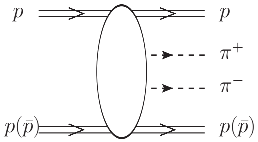
Exclusive production of $\pi^+\pi^-$ pairs in pp and $p\bar{p}$ collisions

Piotr Lebiedowicz
in collaboration with A. Szczurek

Institute of Nuclear Physics PAN, Cracow



11th International Workshop on Meson Production, Properties and Interaction
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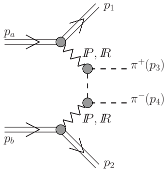
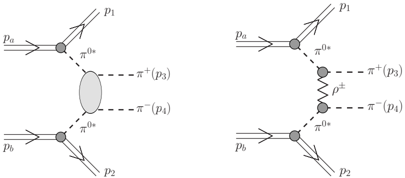
Outline

- πp elastic scattering
- Mechanisms of 4-body processes $pp \to pp\pi^+\pi^-$ and $p\bar{p} \to p\bar{p}\pi^+\pi^-$ (formalism and results)

Central double-difractive mechanism



Pion-pion rescattering mechanisms



with amplitude from PWA

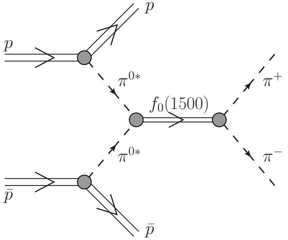
- Conclusions

Background to 3-body processes

The 4-body reactions $pp \to pp\pi^+\pi^-$ and $p\bar{p} \to p\bar{p}\pi^+\pi^-$ constitutes an irreducible background to 3-body processes $pp \to ppM$

M are a broad resonances in the $\pi\pi$ (and/or KK) channels

M	decay modes	
σ	$\pi\pi$	dominant
$\rho^0(770)$	$\pi^+\pi^-$	$\sim$ 98.9%
$f_0(980)$	$\pi\pi$	dominant
$\phi(1020)$	K^+K^-	$\sim$ 48.9%
$f_2(1275)$	$\pi\pi$	$\sim$ 84.8%
$f_0(1500)$	$\pi\pi$	$\sim$ 34.9%
	4π	$\sim$ 49.5%
	$K\bar{K}$	$\sim$ 8.6%
...		

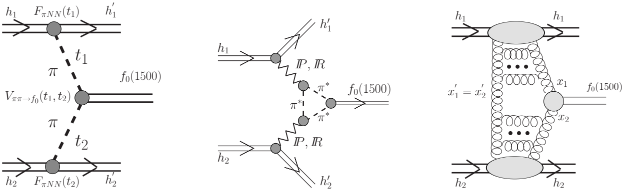


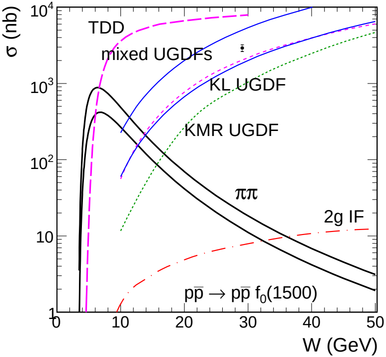
A. Szczurek and P. Lebiedowicz, Nucl. Phys. A826 (2009) 101

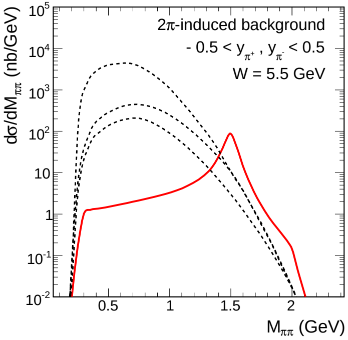
(production of $f_0(1500)$ meson and estimate the $\pi\pi$ background at PANDA energy)

Exclusive production of $f_0(1500)$ meson

$\pi\pi$ fusion, double-diffractive with pionic loop and gluon induced diff. (QCD) mechanisms

