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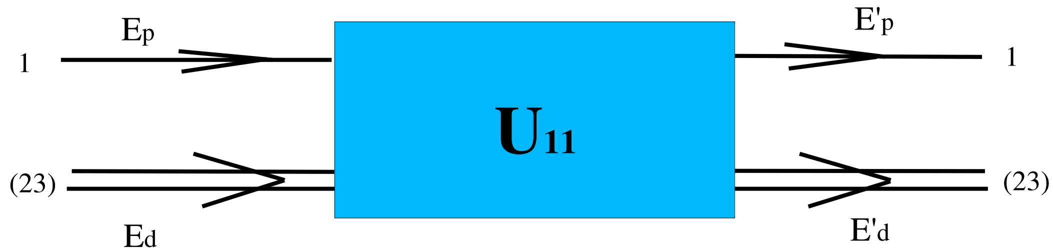
Rescattering effects in deuteron-proton elastic scattering at intermediate energies

- $dp \longrightarrow dp$ reaction is considered at the deuteron kinetic energy from 500 MeV up to 2000 MeV.
- The theoretical model is suggested to describe differential cross sections and polarization observables in this energy range.
- The calculation results are presented in comparison with the data.

dp-elastic scattering



The matrix element of the transition operator U_{11} defines reaction amplitude

$$U_{dp \to dp} = \delta(E_d + E_p - E'_d - E'_p)\mathcal{J} =< 1(23)|[1 - P_{12} - P_{13}]U_{11}|1(23) >$$

Alt-Grassberger-Sandhas equations for rearrangement operators:

$$\begin{aligned} U_{11} &= \qquad\qquad t_2 g_0 U_{21} + t_3 g_0 U_{31} \\ U_{21} &= g_0^{-1} + t_1 g_0 U_{11} + t_3 g_0 U_{31} \\ U_{31} &= g_0^{-1} + t_1 g_0 U_{11} + t_2 g_0 U_{21} \end{aligned}$$

Iterating AGS-equations up to second order terms over t one obtains

$$\mathcal{J}_{dp\to dp} \ = \ \mathcal{J}_{ONE} + \mathcal{J}_{SS} + \mathcal{J}_{DS}$$

One-Nucleon-Exchange

$$\mathcal{J}_{ONE} = -2 < 1(23)|P_{12}g_0^{-1}|1(23) >$$

Single-Scattering

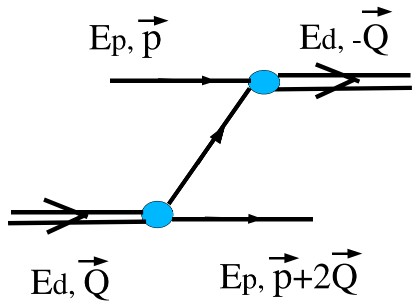
$$\mathcal{J}_{SS} = 2 < 1(23)|t_3^{sym}|1(23) >$$

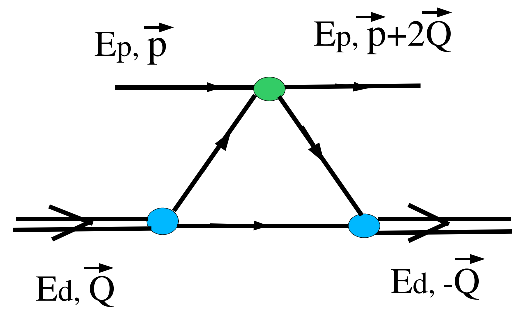
Double-Scattering

$$\mathcal{J}_{DS} = 2 < 1(23)|t_3^{sym}g_0t_2^{sym}|1(23) >,$$

where notations for antisymmetrized operators have been introduced

$t_2^{sym} = [1 - P_{13}]t_2$ and $t_3^{sym} = [1 - P_{12}]t_3$.


$E_p, \vec{p}$
$E_d, -\vec{Q}$
$E_d, \vec{Q}$
$E_p, \vec{p}+2\vec{Q}$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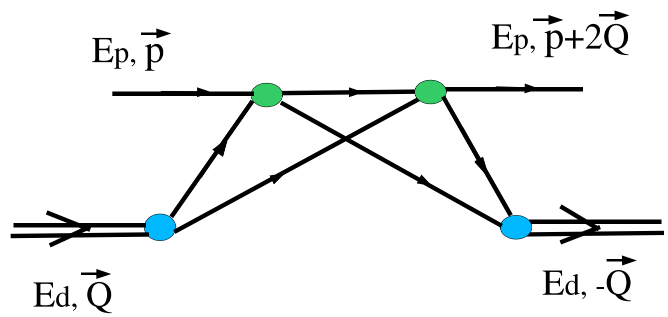
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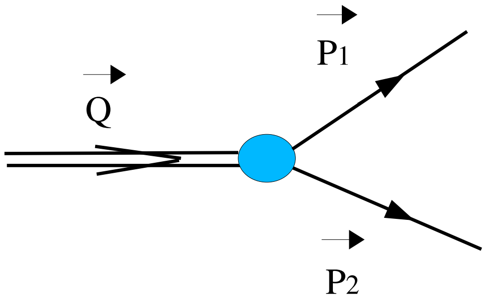


