Light hadron spectrum from lattice QCD [N(939)]

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what is the source of the mass of ordinary matter?

(a lattice field theory talk)

SFB
TR55

The vast majority of the mass of ordinary matter

ultimate (Higgs or alternative) mechanism: responsible for the mass of the leptons and for the mass of the quarks

interestingly enough: just a tiny fraction of the visible mass (such as stars, the earth, the audience, atoms)
electron: almost massless, $\approx$1/2000 of the mass of a proton
quarks (in ordinary matter): also almost massless particles

the vast majority (about 95%) comes through another mechanism
$\Longrightarrow$ this mechanism and this 95% will be the main topic of this talk

quantum chromodynamics (QCD, strong interaction) on the lattice

Lattice field theory

systematic non-perturbative approach (numerical solution):

quantum fields on the lattice

quantum theory: path integral formulation with $S = E_{kin} - E_{pot}$

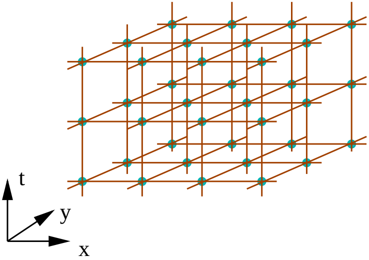
quantum mechanics: for all possible paths add exp(iS)
quantum fields: for all possible field configurations add exp(iS)

Euclidean space-time ($t = i\tau$): exp(-S) sum of Boltzmann factors

we do not have infinitely large computers $\Rightarrow$ two restrictions

a. put it on a space-time grid (proper approach: asymptotic freedom)
formally: four-dimensional statistical system
b. finite size of the system (can be also controlled)

$\Rightarrow$ stochastic approach, with reasonable spacing/size: solvable

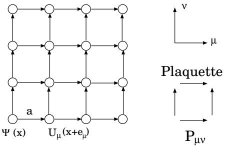


fine lattice to resolve the structure of the proton ($\lesssim$0.1 fm)
few fm size is needed
50-100 points in 'xyz/t' directions
$a \Rightarrow a/2$ means 100-200$\times$CPU

mathematically
10^9 dimensional integrals

advanced techniques,
good balance and
several Tflops are needed

Lattice Lagrangian: gauge fields



$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi}(D_\mu \gamma^\mu + m)\psi$$

anti-commuting $\psi(x)$ quark fields live on the sites
gluon fields, $A^a_\mu(x)$ are used as links and plaquettes

$$U(x,y) = \exp\left(ig_s \int_x^y dx'^\mu \, A^a_\mu(x')\lambda_a/2\right)$$

$$P_{\mu\nu}(n) = U_\mu(n) U_\nu(n+e_\mu) U^\dagger_\mu(n+e_\nu) U^\dagger_\nu(n)$$

$S = S_g + S_f$ consists of the pure gluonic and the fermionic parts

$$S_g = 6/g_s^2 \cdot \sum_{n,\mu,\nu} [1 - \mathrm{Re}(P_{\mu\nu}(n))]$$

Lattice Lagrangian: fermionic fields

quark differencing scheme:

$$\bar\psi(x)\gamma^\mu\partial_\mu\psi(x) \to \bar\psi_n\gamma^\mu(\psi_{n+e_\mu} - \psi_{n-e_\mu})$$

$$\bar\psi(x)\gamma^\mu D_\mu\psi(x) \to \bar\psi_n\gamma^\mu U_\mu(n)\psi_{n+e_\mu} + \dots$$

fermionic part as a bilinear expression: $S_f = \bar\psi_n M_{nm}\psi_m$

we need 2 light quarks (u,d) and the strange quark: $n_f = 2+1$

(complication: fermion doubling $\Rightarrow$ staggered/Wilson)

Euclidean partition function gives Boltzmann weights

$$Z = \int \prod_{n,\mu}[dU_\mu(x)][d\bar\psi_n][d\psi_n]e^{-S_g-S_f} = \int \prod_{n,\mu}[dU_\mu(n)]e^{-S_g}\det(M[U])$$

Historical background

1972 Lagrangian of QCD (H. Fritzsch, M. Gell-Mann, H. Leutwyler)

1973 asymptotic freedom (D. Gross, F. Wilczek, D. Politzer)
at small distances (large energies) the theory is "free"

