

Kaon Fragmentation Function from NJL-Jet

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Collaborators:

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Outlook

- ✦ Motivation
- ✦ Strange NJL-jet: Distribution & Fragmentation Functions
- ✦ Monte-Carlo simulations and inclusion of $p_{\perp}$

Hadron Structure with SIDIS

A. Kotzinian, Nucl. Phys. B441, 234 (1995).

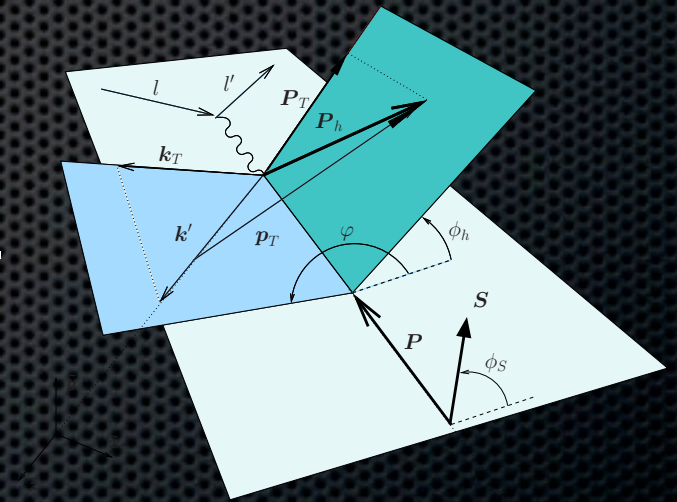
- Semi-inclusive deep inelastic scattering (SIDIS): $e N \rightarrow e h X$
- Cross-section factorizes into parton distribution and fragmentation functions.

Access to hadron structure:

- Ex., unpolarized cross section is $\sim$

$$\sum_q e_q^2 \int d^2 \mathbf{k}_\perp f_1^q(x, k_\perp) \pi y^2 \frac{\hat{s}^2 + \hat{u}^2}{Q^4} D_q^h(z, p_\perp)$$

- NJL provides a sound framework for calculating both!



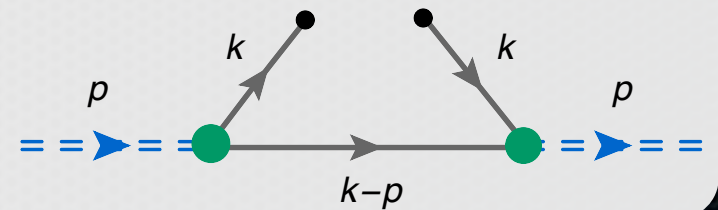
Distribution and Fragmentation

Ito et al. Phys.Rev.D80:074008,2009

Probability of finding quark $q(x)$ in hadron h :

$$f_q^h(x) = p_- \int d^2 k_T \sum_{\alpha} \frac{\langle p | b_{\alpha}^{\dagger}(p) b_{\alpha}(p) | p \rangle}{\langle p | p \rangle}$$

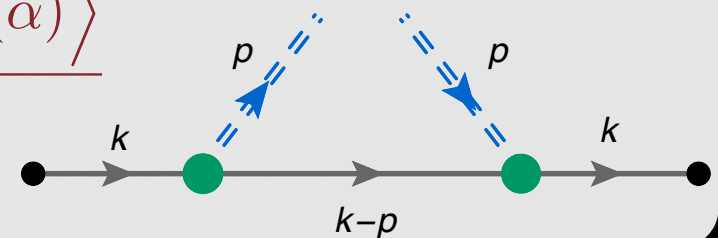
$$x = k_- / p_-$$



Probability of finding hadrons $h(z)$ in cloud of q :

$$D_q^h(z) = \frac{1}{6} dp_- \int d^2 p_{\perp} \sum_{\alpha} \frac{\langle k(\alpha) | a_h^{\dagger}(p) a_h(p) | k(\alpha) \rangle}{\langle k(\alpha) | k(\alpha) \rangle}$$

$$z = p_- / k_-$$

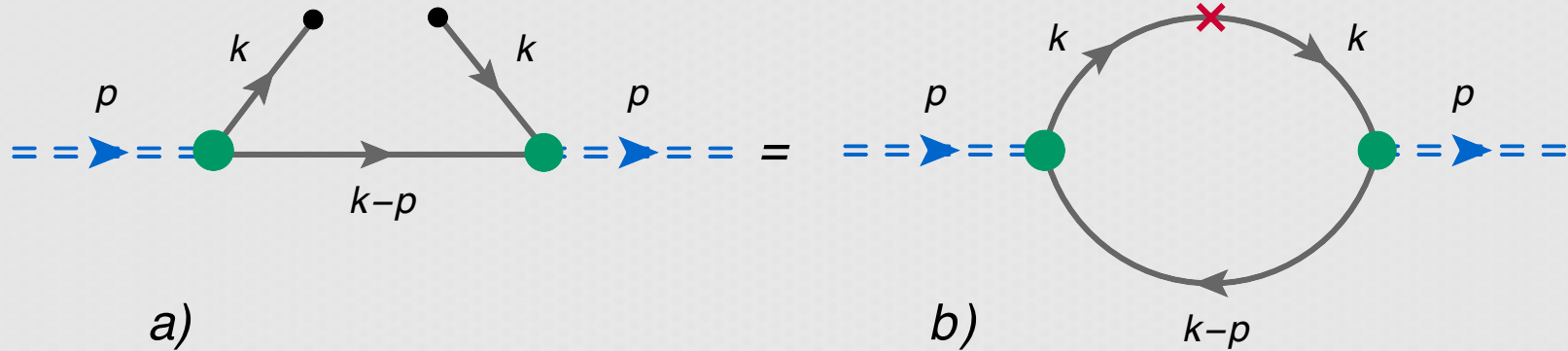


Motivation

- ✦ Effective quark model descriptions of fragmentation functions usually employ “elementary” one-step process.
- ✦ The resulting fragmentation functions are too small compared to data (e.g. M. Hirai et al: PRD 75 (2007) 094009.).
- ✦ Multiple hadron emission is mimicked by introducing “normalization factors”.