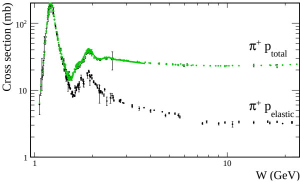


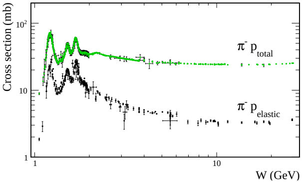




The πp total cross section

The optical theorem: $\sigma_{tot}^{\pi p} \simeq \frac{1}{s} Im\mathcal{M}_{el}^{\pi p}(s, t=0)$ (when s is large)





The Donnachie-Landshoff fits to the $\pi^+ p$ and $\pi^- p$ total cross sections ($\sigma_{tot}^{\pi p} = C_i s^{\alpha_i(0)-1}$):

$$\sigma_{tot}^{\pi^+ p}: \; 13.63 s^{0.0808} + 27.56 s^{-0.4525}$$
$$\sigma_{tot}^{\pi^- p}: \; 13.63 s^{0.0808} + 36.02 s^{-0.4525}$$

imply the values of pomeron and reggeon couplings to πp:
$C_{I\!P} = 13.63 mb$, $C_{f_2} = 31.79 mb$, $C_\rho = 4.23 mb$

$$\mathcal{M}(\pi^+ p \to \pi^+ p) = M_{I\!P} + M_{f_2} - M_\rho$$
$$\mathcal{M}(\pi^- p \to \pi^- p) = M_{I\!P} + M_{f_2} + M_\rho$$
$$M_{f_2}(C=+1) > M_\rho(C=-1)$$

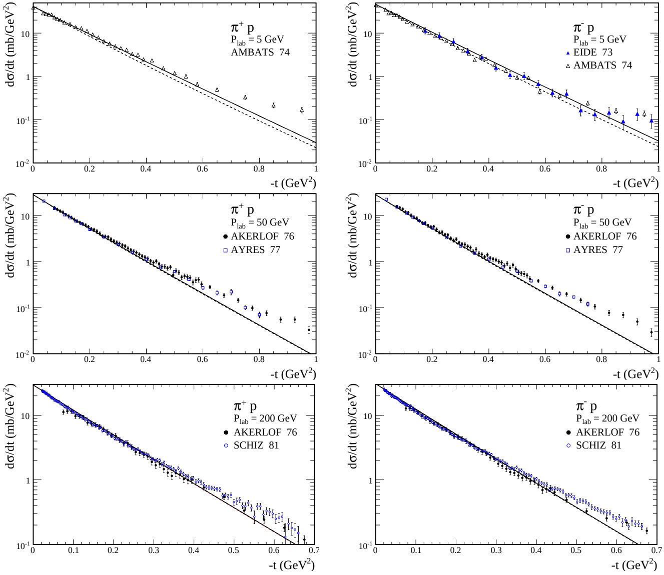
The πp elastic scattering

$$
\begin{aligned}
\mathcal{M}_{\pi^\pm p\to\pi^\pm p}(s,t) \;=\;& \mathrm{i}\, s\, C_{I\!P} \left(\frac{s}{s_0}\right)^{\alpha_{I\!P}(t)-1} \exp\left(\frac{B^{I\!P}_{\pi p}}{2}t\right) \\
&+\; (\mathbf{a}_{\mathrm{f}_2}+\mathrm{i})\, s\, C_{f_2} \left(\frac{s}{s_0}\right)^{\alpha_{I\!R}(t)-1} \exp\left(\frac{B^{I\!R}_{\pi p}}{2}t\right) \\
&\pm\; (\mathbf{a}_{\rho}-\mathrm{i})\, s\, C_{\rho} \left(\frac{s}{s_0}\right)^{\alpha_{I\!R}(t)-1} \exp\left(\frac{B^{I\!R}_{\pi p}}{2}t\right)
\end{aligned}
$$