Lorenz transformation

$$L(\vec{u})p_1 = (E^*, \vec{p})$$
$$L(\vec{u})p_2 = (E^*, -\vec{p})$$

with velocity

$$\vec{u} = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}$$

The c.m. energy of one of the nucleons E^* is related with Mandelstam variable s by

$$E^* = \sqrt{s}/2 \quad .$$

Let's introduce new variables $\vec{Q}$ and $\vec{k}$ which can be expressed through $\vec{p}_1$ and $\vec{p}_2$

$$\vec{Q} = \vec{p}_1 + \vec{p}_2$$
$$\vec{k} = \frac{(E_2 + E^*)\vec{p}_1 - (E_1 + E^*)\vec{p}_2}{E_1 + E_2 + 2E^*} \quad .$$

A two-nucleon state in the $(\vec{p}_1, \vec{p}_2)$ system is connected with a two-nucleon state in the center-of-mass by the relation

$$|\vec{p}_1, \vec{p}_2 >= J^{-1/2}(\vec{p}_1, \vec{p}_2) W_{1/2}(\vec{p}_1, \vec{u}) W_{1/2}(\vec{p}_2, \vec{u}) |\vec{k}, \vec{Q} > \quad ,$$

where $W_{1/2}$ Wigner rotation operator

$$W_{1/2}(\vec{p}_i, \vec{u}) = exp\{-i\omega_i(\vec{n}_i\vec{\sigma}_i)/2\} = cos(\omega_i/2)[1 - i(\vec{n}_i\vec{\sigma}_i)tg(\omega_i/2)]$$

The wave function of the bound state transforms in the same way as the state of a single particle

$$W(L_{\vec{u}'})|\vec{P} >= \sqrt{\frac{E_{\vec{Q}}}{E_{\vec{P}}}} \;\; W_1(\vec{Q}, \vec{u}')|\vec{Q} > \quad ,$$

where the W_1 is the Wigner rotation operator for spin 1 particle

$$W_1(\vec{Q}, \vec{u}') = exp\left\{-i\omega(\vec{n}'\vec{S})\right\}$$

and

$$L(\vec{u}')\vec{P} = \vec{Q}$$

The deuteron wave function in the rest has the standard form

$$< m_p m_n|\Omega_d|\mathcal{M}_d >= \frac{1}{\sqrt{4\pi}} < m_p m_n| \left\{ u(k) + \frac{w(k)}{\sqrt{8}}[3(\vec{\sigma}_1\hat{k})(\vec{\sigma}_2\hat{k}) - (\vec{\sigma}_1\vec{\sigma}_2)] \right\} |\mathcal{M}_d >$$

$u(k)$ and $w(k)$ - $S-$ and $D-$ components of the deuteron.

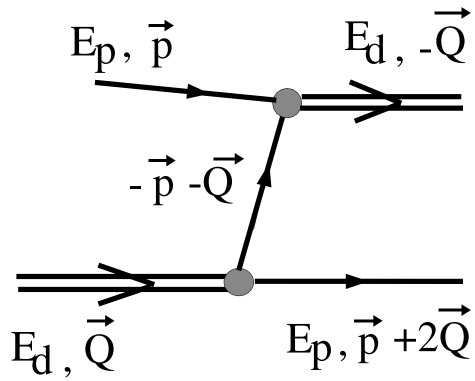
Then the deuteron wave function in the moving frame is

$$< \vec{p}_1\ \vec{p}_2, m_1 m_2|\Omega_d|\vec{Q}, \mathcal{M}_d > \ \sim\ < \vec{k}\vec{Q}, m_1' m_2'|W^{\dagger}_{1/2}(\vec{p}_1, \vec{u})W^{\dagger}_{1/2}(\vec{p}_2, \vec{u})\Omega_d|\vec{0}, \mathcal{M}_d >$$

Deuteron wave function in the moving frame

$$
\begin{aligned}
< \vec{p}_1\ \vec{p}_2, m_1 m_2 | \Omega_d | \vec{Q}, \mathcal{M}_d > \ = \ & < \vec{p}_1\ \vec{p}_2, m_1 m_2 | g_1(\vec{k}, \vec{Q}) + g_2(\vec{k}, \vec{Q})(\vec{\sigma}_1 \vec{n})(\vec{\sigma}_2 \vec{n}) + \\
& + \ g_3(\vec{k}, \vec{Q})(\vec{\sigma}_1 \vec{\sigma}_2) + g_4(\vec{k}, \vec{Q})(\vec{\sigma}_1 \hat{k})(\vec{\sigma}_2 \hat{k}) + \\
& + \ g_5(\vec{k}, \vec{Q})[(\vec{\sigma}_1 + \vec{\sigma}_2)\vec{n}] + \\
& + \ g_6(\vec{k}, \vec{Q})[(\vec{\sigma}_1 \hat{k})(\vec{\sigma}_2 \vec{n} \times \hat{k}) + (\vec{\sigma}_1 \vec{n} \times \hat{k})(\vec{\sigma}_2 \hat{k})] | \vec{Q}, \mathcal{M}_d >
\end{aligned}
$$