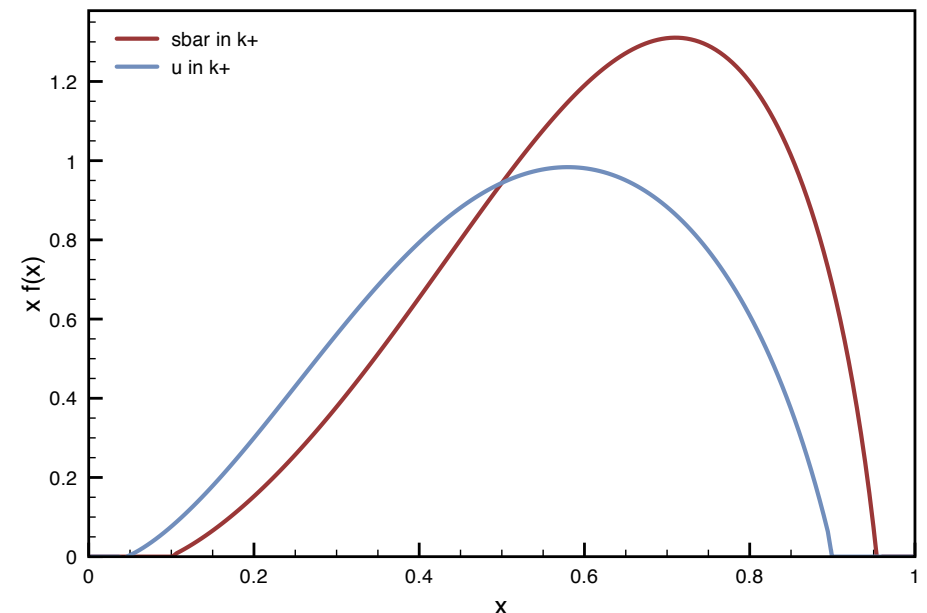
K Distribution Function in NJL

Bentz et al.: Nucl.Phys.A651:143-173,1999.



✧ Regularization:

Lepage-Brodsky (LB)
Invariant Mass Cutoff

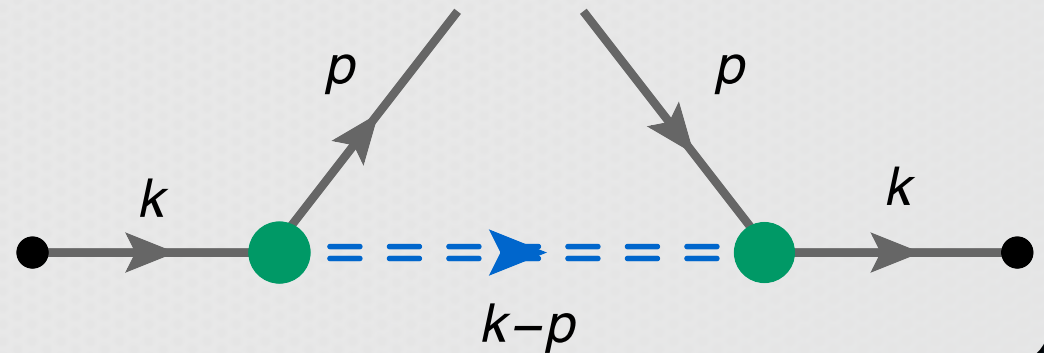


Splitting Functions

- One-quark truncation of the wavefunction:

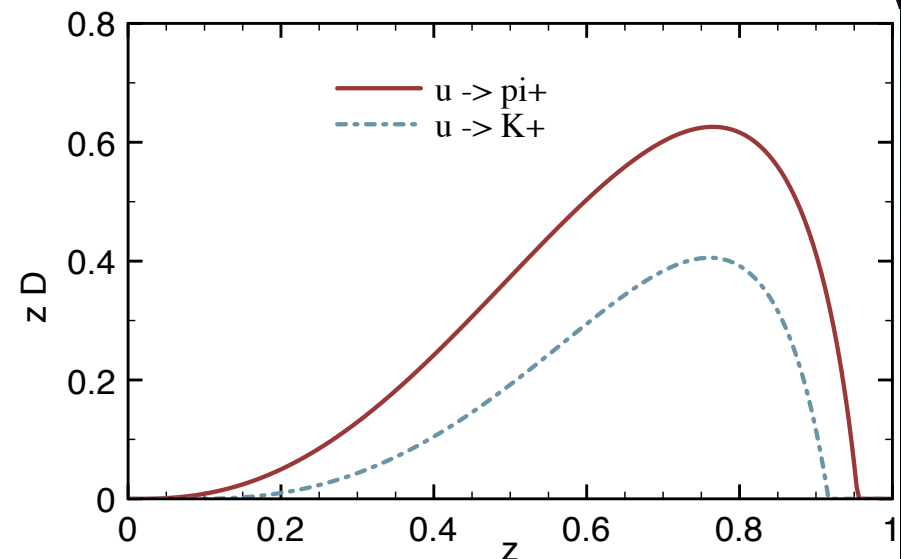
$$d_q^m(z) : q \rightarrow Qm$$

$$m = q\bar{Q}$$



$$u \rightarrow d\pi^+$$

$$u \rightarrow sK^+$$

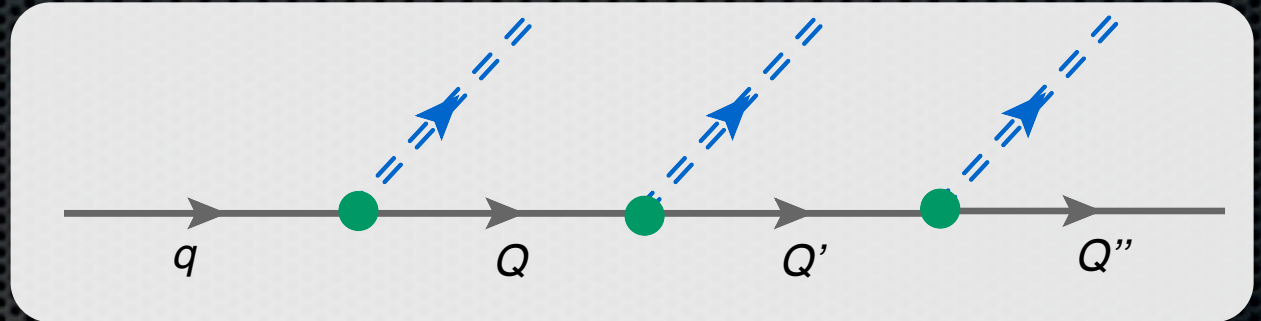


NJL-jet Model for Fragmentation Function

Field, Feynman.Nucl.Phys.B136:1,1978.

✦ Chain Decay:

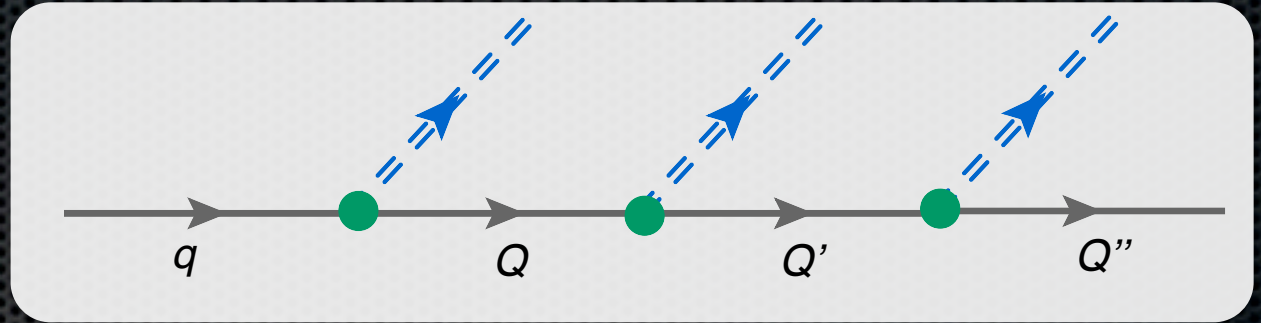
No re-absorption



$$D_q^m(z) = \hat{d}_q^m(z) + \int_z^1 \frac{dy}{y} \hat{d}_q^Q\left(\frac{z}{y}\right) \cdot D_Q^m(y)$$

$$\hat{d}_q^m(z) = \hat{d}_q^{Q'}(1-z)|_{m=\bar{Q}'q}$$

Probabilistic Interpretation of Integral Equation



The probability of finding mesons m with mom. fraction z in a jet of quark q

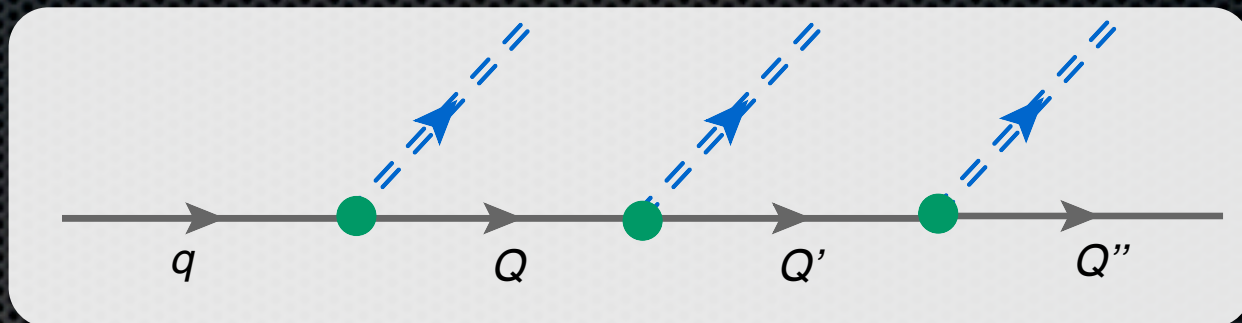
$$D_q^m(z)dz = \hat{d}_q^m(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^m\left(\frac{z}{y}\right)\frac{dz}{y}$$

