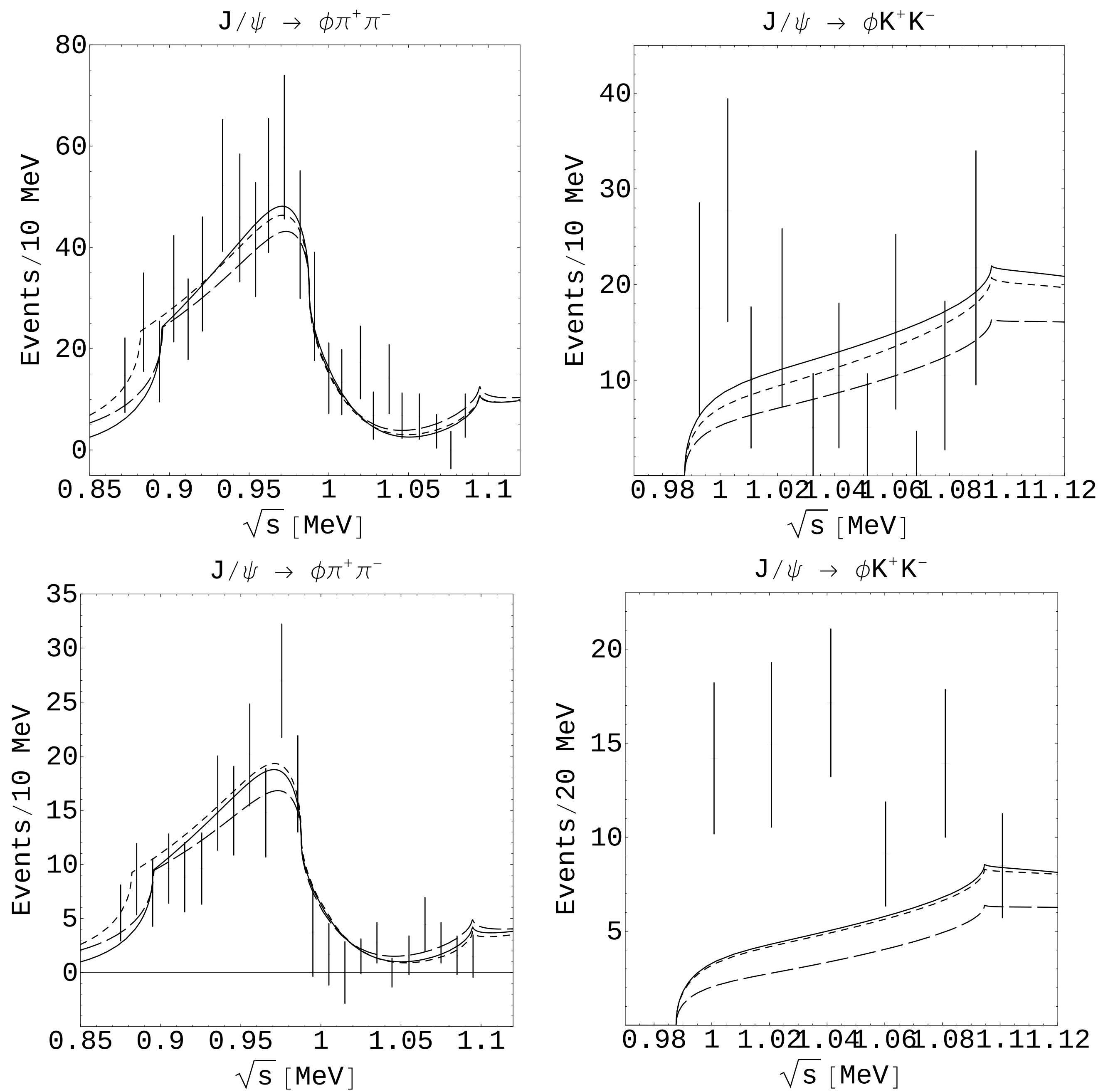


Figure 5: The $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ decays. The upper panel shows the fit to data of Mark III, the lower to DM2. The solid line corresponds to the "up" solution, the short-dashed to the "down" and the long-dashed to the "up" without $f_0(1370)$.