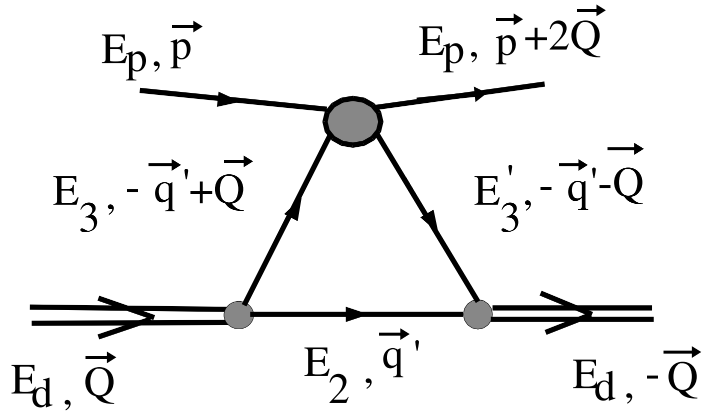
g_i are combinations of the $S-$ and $D-$ components of the deuteron wave function (u and w)



(a)

One nucleon exchange scattering

$$\mathcal{J}_{ONE} = -\frac{1}{2}(E_d - E_p - \sqrt{m_N^2 + \vec{p}^{\,2} - \vec{Q}^2}) \cdot$$
$$< \vec{p}' m'; -\vec{Q}\mathcal{M}_d' | \Omega_d^\dagger(23)[1 + (\vec{\sigma}_1\vec{\sigma}_2)]\Omega_d(23)|\vec{Q}\mathcal{M}_d; \vec{p}m >$$



(b)

Single scattering

$$\mathcal{J}_{SS} = \int d\vec{q}\,' < -\vec{Q}\mathcal{M}'_d|\Omega_d^\dagger|\vec{q}\,'m'', -\vec{Q} - \vec{q}\,'m'_3 >$$
$$< \vec{p}\,'m', -\vec{Q} - \vec{q}\,'|\frac{3}{2}t^1_{12} + \frac{1}{2}t^0_{12}|\vec{p}m, \vec{Q} - \vec{q}\,'m'_2 >< \vec{q}\,'m'', \vec{Q} - \vec{q}\,'m'_2|\Omega_d|\vec{Q}\mathcal{M}_d >$$

Nucleon-Nucleon t-matrix

W.G.Love, M.A.Franey, Phys.Rev.C24, 1073 (1981)

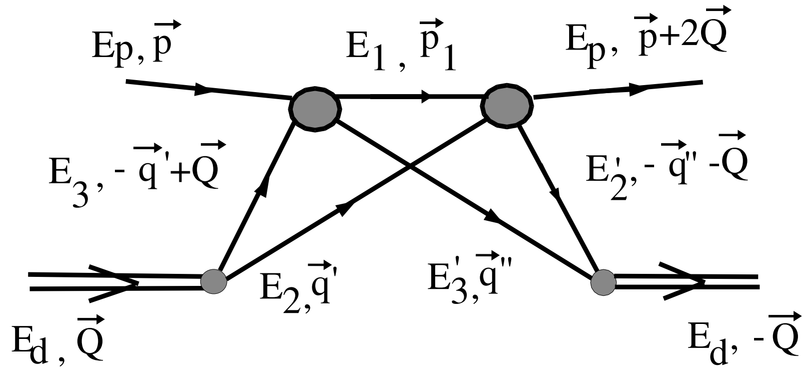
N.B.Ladygina,nucl-th/0805.3021

$$< \kappa' m_1' m_2' | t | \kappa m_1 m_2 > \; = \; < \vec{\kappa}' m_1' m_2' | A + B(\vec{\sigma}_1 \hat{N}^*)(\vec{\sigma}_2 \hat{N}^*) + C(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{N}^* + D(\vec{\sigma}_1 \hat{q}^*)(\vec{\sigma}_2 \hat{q}^*) + F(\vec{\sigma}_1 \hat{Q}^*)(\vec{\sigma}_2 \hat{Q}^*) | \vec{\kappa} m_1 m_2 >$$

where the orthonormal basis is combinations of the nucleons relative momenta in the initial $\vec{\kappa}$ and final $\vec{\kappa}'$ states

$$\hat{q}^* = \frac{\vec{\kappa} - \vec{\kappa}'}{|\vec{\kappa} - \vec{\kappa}'|} \quad , \quad \hat{Q}^* = \frac{\vec{\kappa} + \vec{\kappa}'}{|\vec{\kappa} + \vec{\kappa}'|} \quad , \quad \hat{N}^* = \frac{\vec{\kappa} \times \vec{\kappa}'}{|\vec{\kappa} \times \vec{\kappa}'|}$$

$$< \vec{p}' \vec{p}_3'; m' m_3' | t | \vec{p} \vec{p}_3; m m_3 > \sim < \kappa' m_1' m_2' | W^{\dagger}_{1/2}(\vec{p}') W^{\dagger}_{1/2}(\vec{p}_3') \; t \; W_{1/2}(\vec{p}) W_{1/2}(\vec{p}_3) | \kappa m_1 m_2 >$$