- where $\mathbf{a}_{\mathrm{f}_2}$ = -0.860895 and $\mathbf{a}_\rho$ = -1.16158 are signature factors
- $s_0 = 1\ \mathrm{GeV}^2$
- the parameters of pomeron and reggeons couplings are taken from the Donnachie-Landshoff model for total cross section ($C_{I\!P} = 13.63mb$, $C_{f_2} = 31.79mb$, $C_\rho = 4.23mb$)
- the trajectories determined from total and elastic cross sections ($\alpha_i(t) = \alpha_i(0) + \alpha'_i t$):

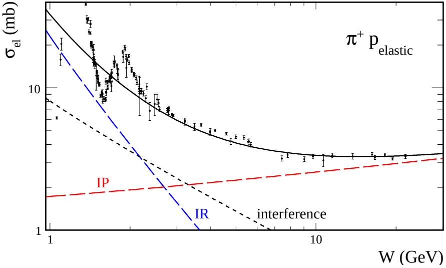
$$\alpha_{I\!P}(t) = 1.0808 + 0.25t, \quad \alpha_{I\!R}(t) = 0.5475 + 0.93t$$

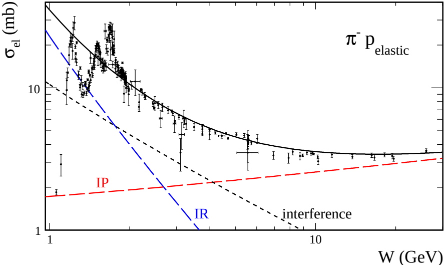
- the effective slope parameter $B(s) = B_{\pi p} + 2\alpha'_i \ln(\frac{s}{s_0})$



- solid lines: $B_{I\!P} = 5.5$ GeV^{-2} and $B_{I\!R} = 4$ GeV^{-2}
- dashed lines: $B_{I\!P} = B_{I\!R} = 5.5$ GeV^{-2}

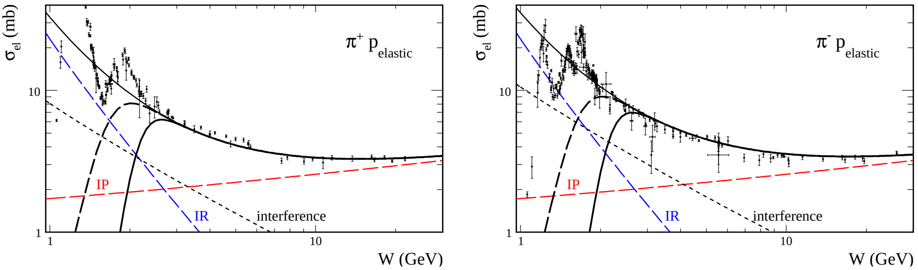
The πp elastic scattering





- nicely describes the πp data for $\sqrt{s} > 2.5$ GeV (with $B_{I\!P} = 5.5$ GeV^{-2} and $B_{I\!R} = 4$ GeV^{-2})

The πp elastic scattering



- smooth cut-off correction factor

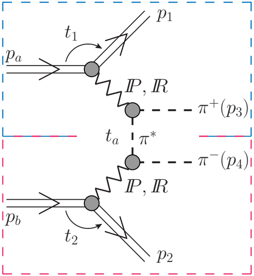
$$f_{cont}^{\pi N}(W) = \frac{\exp\left(\frac{W - W_{cut}}{a_{cut}}\right)}{1+\exp\left(\frac{W-W_{cut}}{a_{cut}}\right)}, \quad W_{cut} = 1.5 \text{ GeV (dashed line) and } 2 \text{ GeV (solid line)}$$

The parameter W_{cut} gives the position of the cut and parameter $a_{cut} = 0.2$ GeV describes how sharp is the cut off.

