



# Heavy flavour hadronic molecules

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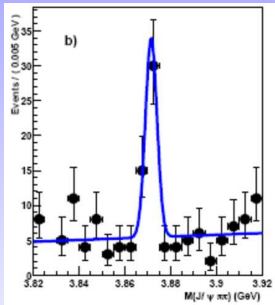


# Outline

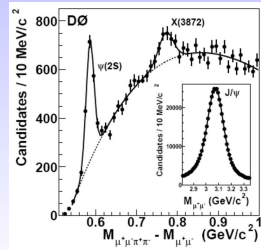
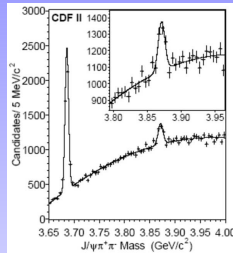
- 1 Introduction
- 2  $X(3872)$ . Decays and Lineshapes
- 3 Other  $XYZ$  mesons
- 4 Summary

## Experimental situation of $X(3872)$

Narrow state seen in  $B$  decays and  $p\bar{p}$  collision decaying to  $\pi\pi J/\psi$ ,  $\pi\pi\pi J/\psi$ ,  $\gamma J/\psi$  and  $D^0\bar{D}^0\pi^0$ .



Belle



BaBar

## Measured Properties of $X(3872)$

- Quantum Numbers compatible with  $J^{PC} = 1^{++}$  (strongly preferred by the data) and  $J^{PC} = 2^{-+}$ .
- Width :  $\Gamma = 3.0 \begin{cases} +2.1 \\ -1.7 \end{cases} \text{ MeV}$
- Mass :  $M_X = 3871.55 \pm 0.20 \text{ MeV}/c^2 \rightarrow$  below  $D^0 \bar{D}^{*0}$  mass threshold of  $3871.80 \pm 0.35 \text{ MeV}/c^2$
- $R_1 = \frac{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 1.0 \pm 0.4 \pm 0.3 \text{ (Belle)} \\ 0.8 \pm 0.3 \text{ (BaBar)} \end{cases},$
- $R_2 = \frac{\mathcal{B}(X \rightarrow J/\psi \gamma)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 0.33 \pm 0.12 \text{ (BaBar)} \\ 0.14 \pm 0.05 \text{ (Belle)} \end{cases},$
- $R_3 = \frac{\mathcal{B}(X \rightarrow \psi(2S) \gamma)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = 1.1 \pm 0.4 \text{ (BaBar)}.$

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*Experimental data suggest a weakly-bound  $D^0 D^{*0}$  molecule coupled to  $2P \ c \bar{c}$  states.*

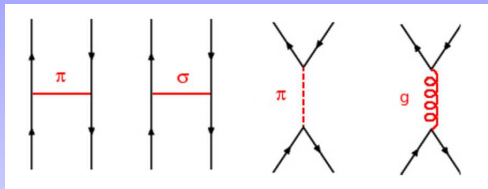
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*P. G. Ortega, J. Segovia, D.R. Entem & F. Fernández Phys. Rev. D* **81**, 054023 (2010)

# Ingredients of constituent quark model

- Model includes:
  - Chiral symmetry breaking → Pseudo-Goldstone Bosons.



- QCD perturbative effects → One Gluon Exchange.
  - Confinement → Non necessary for Meson-Antimeson interaction.
- Interactions:

$$V_{q_i q_j} = \begin{cases} q_i q_j = nn \Rightarrow V_{CON} + V_{OGE} + V_{\pi} + V_{\sigma} \\ q_i q_j = nQ \Rightarrow V_{CON} + V_{OGE} \\ q_i q_j = QQ \Rightarrow V_{CON} + V_{OGE} \end{cases}$$

## Model Results for $1^{--}$ sector.

| (nL)                                         | States       | QM                                           | Exp.                 |
|----------------------------------------------|--------------|----------------------------------------------|----------------------|
| (1S)                                         | $J/\psi$     | 3096                                         | $3096.916 \pm 0.011$ |
| (2S)                                         | $\psi(2S)$   | 3703                                         | $3686.09 \pm 0.04$   |
| (1D)                                         | $\psi(3770)$ | 3796                                         | $3772 \pm 1.1$       |
| (3S)                                         | $\psi(4040)$ | 4097                                         | $4039 \pm 1$         |
| (2D)                                         | $\psi(4160)$ | 4153                                         | $4153 \pm 3$         |
| (4S)                                         | $\psi(4360)$ | 4389                                         | $4361 \pm 9$         |
| (3D)                                         | $\psi(4415)$ | 4426                                         | $4421 \pm 4$         |
| $\begin{bmatrix} (5S) \\ (4D) \end{bmatrix}$ | $\psi(4660)$ | $\begin{bmatrix} 4614 \\ 4641 \end{bmatrix}$ | $4664 \pm 11$        |

**Table:** Masses in MeV of  $J^{PC} = 1^{--}$   $c\bar{c}$  mesons (nL) refers to the dominant partial wave and QM denotes the results of the model.