Double scattering

(c)

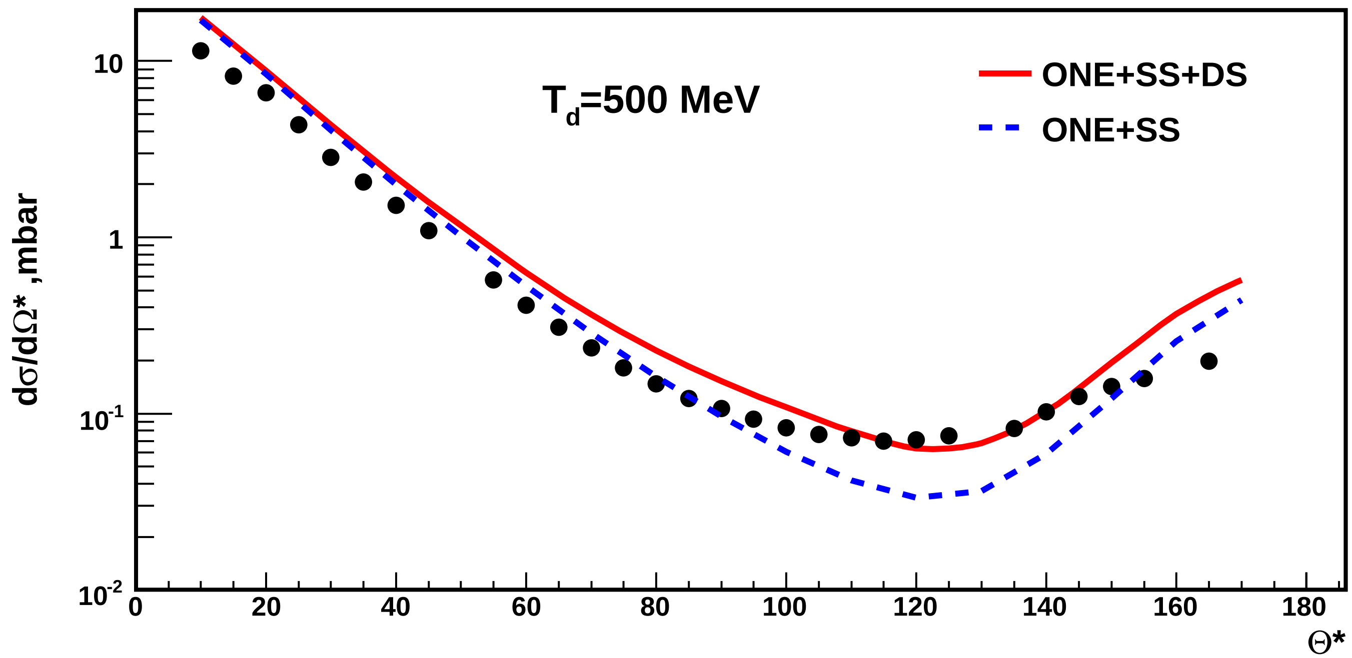
$$\mathcal{J}_{DS} = NN' \int d\vec{q}\,' \int d\vec{q}'' < -\vec{Q}\mathcal{M}_d'|\Omega_d^\dagger|\vec{q}\,'m_2', \vec{Q} - \vec{q}\,'\, m_3' >$$

$$< m'm_2'm_3'|\Big\{ t_{12}^1(\sqrt{s_{12}'}, \vec{\kappa}, \vec{\kappa}\,')t_{13}^1(\sqrt{s_{13}}, \vec{k}, \vec{k}') +$$