Probability of emitting the meson at link 1

Probability that Momentum fraction y is transferred to jet at step 1

The probability scales with mom. fraction

Probabilistic Interpretation of Integral Equation

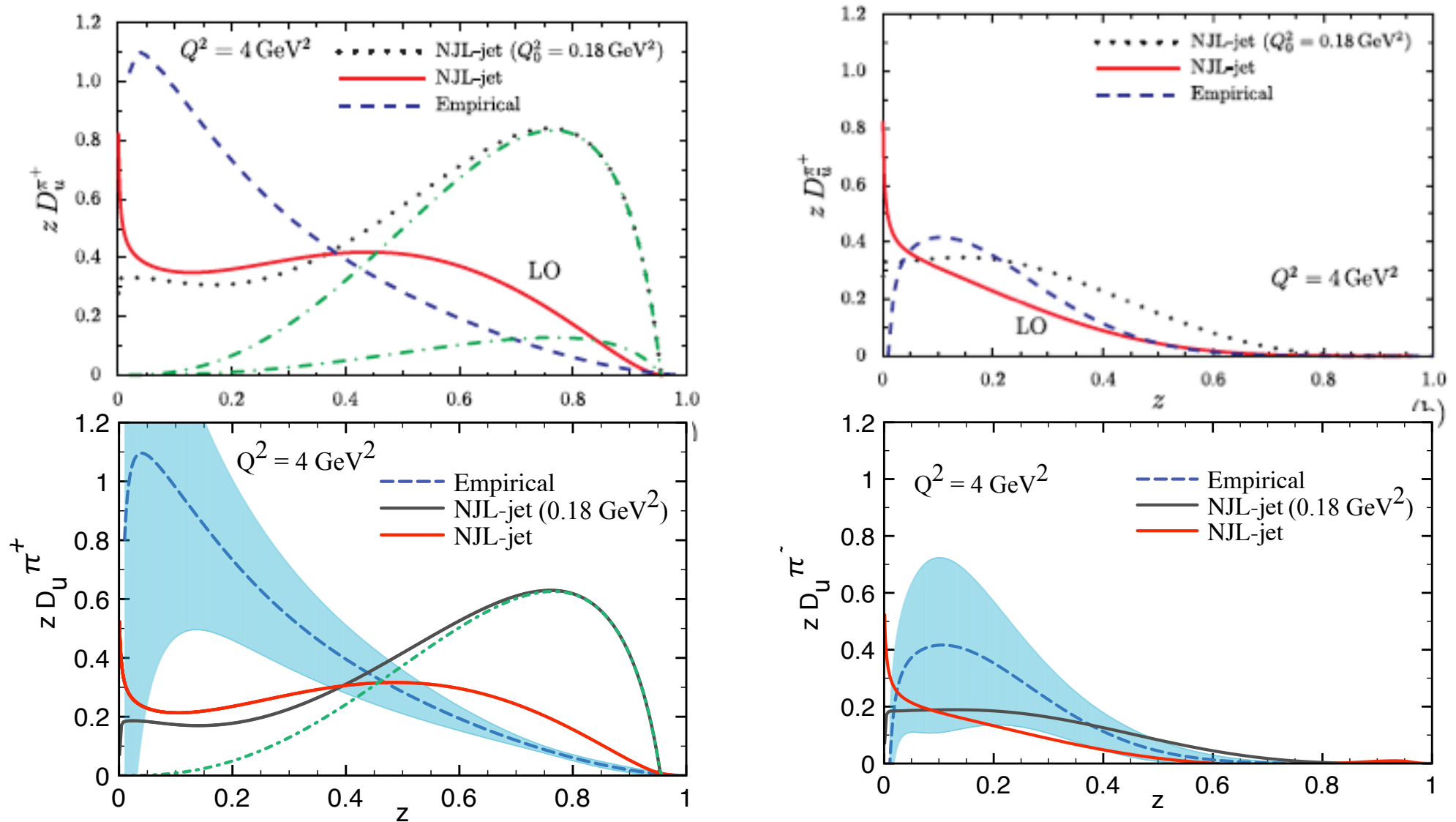