Reference : *Phys. Rev. D* **78**, 114033 (2008).

## XYZ Mesons

| Meson   | Mass (Exp)    | Candidate?                                                                      | $J^{PC}$ | Mass (Th)                                                           |
|---------|---------------|---------------------------------------------------------------------------------|----------|---------------------------------------------------------------------|
| Y(4360) | $4361 \pm 9$  | $\psi(4360)$                                                                    | $1^{--}$ | 4389                                                                |
| Y(4660) | $4664 \pm 11$ | $\left[ \begin{smallmatrix} (5S) \\ (4D) \end{smallmatrix} \right], \psi(4660)$ | $1^{--}$ | $\left[ \begin{smallmatrix} 4614 \\ 4641 \end{smallmatrix} \right]$ |
| X(4160) | $4156 \pm 15$ | $\eta_{c2}$                                                                     | $2^{-+}$ | 4166                                                                |
| Z(3930) | $3929 \pm 5$  | $\chi_{c2}$                                                                     | $2^{++}$ | 3968                                                                |

**Table:** Candidates for some XYZ mesons in our CQM  $c\bar{c}$  spectrum.

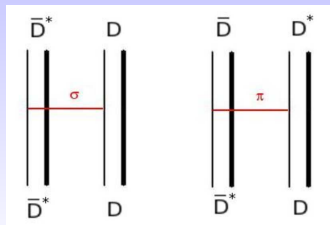
*No  $c\bar{c}$  candidates for X(4260), Y(4008), X(3940), Y(3940), X(3872)*

# Resonating Group Method and Gaussian Expansion Method

- Quark interactions  $\rightarrow$  Cluster interaction.
- Direct RGM Potential:

$${}^{RGM}V_D(\vec{P}', \vec{P}_i) = \sum_{i \in A, j \in B} \int d\vec{p}_{\xi'_A} d\vec{p}_{\xi'_B} d\vec{p}_{\xi_A} d\vec{p}_{\xi_B} \phi_A^*(\vec{p}_{\xi'_A}) \phi_B^*(\vec{p}_{\xi'_B}) V_{ij}(\vec{P}', \vec{P}_i) \phi_A(\vec{p}_{\xi_A}) \phi_B(\vec{p}_{\xi_B})$$

- $\phi_C(\vec{p}_C)$  is the wave function for cluster  $C$  solution of Schrödinger's equation using Gaussian Expansion Method.



## $^3P_0$ Interaction

- Pair creation Hamiltonian:

$$\mathcal{H} = g \int d^3x \bar{\psi}(x) \psi(x)$$

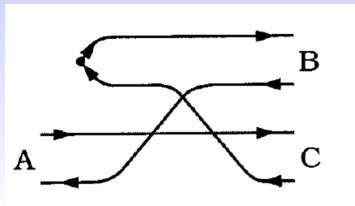
- Non relativistic reduction:

$$T = -3\sqrt{2}\gamma' \sum_{\mu} \int d^3p d^3p' \delta^{(3)}(p+p') \left[ \gamma_1 \left( \frac{p-p'}{2} \right) b_{\mu}^{\dagger}(p) d_{\nu}^{\dagger}(p') \right]^{C=1, I=0, S=1, J=0}$$

with  $\gamma' = 2^{5/2} \pi^{1/2} \gamma$  and  $\gamma = \frac{g}{2m}$

- Transition potential:

$$\langle \phi_{M_1} \phi_{M_2} \beta | T | \psi_{\alpha} \rangle = P h_{\beta\alpha} \delta^{(3)}(\vec{P}_{cm})$$