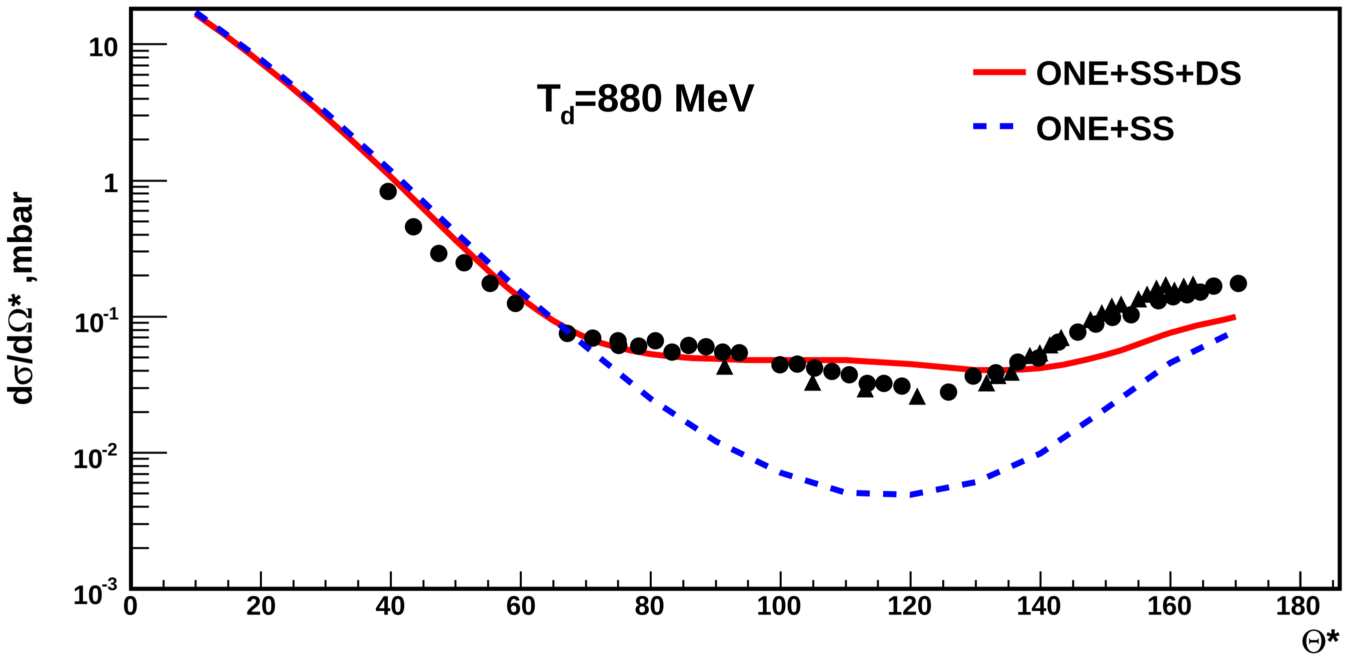
$$\frac{1}{4}[t_{12}^1(\sqrt{s_{12}'}, \vec{\kappa}, \vec{\kappa}\,') + t_{12}^0(\sqrt{s_{12}'}, \vec{\kappa}, \vec{\kappa}\,')][t_{13}^1(\sqrt{s_{13}}, \vec{k}, \vec{k}') + t_{13}^0(\sqrt{s_{13}}, \vec{k}, \vec{k}')]\Big\}$$

$$\frac{1}{E_d + E_p - E_1' - E_2' - E_3' + i\varepsilon}|mm_2m_3 >< -\vec{Q} - \vec{q}\,''\, m_2, \vec{q}\,''\, m_3|\Omega_d|\mathcal{M}_d >$$

$$\frac{1}{E - E' + i\varepsilon} = \mathcal{P}\frac{1}{E - E'} - i\pi\delta(E - E')$$

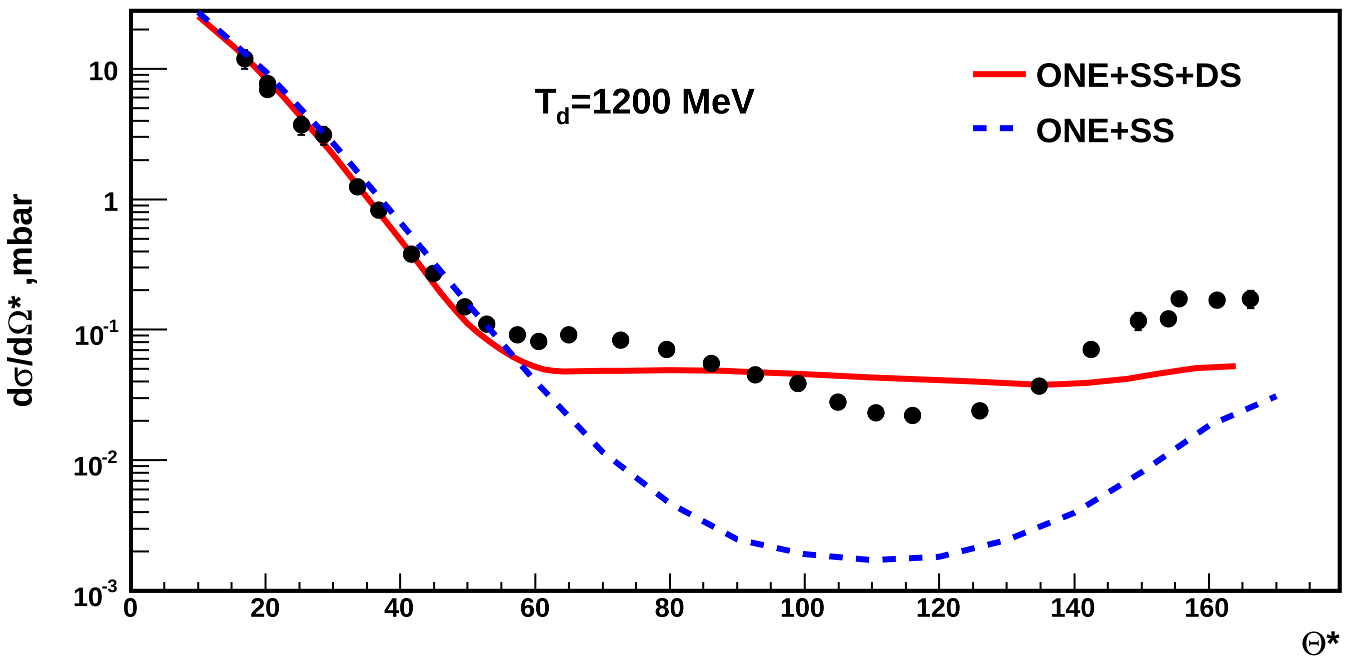


●- K.Hatanaka et al., Phys.Rev.C66, 044002 (2002)