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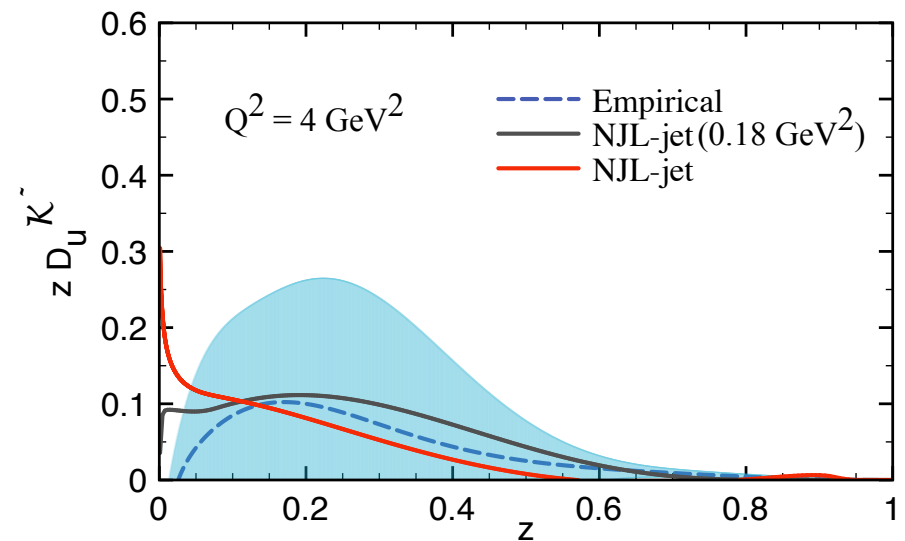
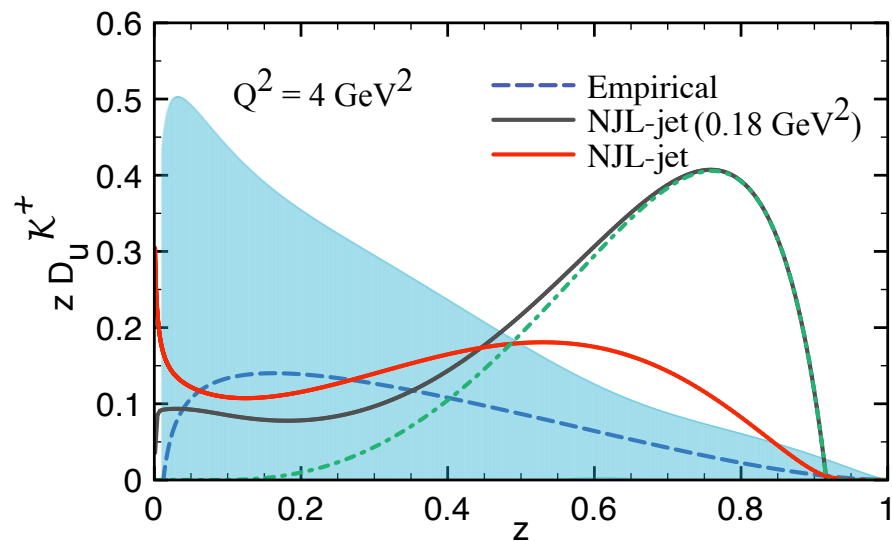
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Strangeness Effect in Pion

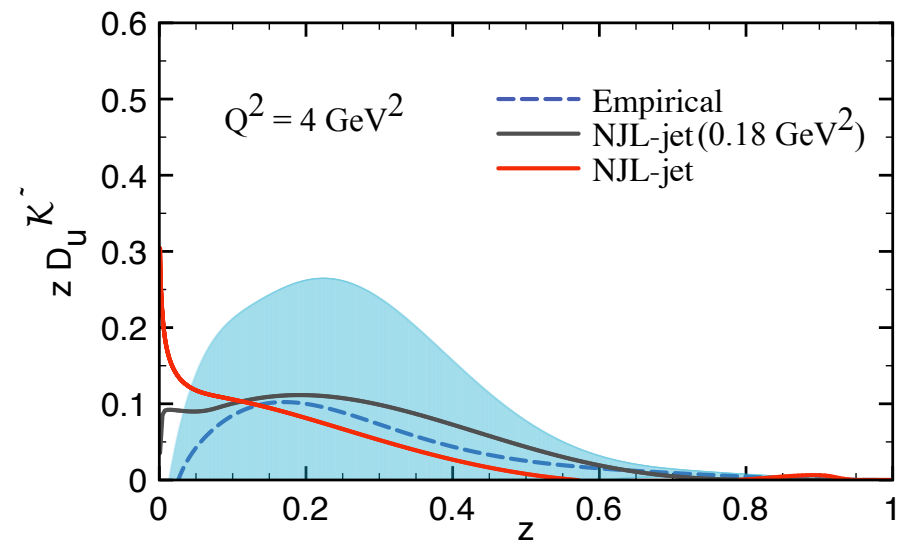
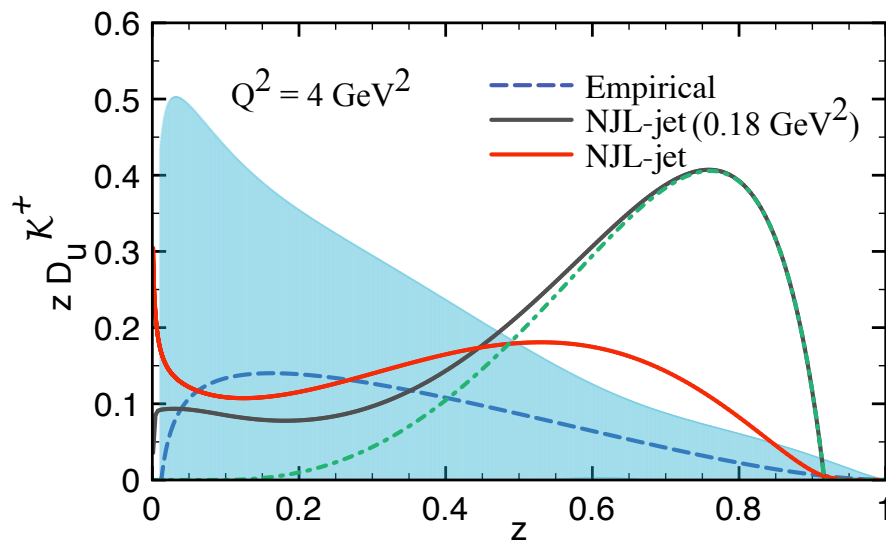
Ito et al. Phys.Rev.D80:074008,2009