- model includes absorption effects in an effective way

Central double diffractive contribution

$$\mathcal{M}^{pp\to pp\pi\pi} = M_{13}(t_1, s_{13}) F_{off}(t_a) \frac{1}{t_a - m_\pi^2} F_{off}(t_a) M_{24}(t_2, s_{24}) \times f_{cont}^{\pi p}(W_{13}) \times f_{cont}^{\pi p}(W_{24})$$
$$+ M_{14}(t_1, s_{14}) F_{off}(t_b) \frac{1}{t_b - m_\pi^2} F_{off}(t_b) M_{23}(t_2, s_{23}) \times f_{cont}^{\pi p}(W_{14}) \times f_{cont}^{\pi p}(W_{23})$$



M_{ik} – πp "interaction" in (i,k) subsystems between forward nucleon $(i=1)$ or backward nucleon $(i=2)$ and one of two pions π^+ $(k=3)$, π^- $(k=4)$

+ crossed diagram $(3 \leftrightarrow 4)$

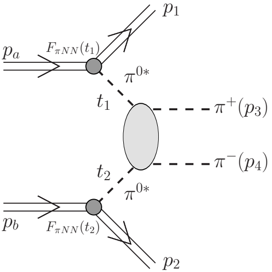
(J. Pumplin and F.S. Henyey)

$$M_{ik}(t_i, s_{ik}) = \mathrm{i}\, s_{ik}\, C_{I\!P} \left(\frac{s_{ik}}{s_0}\right)^{\alpha_{I\!P}(t_i)-1} \exp\left(\frac{B_{I\!P}}{2}\, t_i\right)$$
$$+ (a_{f_2} + \mathrm{i})\, s_{ik}\, C_{f_2} \left(\frac{s_{ik}}{s_0}\right)^{\alpha_{I\!R}(t_i)-1} \exp\left(\frac{B_{I\!R}}{2}\, t_i\right)$$
$$\pm (a_\rho - \mathrm{i})\, s_{ik}\, C_\rho \left(\frac{s_{ik}}{s_0}\right)^{\alpha_{I\!R}(t_i)-1} \exp\left(\frac{B_{I\!R}}{2}\, t_i\right)$$

above $s_{ik} = W_{ik}^2$, where W_{ik} is the center-of-mass energy

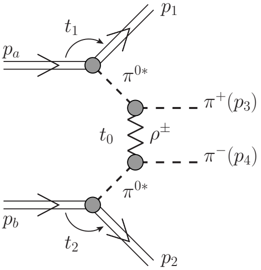
Pion-pion rescattering contributions

low $\pi\pi$-mass rescattering
(partial wave analysis)



$+ \; (3 \leftrightarrow 4)$

high $\pi\pi$-mass rescattering
(Regge phenomenology)



$+ \; (3 \leftrightarrow 4)$

$$\overline{|\mathcal{M}|^2} = (E_a+m)\,(E_1+m)\left(\frac{\mathbf{p}_a^2}{(E_a+m)^2}+\frac{\mathbf{p}_1^2}{(E_1+m)^2}-\frac{2\mathbf{p}_a\cdot\mathbf{p}_1}{(E_a+m)(E_1+m)}\right)$$

$$\times \frac{g^2_{\pi NN}}{(t_1-m_\pi^2)^2}F^2_{\pi NN}(t_1) \times |\mathcal{M}_{2\to 2}(s_{34},t_0;t_1,t_2)|^2 \times \frac{g^2_{\pi NN}}{(t_2-m_\pi^2)^2}F^2_{\pi NN}(t_2)$$

$$\times (E_b+m)\,(E_2+m)\left(\frac{\mathbf{p}_b^2}{(E_b+m)^2}+\frac{\mathbf{p}_2^2}{(E_2+m)^2}-\frac{2\mathbf{p}_b\cdot\mathbf{p}_2}{(E_b+m)(E_2+m)}\right)$$

Amplitudes of subprocess $\pi^{0*}\pi^{0*} \to \pi^+\pi^-$

- **low** $\pi\pi$**-mass rescattering** amplitude (from partial wave analysis) [*P. Lebiedowicz, A. Szczurek and R. Kamiński, Phys. Lett. B680 (2009) 459*]

$$\mathcal{M}_{2\to 2}(s_{34}, cos\theta^*; t_1, t_2) = 16\pi\sqrt{\frac{s_{34}}{s_{34}-4m_\pi^2}}\sum_I\sum_L(2L+1)P_L(cos\theta^*)\mathbf{f_L^I}(\mathbf{s_{34}}) \times F_{off}(t_1)F_{off}(t_2)$$