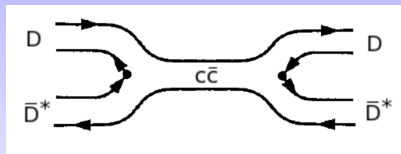


## Modeling the $1^{++}$ sector

- Hadronic state:  $|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\psi\rangle + \sum_{\beta} \chi_{\beta}(P) |\phi_{M1} \phi_{M2} \beta\rangle$
- Solving the coupling with  $c\bar{c}$  states  $\rightarrow$  Schrödinger type equation:

$$\sum_{\beta} \int \left( H_{\beta'\beta}^{M_1 M_2}(P', P) + V_{\beta'\beta}^{\text{eff}}(P', P) \right) \chi_{\beta}(P) P^2 dP = E \chi_{\beta'}(P')$$

with  $V_{\beta'\beta}^{\text{eff}}(P', P)$ :

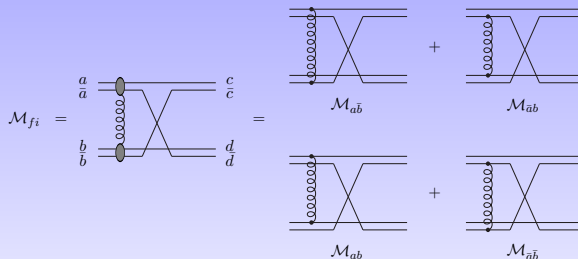


- The  $c\bar{c}$  amplitudes are given by,

$$c_{\alpha} = \frac{1}{E - M_{\alpha}} \sum_{\beta} \int h_{\alpha\beta}(P) \chi_{\beta}(P) P^2 dP$$

## Modeling the $1^{++}$ sector

- Inclusion of  $J/\psi\rho$  and  $J/\psi\omega$  channels necessary for strong decay description  $\rightarrow$  Rearrangement diagrams
- Small contribution to the mass



**Figure:** Diagrams included in the quark rearrangement process  $DD^* \rightarrow \rho(\omega) J/\psi$ .

## Results for $X(3872)$ in a coupled channel approach

| $\gamma$ | $E_{bind}$ | $c\bar{c}(2^3P_1)$ | $D^0 D^{*0}$ | $D^\pm D^{*\mp}$ | $J/\psi \rho$ | $J/\psi \omega$ |
|----------|------------|--------------------|--------------|------------------|---------------|-----------------|
| 0.231    | -0.60      | 12.40              | 79.24        | 7.46             | 0.49          | 0.40            |
| 0.226    | -0.25      | 8.00               | 86.61        | 4.58             | 0.53          | 0.29            |

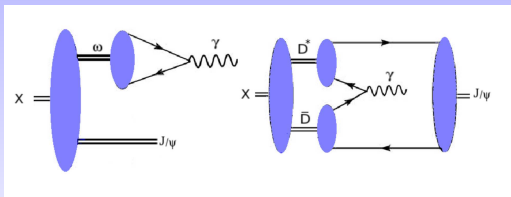
**Table:** Binding energy (in MeV) and channel probabilities (in %) for the  $X(3872)$  state for two values of the  $^3P_0$  model  $\gamma$  parameter.

## Radiative decay description

- Decay through molecular component:

$$\Gamma_{ANN} = \frac{4}{27} \alpha \frac{qE_{\Psi}}{M_X} e^{-\frac{q^2}{2\beta_D^2}} \left( \eta_{00} - \frac{1}{2} \eta_{+-} \right)^2$$

$$\Gamma_{VMD} = \frac{4}{27} \alpha \frac{qE_{\Psi}}{M_X} (3|\phi_{\rho}(r=0)|\chi_{\rho J/\Psi}(q) + |\phi_{\omega}(r=0)|\chi_{\omega J/\Psi}(q))^2$$



- Decay through  $c\bar{c}$  component:

$$\Gamma_{E1} \left( n^{2S+1} L_J \rightarrow n'^{2S'+1} L'_{J'} \right) = \frac{4\alpha e_c^2 q^3}{3} (2J' + 1) S_{fi}^E \delta_{SS'} |\mathcal{E}_{fi}|^2 \frac{E_f}{M_i}$$

## Strong decay description

$$\Gamma_{\pi^+\pi^-J/\psi} = \sum_{J,L} \int_0^{k_{max}} dk \frac{\Gamma_\rho}{(M_X - E_\rho - E_{J/\psi})^2 + \frac{\Gamma_\rho^2}{4}} |\mathcal{M}_{\rho J/\psi}(k)|^2.$$

where

$$\mathcal{M}_{\rho J/\psi} = \int d^3P \chi_{D\bar{D}^*}(P) h_{D\bar{D}^* \rightarrow \rho J/\psi}(P, P').$$