Discussion and conclusions

- In the combined model-independent analysis of data on the $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ processes in the $I^G J^{PC} = 0^+0^{++}$ channel and on the $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ decays, an additional confirmation of the σ-meson with mass $m_\sigma \approx m_\rho$ is obtained. This value remarkably accords with prediction on the basis of mended symmetry by Weinberg (*S.Weinberg, PRL **65** (1990) 1177*). For Γ_{tot}, there are obtained two values, equal to about 0.5 GeV and 1 GeV, corresponding two possible solutions for the $\pi\pi$-scattering phase shift below 1 GeV – "up" and "down", respectively.

- An indication for $f_0(980)$ ($m_{res} = 1008$ MeV, $\Gamma_{tot} = 75$ MeV) is obtained to be, e.g., the bound $\eta\eta$ state.

- **The $f_0(1370)$ and $f_0(1710)$ have the dominant $s\bar{s}$ component. Conclusion about the $f_0(1370)$ quit agrees with the one of work of Crystal Barrel Collaboration (*C.Amsler et al., PL B **355** (1995) 425*) where the $f_0(1370)$ is identified as $\eta\eta$ resonance in the $\pi^0\eta\eta$ final state of the $\bar{p}p$ annihilation at rest. Conclusion about the $f_0(1710)$ is quite consistent with the experimental facts that this state is observed in $\gamma\gamma \to K_S K_S$ (*S.Braccini, Frascati Phys. Series **XV**, 53 (1999)*) and not observed in $\gamma\gamma \to \pi^+\pi^-$ (*R.Barate et al., PL B **472**, 189 (2000)*).**

- **In the 1500-MeV region, indeed, there are two states: the $f_0(1500)$ ($m_{res} \approx 1500$ MeV, $\Gamma_{tot} \approx 116$ MeV) and the $f_0'(1500)$ ($m_{res} \approx 1540$ MeV, $\Gamma_{tot} \approx 660$ MeV). The $f_0'(1500)$ is interpreted as the glueball on the basis of indications about its couplings with the considered channels. Its biggest width among enclosing states tells also in favour of its glueball nature (*V.V.Anisovich et al., NP Proc.Suppl. A**56** (1997) 270*).**

- **We propose a following assignment of the scalar mesons below 1.9 GeV to lower nonets, when excluding the $f_0(980)$ as the non-$q\bar{q}$ state. The lowest nonet: the isovector $a_0(980)$, the isodoublet $K_0^*(930)$, and $f_0(600)$ and $f_0(1370)$ as mixtures of the 8th component of octet and the SU(3) singlet. The Gell-Mann–Okubo (GM-O) formula**

$$3m_{f_8}^2 = 4m_{K_0^*}^2 - m_{a_0}^2$$

gives $m_{f_8} = 910$ MeV.

In relation for masses of nonet

$$m_\sigma + m_{f_0(1370)} = 2m_{K_0^*}$$

the left-hand side is by about 16 % bigger than the right-hand one.

- **For the next nonet we find: $a_0(1450)$, $K_0^*(1450)$, and $f_0(1500)$ and $f_0(1710)$. From the GM-O formula, $m_{f_8} \approx 1453$ MeV. In formula**

$$m_{f_0(1500)} + m_{f_0(1710)} = 2m_{K_0^*(1450)}$$

the left-hand side is by about 12 % bigger than the right-hand one.

This assignment moves off a number of questions, stood earlier, and does not put the new ones. The mass formulas indicate to non-simple mixing scheme. The breaking of 2nd relations tells us that the $\sigma - f_0(1370)$ and $f_0(1500) - f_0(1710)$ systems get additional contributions absent in the $K_0^*(900)$ and $K_0^*(1450)$, respectively. A search of the adequate mixing scheme is complicated by the circumstance that here there is also a remainder chiral symmetry, though, on the other hand, this permits one to predict correctly, *e.g.*, the σ-meson mass.

Appendices

There is a number of properties of the scalar mesons, which do not allow one satisfactorily to make up the lowest nonet. The main of them is inaccordance of the approximately equal masses of the $f_0(980)$ and $a_0(980)$ and the found $s\bar{s}$ dominance in the wave function of the $f_0(980)$. If these states are in the same nonet, the $f_0(980)$ must be heavier than $a_0(980)$ for 250-300 MeV, because a difference of masses of s- and u-quarks is 120-150 MeV. We proposed our way to solve this problem.

The discovery of the κ-doublet (if it will be confirmed) moves off some more a number of problems.