Results for Kaon

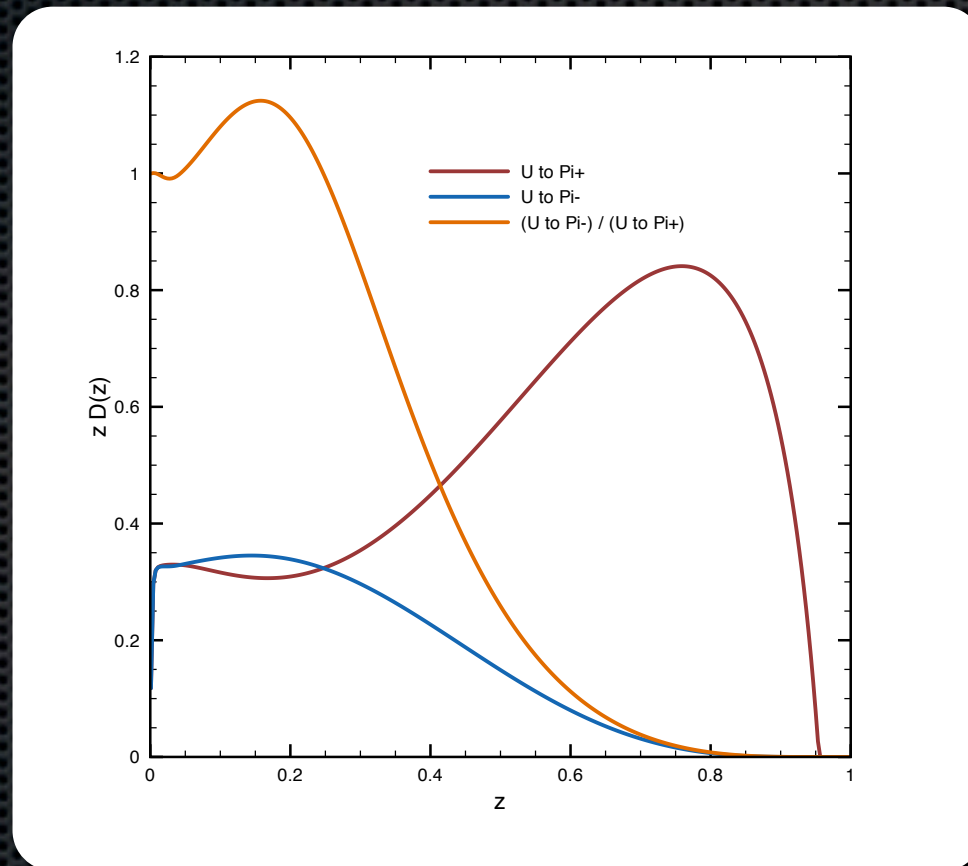


Results for Kaon



Momentum and Isospin Sum Rules Satisfied!

Unfavored to Favored Ratio

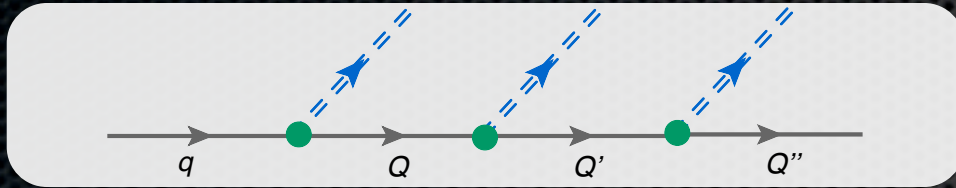


Monte-Carlo (MC) Simulations

- ✦ Simulate decay chains to extract probabilities
- ✦ Allows for inclusion of $p_{\perp}$
- ✦ Numerically trivially parallelizeable (MPI, GPGPU)

Fragmentations from MC

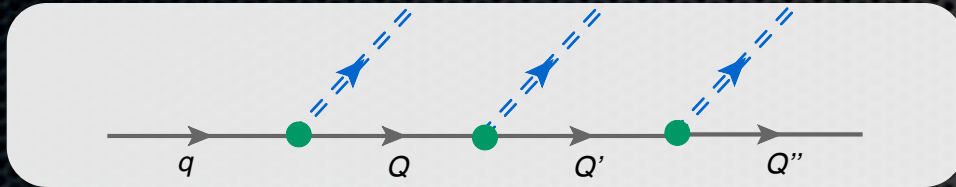
- ✧ Assume Cascade process:



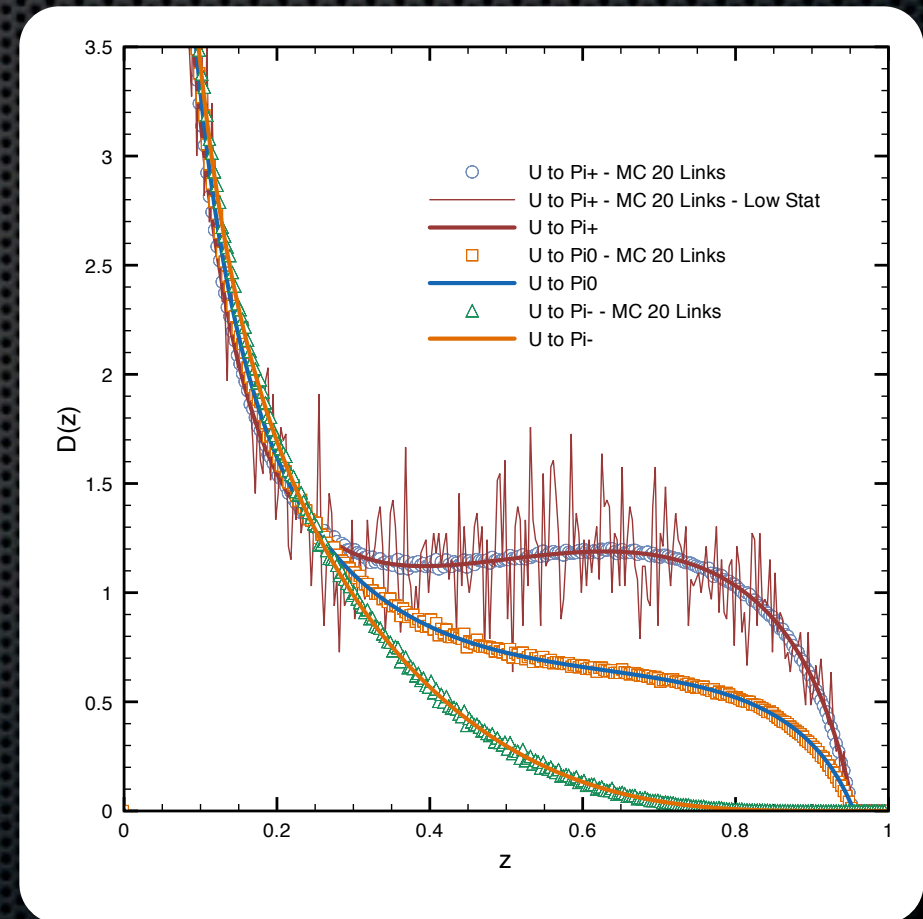
- ✧ Randomly sample z from input splittings.
- ✧ Evolve to sufficiently large number of decay links.
- ✧ Repeat for decay chains with the same initial quark.