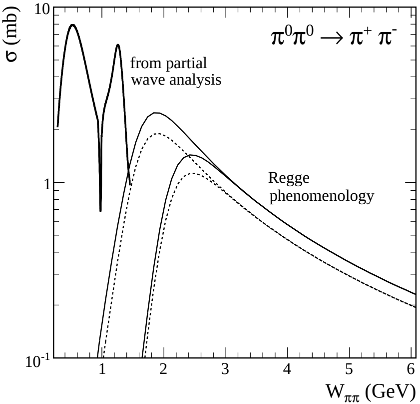
$\mathbf{f_L^I}(\mathbf{s_{34}}) = \frac{\eta_L^I(s_{34})e^{2i\delta_L^I(s_{34})}-1}{2i}$ *R. Kamiński, J.R. Pelaez and F.J. Yndurain*

$$cos\theta^* = \frac{1}{2}(cos\theta^*_{\hat{t}} + cos\theta^*_{\hat{u}}) = \frac{\hat{t}-\hat{u}}{s_{34}-m_\pi^2-m_\pi^2-t_1-t_2}$$

- **high** $\pi\pi$**-mass rescattering** amplitude (similarly as for πp scattering)

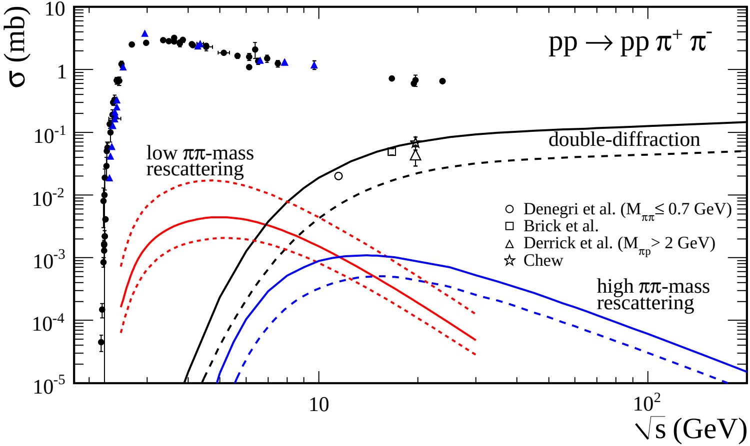
$$\mathcal{M}_{2\to 2}(s_{34}, \hat{t}, \hat{u}; t_1, t_2) = \Big[(a_\rho - \mathrm{i})\, s_{34}\, C_\rho^{\pi\pi}\left(\frac{s_{34}}{s_0}\right)^{\alpha_{IR}(\hat{t})-1}\exp\left(\frac{B_{\pi\pi}}{2}\hat{t}\right) + (a_\rho - \mathrm{i})\, s_{34}\, C_\rho^{\pi\pi}\left(\frac{s_{34}}{s_0}\right)^{\alpha_{IR}(\hat{u})-1}\exp\left(\frac{B_{\pi\pi}}{2}\hat{u}\right) \Big] \times F_{off}(t_1)F_{off}(t_2) \times f_{cont}^{\pi\pi}(W_{34})$$

$C_\rho^{\pi\pi} = (C_\rho^{\pi N})^2/C_\rho^{NN} \approx 16.38$ mb *A. Szczurek, N.N. Nikolaev and J. Speth*
$B_{\pi\pi} \sim 4 - 6\ \mathrm{GeV}^{-2}$



the Regge contributions ($W_{cut} = 1.5$, 2 GeV, $a_{cut} = 0.2$ GeV)
$B_{\pi\pi} = 4$ GeV^{-2} (solid lines)
$B_{\pi\pi} = 6$ GeV^{-2} (dotted lines)