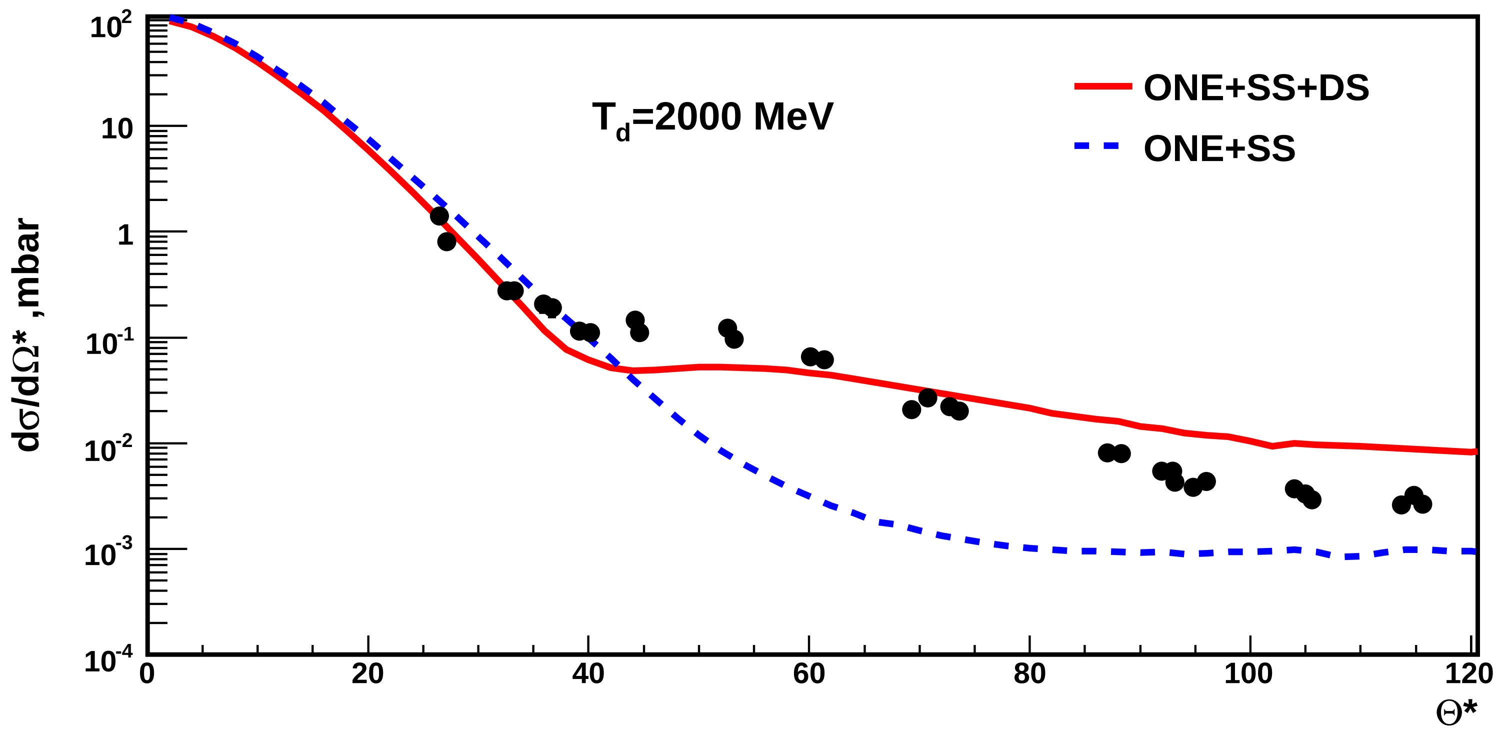


▲ - N.E.Booth et al., Phys.Rev.D4, p.1261 (1971), $\boldsymbol{T_d = 850 MeV}$

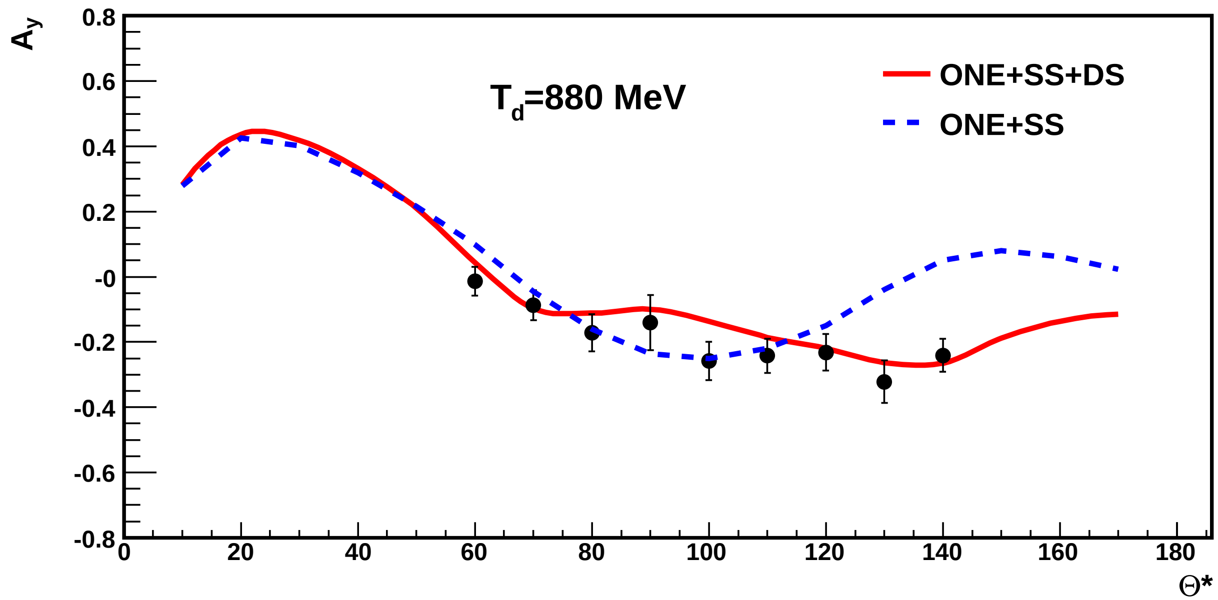