Fragmentations from MC

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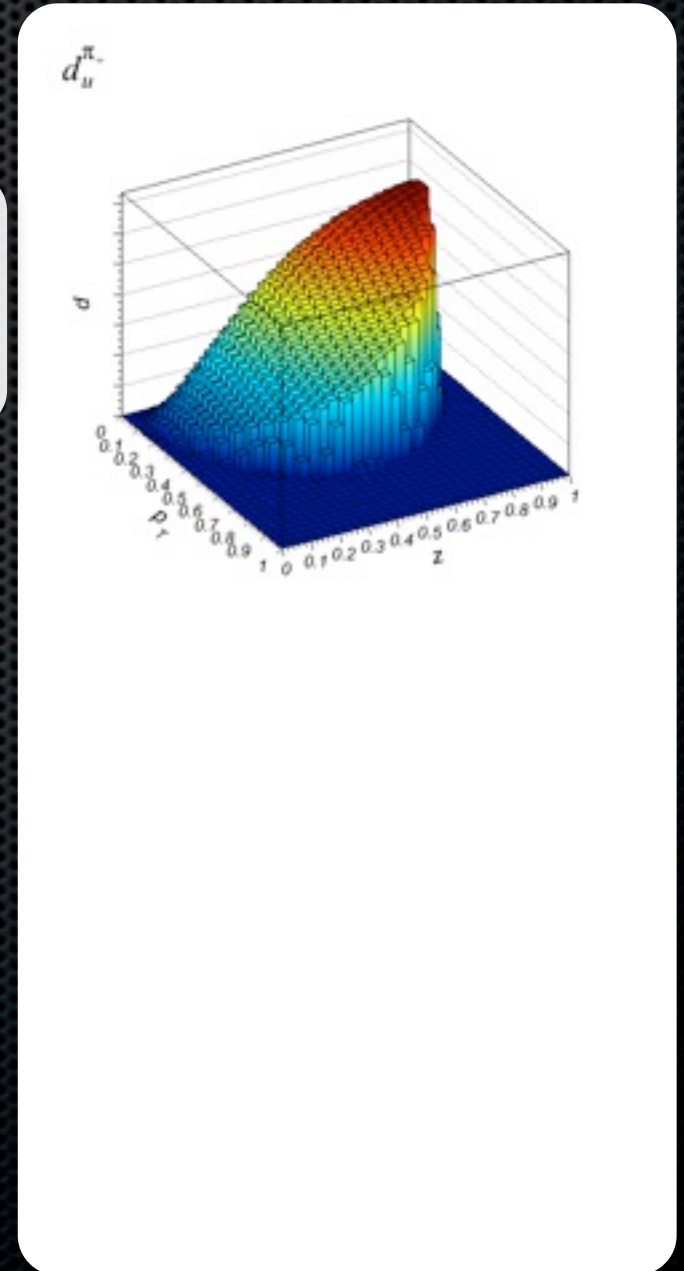


Including Transverse Momenta

- ✦ $p_{\perp}$ -dependent splittings: $d_q^m(z, p_{\perp})$

$$\frac{C_I}{2} g_{mqq}^2 z \frac{p_{\perp}^2 + ((z-1)M_1 + M_2)^2}{(p_{\perp}^2 + z(z-1)M_1^2 + zM_2^2 + (1-z)m_m^2)^2}.$$

- ✦ Conserve transverse momenta at each link.