- can see two characteristic bumps corresponding to scalar σ and tensor $f_2(1270)$ mesons as well as a dip from the destructive interference with $f_0(980)$ and σ

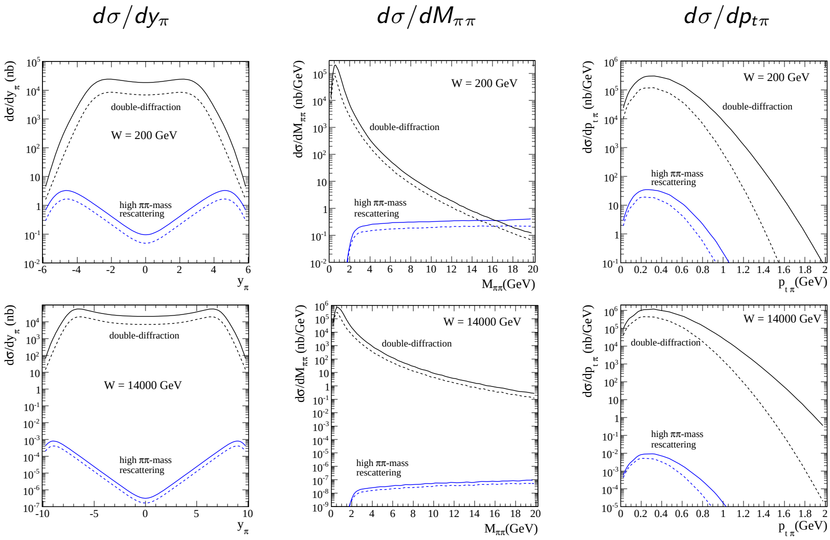


We predicted cross sections for RHIC and LHC energy (without absorption effects)

Λ_{off}^2 (GeV^2)	W = 500 GeV	W = 14 TeV
1 (solid lines)	0.2 mb	0.5 mb
0.5 (dashed lines)	0.08 mb	0.2 mb

$$F_{off}(k^2) = \exp\left((k^2 - m_\pi^2)/\Lambda_{off}^2\right)$$

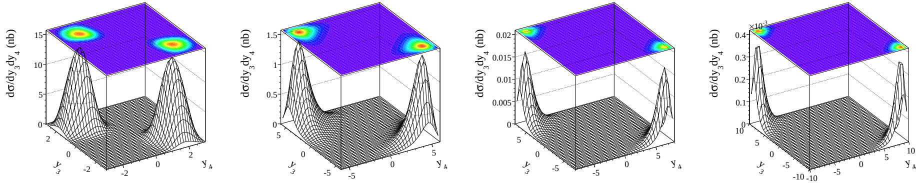
Double diffractive vs $\pi\pi$ rescattering contributions



solid lines: $\Lambda^2_{off,E} = 1$ GeV^2, dashed lines: $\Lambda^2_{off,E} = 0.5$ GeV^2

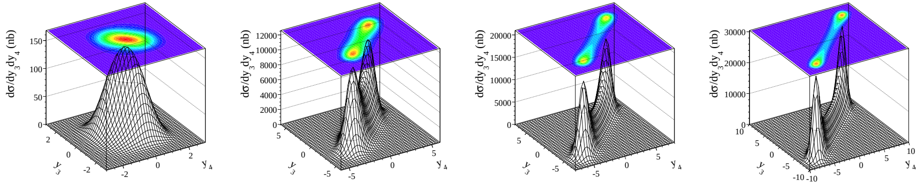
Differential cross section in rapidity space

The high $\pi\pi$-mass rescattering contribution



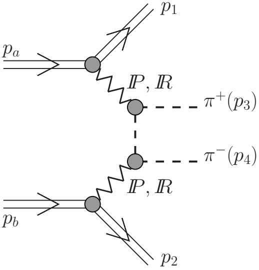
$W = 5.5$ GeV $\quad W = 200$ GeV $\quad W = 1960$ GeV $\quad W = 14$ TeV