## Decay results

### Strong decay results

- Experimental results:

$$R_1 = \frac{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 1.0 \pm 0.4 \pm 0.3 \\ 0.8 \pm 0.3 \end{cases},$$

- Theoretical results:

| $E_{bind}(\text{MeV})$ | $\Gamma_{\pi^+\pi^-J/\psi}(\text{KeV})$ | $\Gamma_{\pi^+\pi^-\pi^0J/\psi}(\text{KeV})$ | $R_1$ |
|------------------------|-----------------------------------------|----------------------------------------------|-------|
| -0.60                  | 27.61                                   | 14.40                                        | 0.52  |
| -0.25                  | 24.18                                   | 10.64                                        | 0.44  |

# Decay results

## Radiative decay results

- Experimental results:

$$R_2 = \frac{\mathcal{B}(X \rightarrow J/\psi \gamma)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 0.33 \pm 0.12 \\ 0.14 \pm 0.05 \end{cases},$$

- Theoretical results:

| $E_{bind}(\text{MeV})$ | $\Gamma_{J/\psi \gamma}^{VMD}$ | $\Gamma_{J/\psi \gamma}^{ANN}$ | $R_2^M$             | $\Gamma_{J/\psi \gamma}^{c\bar{c}}$ | $R_2^{c\bar{c}}$ | $R_2$ |
|------------------------|--------------------------------|--------------------------------|---------------------|-------------------------------------|------------------|-------|
| -0.60                  | 0.014                          | 0.056                          | $2.5 \cdot 10^{-3}$ | 8.15                                | 0.29             | 0.30  |
| -0.25                  | 0.011                          | 0.045                          | $2.3 \cdot 10^{-3}$ | 5.25                                | 0.22             | 0.22  |

**Table:** Decays in KeV.

Molecular component  $\rightarrow$  Vector meson dominance (VMD) and Annihilation (ANN) mechanisms.

$R_2^M$  is the ratio including only the contributions from the molecular part.

$R_2^{c\bar{c}}$  from the  $c\bar{c}$  component and  $R_2$  is the full result.

# Decay results

## Radiative decay results

- Experimental results:

$$R_3 = \frac{\mathcal{B}(X \rightarrow \psi(2S)\gamma)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = 1.1 \pm 0.4.$$

- Theoretical results:

| $E_{bind}(\text{MeV})$ | $\Gamma_{\psi(2S)\gamma}^{ANN}$ | $R_3^M$             | $\Gamma_{\psi(2S)\gamma}^{c\bar{c}}$ | $R_3^{c\bar{c}}$ | $R_3$ |
|------------------------|---------------------------------|---------------------|--------------------------------------|------------------|-------|
| -0.60                  | 0.134                           | $4.8 \cdot 10^{-3}$ | 9.80                                 | 0.35             | 0.34  |
| -0.25                  | 0.101                           | $4.2 \cdot 10^{-3}$ | 6.31                                 | 0.26             | 0.26  |

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$R_3^{c\bar{c}}$  from the  $c\bar{c}$  component and  $R_3$  is the full result.

## X(3872) as a $J^{PC} = 2^{-+}$ state

- Ref: arxiv/hep-ex:1005.5190
- $DD^*$  P-wave molecule  $\rightarrow$  Need to increase the strength of  $g_{ch}$ .

$$g_{ch}'^2 \approx 21 g_{ch}^2 \quad (1)$$

- Coupling to  $\eta_{c2}(3811 \text{ MeV})$  and  $\eta_{c2}(4166 \text{ MeV})$ ?  $\rightarrow {}^3P_0$  potential smaller than in  $1^{++}$  sector.
- Results for isospin ratio:

$$R_1 = \frac{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = 0.0051,$$

- **BUT...** If we increase  $g_{ch}$  we find 4  $DD^*$  states in  $1^{++}$  !:  
3871.8 MeV, 3852.1 MeV, 3731.6 MeV and 3721.7 MeV.