● - J.C.Alder et al., Phys.Rev.C6, p.2010 (1972), $\boldsymbol{T_d = 940 MeV}$

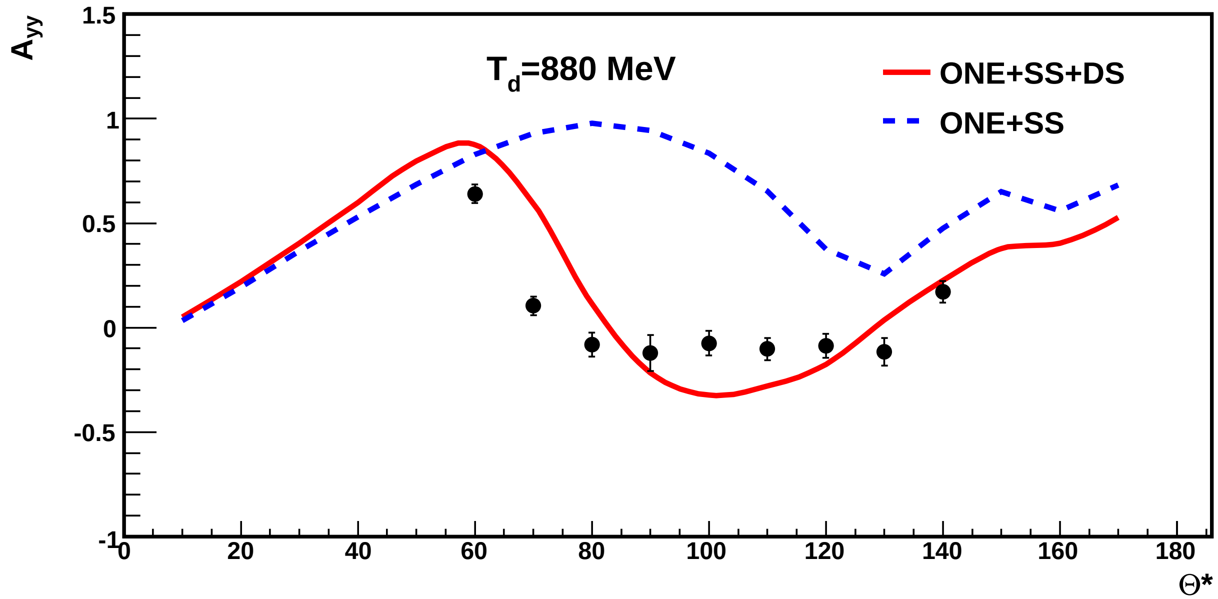


● - E.T.Boschitz et al., Phys.Rev.C6, p.457 (1972)