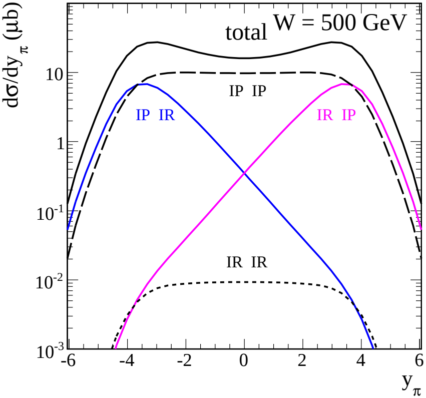
The double-diffractive contribution



Decomposition of cross section in y_π

Rapidity distribution of pions ($y_\pi = y_3 \cong y_4$) at $W = 500$ GeV
when all (solid line) and only some components in the amplitude are included



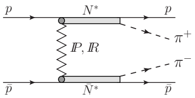


+ crossed diagram ($3 \leftrightarrow 4$)

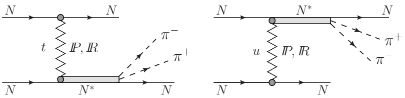
$I\!P \otimes I\!P$ component peaks at midrapidities of pions
$I\!P \otimes I\!R$ and $I\!R \otimes I\!P$ peaks at backward and forward y_π

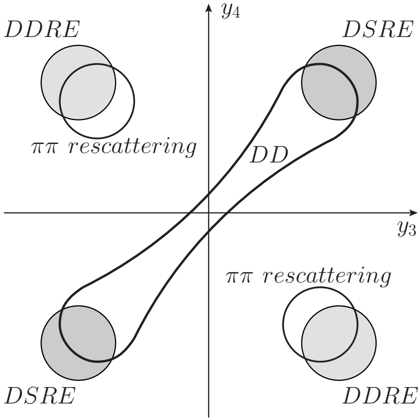
Localization of mechanisms at high energies

double resonance excitation (DDRE)

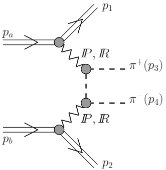


single resonance excitation (DSRE)

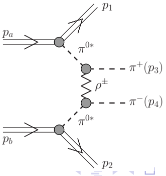




double-diffraction (DD)



high $\pi\pi$-mass rescattering



Summary

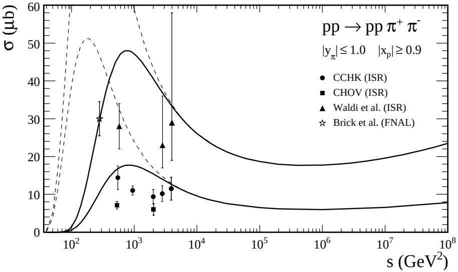
- We have estimated cross section for different exclusive production mechanisms of $\pi^+\pi^-$ pairs (mostly at high energies)
- We considered
  - double-diffractive contribution (both pomeron and reggeon exchanges) - dominates at higher energies and small $M_{\pi\pi}$
  - pion-pion rescattering contributions (using a recent phase shift at low energies and Regge form at high energies)
- The distributions in rapidity space for the double-diffraction at high energies are particularly interesting – preference for the same-hemisphere emission of pions
- The schematic localization of different mechanisms for $pp \to pp\pi^+\pi^-$ and $p\bar{p} \to p\bar{p}\pi^+\pi^-$ at high energies can be resolved experimentally
- The processes considered constitute a sizeable contribution to the total nucleon-nucleon cross section as well as to pion inclusive cross section

We discuss detail of the exclusive production $\pi^+\pi^-$ pairs in papers:
P. Lebiedowicz, A. Szczurek and R. Kamiński, Phys. Lett. B680 (2009) 459;
P. Lebiedowicz and A. Szczurek, Phys. Rev. D81 (2010) 036003
and production of $f_0(1500)$ meson and estimate the $\pi\pi$ background at PANDA energy:
A. Szczurek and P. Lebiedowicz, Nucl. Phys. A826 (2009) 101