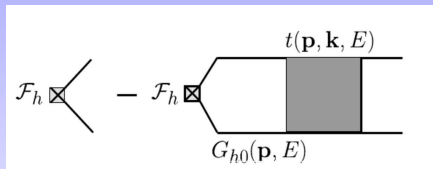
*Unlikely Scenario  $\rightarrow$  not a molecule*

# Lineshapes

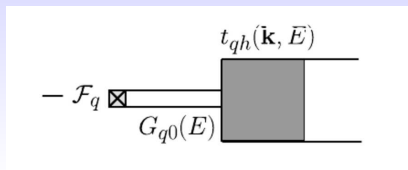
- Lippmann-Schwinger equation  $\rightarrow t^{\beta\beta'}(\vec{p}, \vec{p}', E)$  matrix
- Lineshapes

$$\frac{dB_r((M_1 M_2)^\beta)}{dE} = \text{const} \times k |\mathcal{M}^\beta(E)|^2 \Theta(E)$$

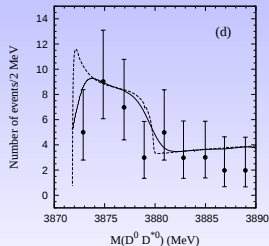
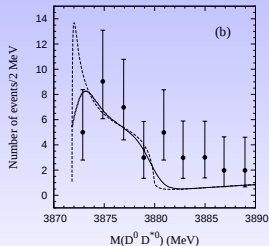
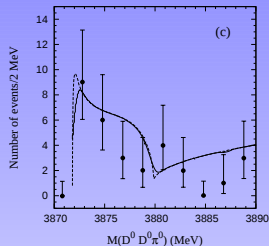
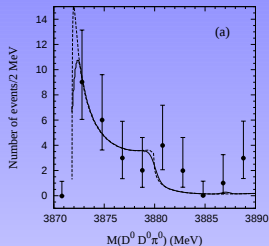
- Hadronic contribution  $\mathcal{M}_h^\beta(E)$



- Mesonic contribution  $\mathcal{M}_q^\beta(E)$



# Lineshapes for $E_b = -0.25 \text{ MeV}$



Belle and BaBar data for the  $B \rightarrow KD^0 \bar{D}^0 \pi^0$  (Belle) and  $B \rightarrow KD^0 \bar{D}^{*0}$  (BaBar) reactions.

# $Y(4008)$

- Mass and Width

$$J^{PC} = 1^{--},$$

$$M_Y = 4008 \pm 40^{+114}_{-28} \text{ MeV},$$

$$\Gamma_Y = 226 \pm 44 \pm 87 \text{ MeV}.$$

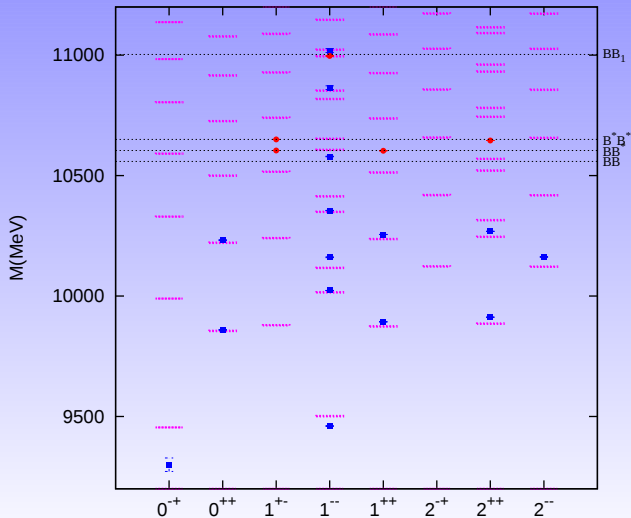
| Mass (MeV) | $c\bar{c}(2^3S_1)$ | $c\bar{c}(3^3D_1)$ | $c\bar{c}(4^3S_1)$ | $D^*D^*$ |
|------------|--------------------|--------------------|--------------------|----------|
| 3650.973   | 92.45 %            | 0.22 %             | 0.01 %             | 7.30 %   |
| 3793.410   | 0.33 %             | 99.11 %            | 0.00 %             | 0.56 %   |
| 4016.423   | 0.59 %             | 0.03 %             | 34.53 %            | 64.53 %  |
| 4036.804   | 0.70 %             | 0.03 %             | 48.73 %            | 50.00 %  |