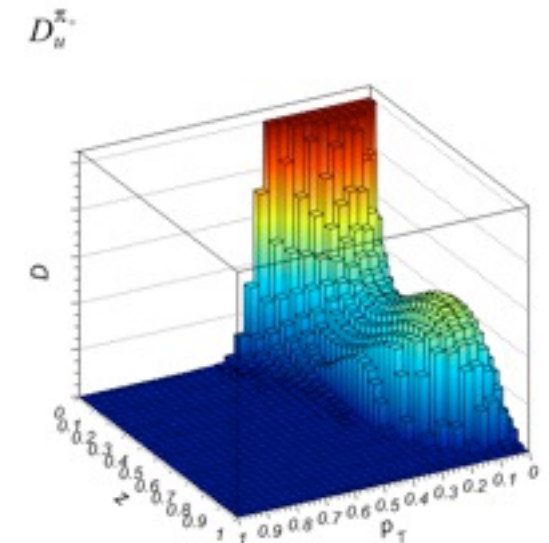
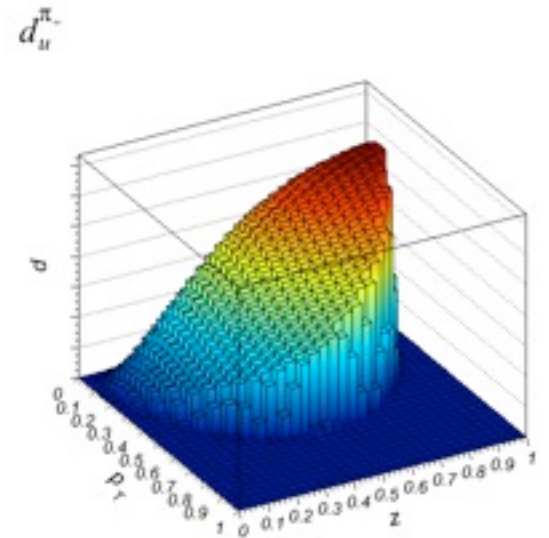


Including Transverse Momenta

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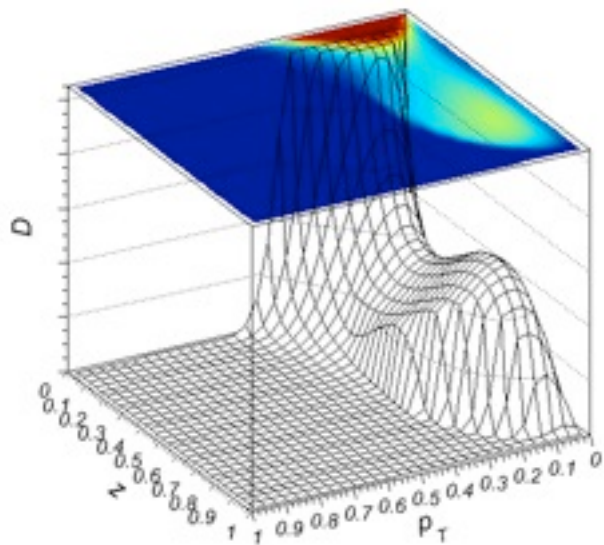
$$\frac{C_I}{2} g_{mq}^2 z \frac{p_{\perp}^2 + ((z-1)M_1 + M_2)^2}{(p_{\perp}^2 + z(z-1)M_1^2 + zM_2^2 + (1-z)m_m^2)^2}.$$

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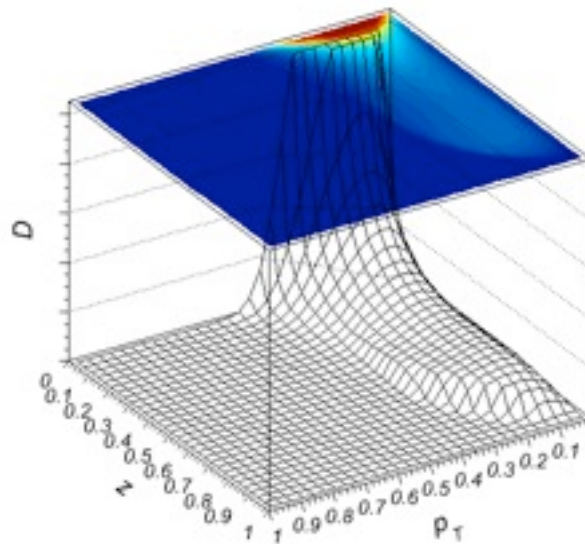


More Results with $p_{\perp}$

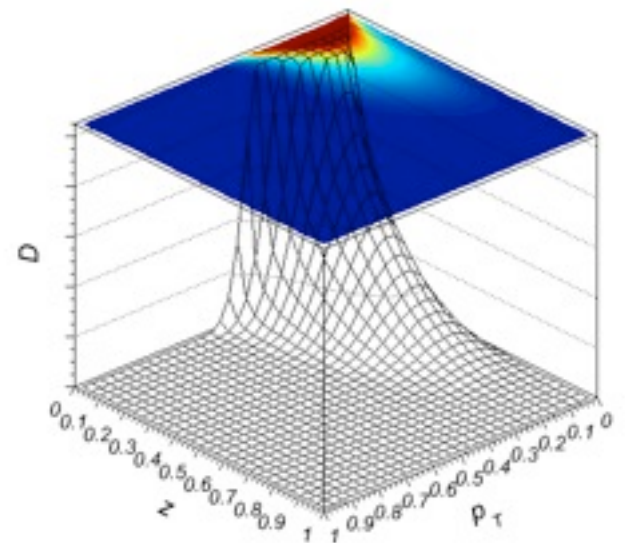
$D_u^{\pi_+}$



$D_u^{\pi_0}$



$D_u^{\pi_-}$



Outlook

- ✦ NJL-jet model: coupled channel cascade description of fragmentation function.
- ✦ Preliminary results: qualitative agreement with empirical parametrizations of experimental data.
- ✦ Improvements: Include vector mesons and ~~baryons~~ in fragmentation process, NLO DGLAP evolution.
- ✦ Improvements in MC.
- ✦ Polarized Fragmentation Functions