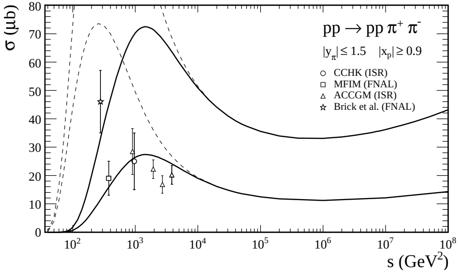
Comparison with experiments

With extra cuts

$|y_\pi| \leqslant 1, \; |x_p| \geqslant 0.9$



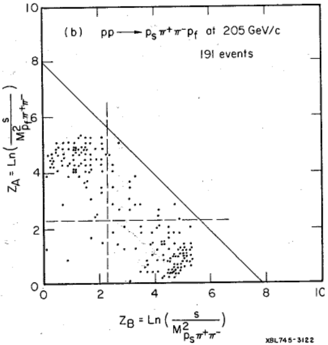
$|y_\pi| \leqslant 1.5, \; |x_p| \geqslant 0.9$



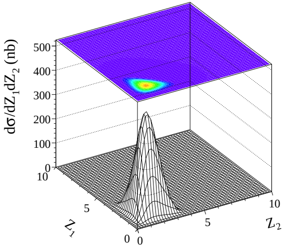
Double-diffractive contribution with

- lower lines: $\Lambda^2_{off,E} = 0.5$ GeV^2
- upper lines: $\Lambda^2_{off,E} = 1$ GeV^2
- dashed lines: without correction factors $f^{\pi p}_{cont}(W_{ik})$ which excluding resonances region
- solid lines: with $f^{\pi p}_{cont}(W_{ik})$, where $W_0 = 2$ GeV

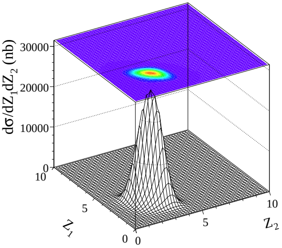
$pp \to pp\pi^+\pi^-$ reaction at $P_{lab} = 205\,GeV$ ($\sqrt{s} \approx 20\,GeV$) – NAL experiment
[*D.M. Chew, Nucl. Phys. B82 (1974) 422*]



high $\pi\pi$-mass rescattering



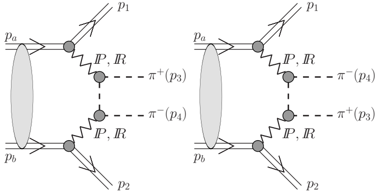
double-diffraction



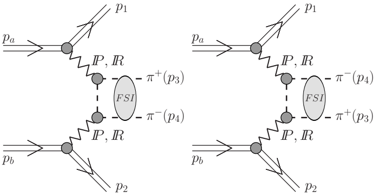
where $Z_1 = \ln(\frac{s}{M^2_{p_1\pi^+\pi^-}})$ and $Z_2 = \ln(\frac{s}{M^2_{p_2\pi^+\pi^-}})$

Beyond the Born approximation

Diagrams representing the absorption effects due to proton-proton interaction



Diagrams representing pion-pion FSI (final state interaction)



When going from the Born to the diagrams with the pion-pion FSI the following replacement is formally required:

$$\frac{F^A_{off}(k)F^B_{off}(k)}{k^2 - m_\pi^2} \to \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_\pi^2}\frac{F^A_{off}(k,k_3)}{k_3^2 - m_\pi^2}\frac{F^B_{off}(k,k_4)}{k_4^2 - m_\pi^2}\sum_{ij}\mathcal{M}^{off-shell}_{\pi_i\pi_j\to\pi^\pm\pi^\mp}(k_3k_4 \to p_3p_4)$$

where the sum runs over different isospin combinations of pions.
In general the integral above is complicated (singularities, unknown elements), the vertex form factors (A and B) with two pions being off-mass-shell are not well known, and even the off-shell matrix element is not fully under control.