**Table:** Mass and probabilities with  $^3P_0$   $\gamma$  fitted to  $\psi(3770) \rightarrow DD$  decay.

# Hidden bottom sector

| Mesons        | Threshold    | $J^{PC}$                                               | $^3P_0$                                                                                          | without $^3P_0$                                                                                  |
|---------------|--------------|--------------------------------------------------------|--------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|
| $BB$          | 10558.56 MeV | $0^{++}$                                               | $g'_{ch}{}^2 = 2.9g_{ch}^2$                                                                      | -0.02 MeV                                                                                        |
| $BB^*$        | 10604.38 MeV | $\begin{cases} 1^{++} \\ 1^{+-} \end{cases}$           | $\begin{cases} -1.31 \text{ MeV} \\ -0.01 \text{ MeV} \end{cases}$                               | $\begin{cases} -8.96 \text{ MeV} \\ -0.05 \text{ MeV} \end{cases}$                               |
| $B^*B^*$      | 10650.20 MeV | $\begin{cases} 0^{++} \\ 1^{+-} \\ 2^{++} \end{cases}$ | $\begin{cases} g'_{ch}{}^2 = 2.70g_{ch}^2 \\ -0.04 \text{ MeV} \\ -4.02 \text{ MeV} \end{cases}$ | $\begin{cases} g'_{ch}{}^2 = 2.80g_{ch}^2 \\ -0.04 \text{ MeV} \\ -9.26 \text{ MeV} \end{cases}$ |
| $BB_1(^1P_1)$ | 11002.5 MeV  | $\begin{cases} 1^{-+} \\ 1^{--} \end{cases}$           | $\begin{cases} - \\ -5.2 \text{ MeV} \end{cases}$                                                | $\begin{cases} g'_{ch}{}^2 = 1.3g_{ch}^2 \\ g'_{ch}{}^2 = 1.13g_{ch}^2 \end{cases}$              |
| $BB_1(^3P_1)$ | 11002.5 MeV  | $\begin{cases} 1^{-+} \\ 1^{--} \end{cases}$           | $\begin{cases} - \\ -4.61 \text{ MeV} \end{cases}$                                               | $\begin{cases} g'_{ch}{}^2 = 1.3g_{ch}^2 \\ -0.002 \text{ MeV} \end{cases}$                      |

# Hidden bottom sector



## Summary

- $X(3872)$  found as a weakly-bound  $D^0 D^{*0}$  molecule coupled to  $2^3P_1 c\bar{c}$  state.
- Good description of the radiative and strong decays. Although  $X(3872) \rightarrow \psi(2S)\gamma$  lower than expected.
- Lineshapes in good agreement with the data, specially for the mesonic production.
- $Y(4008)$  found as molecule in the same formalism, coupled to  $c\bar{c} \rightarrow$  Experimental confirmation needed.
- Rich spectroscopy in the hidden bottom sector.

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## Summary

Thanks for your attention

$D_{s1}(2460)$ 

- Close thresholds

$$D^* K \longrightarrow M = 2504.16 \text{ MeV},$$

$$DK^* \longrightarrow M = 2763.70 \text{ MeV},$$

$$D^* K^* \longrightarrow M = 2904.84 \text{ MeV}.$$

- Close  $D_{s1}$  states

$$1^3P_1 \longrightarrow M = 2571.475 \text{ MeV},$$

$$1^1P_1 \longrightarrow M = 2575.934 \text{ MeV},$$

| $M \text{ (MeV)}$ | $D_{s1}(1^3P_1)$ | $D_{s1}(1^1P_1)$ | $D^* K$ | $DK^*$ | $D^* K^*$ |
|-------------------|------------------|------------------|---------|--------|-----------|
| 2501.628          | 72.03 %          | 0 %              | 23.5 %  | 4.47 % | 0 %       |
| 2430.604          | 0 %              | 75.13 %          | 14.52 % | 4.57 % | 5.78 %    |
| 2494.290          | 52.62 %          | 9.32 %           | 33.92 % | 3.40 % | 0.73 %    |

**Table:** Mass and probabilities with  $^3P_0 \gamma$  fitted to  $\psi(3770) \rightarrow DD$  decay.

## Strong decay description

$$\Gamma_{\pi^+\pi^-J/\psi} = \sum_{J,L} \int_0^{k_{\max}} dk \frac{\Gamma_\rho}{(M_X - E_\rho - E_{J/\psi})^2 + \frac{\Gamma_\rho^2}{4}} |\mathcal{M}_{\rho J/\psi}(k)|^2.$$

where

$$\mathcal{M}_{\rho J/\psi} = \int d^3P \chi_{D\bar{D}^*}(P) h_{D\bar{D}^* \rightarrow \rho J/\psi}(P, P').$$

$$\Gamma_{\pi^+\pi^-J/\psi} = \sum_{J,L} \int_0^{k_{\max}} dk \Gamma_\rho |\chi_{\rho J/\psi}(k)|^2.$$

As the calculation is perturbative for the  $\rho J/\psi$  component we have the relation

$$\chi_{\rho J/\psi} = \frac{\mathcal{M}_{\rho J/\psi}}{M_X - E_\rho - E_{J/\psi}}$$