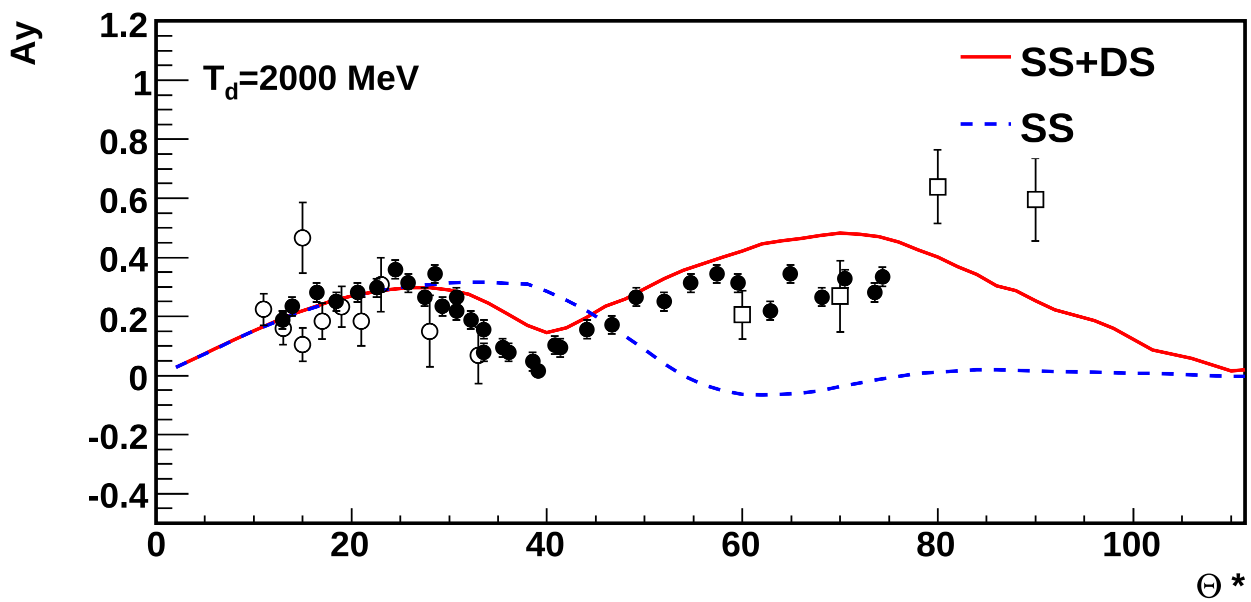


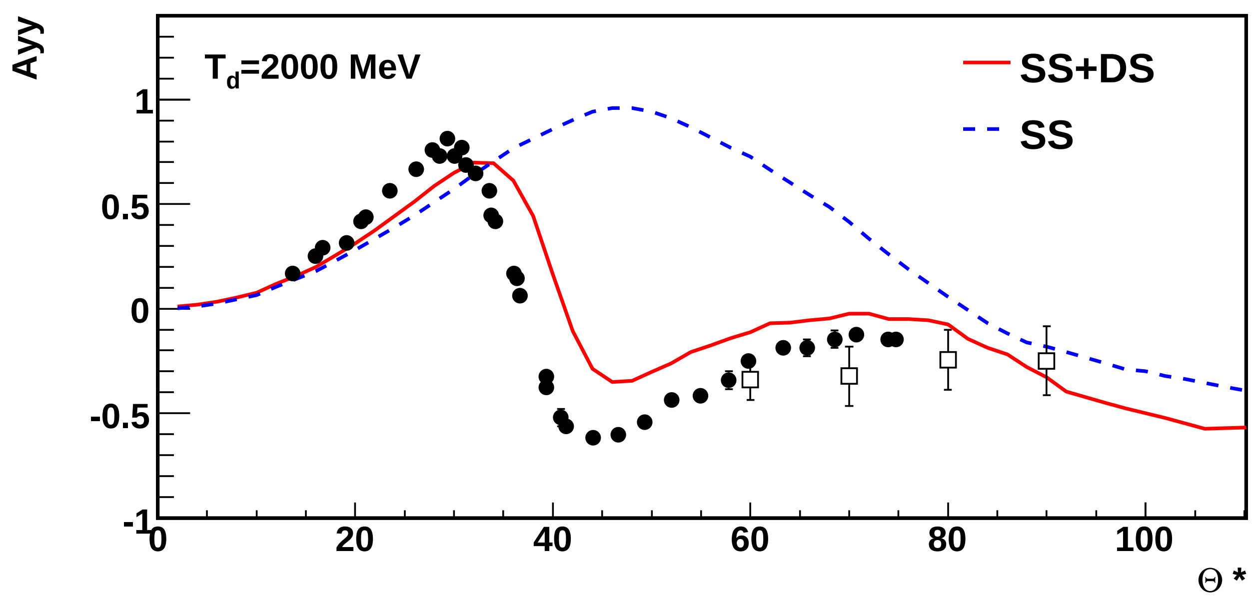
● - G.W.Bennett et al., Phys.Rev.Lett. V19 (1967) p.387





● NUCLOTRON data, LNS collaboration





● M.Haji-Saied et al, Phys.Rev.C36, p. 2010

○ LHE JINR, hydrogen bubble chamber experiment (unpublished)

□ NUCLOTRON data, LNS collaboration

Conclusions

- The deuteron-proton elastic scattering reaction is considered in the deuteron energy range from 500 MeV up to 2000 MeV.

- The theoretical model for description of this process is suggested. This model is based on the multiple scattering expansion formalism taking relativistic kinematics and relativistic spin theory into account.

- It was shown that the double-scattering effect increases with the energy.

- The experimental data on the differential cross section are described at four deuteron energies: 500, 880, 1200, and 2000 MeV. The description of the polarization data on the vector, A_y, and tensor, A_{yy}, analyzing powers are also obtained at 880 MeV and 2000 MeV of the deuteron energies.
A good agreement between the theoretical predictions and experimental data was obtained.