1974 lattice formulation (Kenneth Wilson)
at large distances the coupling is large: non-perturbative

Nobel Prize 2008: Y. Nambu, & M. Kobayashi T. Masakawa

spontaneous symmetry breaking in quantum field theory
strong interaction picture: mass gap is the mass of the nucleon

mass eigenstates and weak eigenstates are different

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Scientific Background on the Nobel Prize in Physics 2008

"Even though QCD is the correct theory for the strong interactions, it can not be used to compute at all energy and momentum scales ... (there is) ... a region where perturbative methods do not work for QCD."

true, but the situation is somewhat better: new era
fully controlled non-perturbative approach works (took 35 years)

Importance sampling

$$Z = \int \prod_{n,\mu} [dU_\mu(n)] e^{-S_g} \det(M[U])$$

we do not take into account all possible gauge configuration

each of them is generated with a probability $\propto$ its weight

importance sampling, Metropolis algorithm:
(all other algorithms are based on importance sampling)

$$P(U \to U') = \min\left[1, \exp(-\Delta S_g) \det(M[U'])/\det(M[U])\right]$$

gauge part: trace of 3$\times$3 matrices (easy, without M: quenched)
fermionic part: determinant of $10^6 \times 10^6$ sparse matrices (hard)

more efficient ways than direct evaluation (Mx=a), but still hard

Hadron spectroscopy in lattice QCD

Determine the transition amplitude between:
having a “particle” at time 0 and the same “particle” at time t
$\Rightarrow$ Euclidean correlation function of a composite operator $\mathcal{O}$:

$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

insert a complete set of eigenvectors $|i\rangle$

$$= \sum_i \langle 0|e^{Ht}\,\mathcal{O}(0)\,e^{-Ht}|i\rangle\langle i|\mathcal{O}^\dagger(0)|0\rangle = \sum_i |\langle 0|\mathcal{O}^\dagger(0)|i\rangle|^2\, e^{-(E_i-E_0)t},$$

where $|i\rangle$: eigenvectors of the Hamiltonian with eigenvalue E_i.

and $$\mathcal{O}(t) = e^{Ht}\,\mathcal{O}(0)\,e^{-Ht}.$$

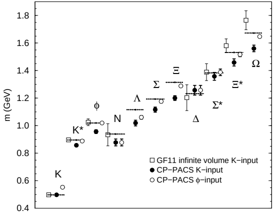
t large $\Rightarrow$ lightest states (created by $\mathcal{O}$) dominate: $C(t) \propto e^{-M\cdot t}$
t large $\Rightarrow$ exponential fits or mass plateaus M_t=log[C(t)/C(t+1)]

Quenched results

QCD is 35 years old $\Rightarrow$ properties of hadrons (Rosenfeld table)

non-perturbative lattice formulation (Wilson) immediately appeared
needed 20 years even for quenched result of the spectrum (cheap)
instead of det(M) of a $10^6 \times 10^6$ matrix trace of 3×3 matrices

always at the frontiers of computer technology:
GF11: IBM "to verify quantum chromodynamics" (10 Gflops, '92)
CP-PACS Japanese purpose made machine (Hitachi 614 Gflops, '96)

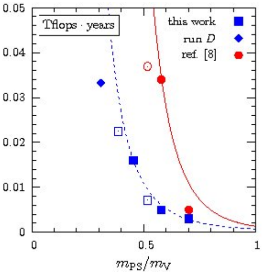


the ≈10% discrepancy was believed to be a quenching effect

Difficulties of full dynamical calculations

though the quenched result is qualitatively correct
uncontrolled systematics $\Rightarrow$ full "dynamical" studies
by two-three orders of magnitude more expensive (balance)
present day machines offer several hundreds of Tflops

no revolution but evolution in the algorithmic developments
Berlin Wall '01: it is extremely difficult to reach small quark masses:



hadron masses (and other questions) many results in the literature

JLQCD, PACS-SC (Japan), MILC (USA), QCDSF (Germany-UK),
RBC & UKQCD (USA-UK), ETM (Europe), Alpha(Europe)
JLAB (USA), CERN-Rome (Swiss-Italian)

note, that all of them neglected one or more of the ingredients required for controlling all systematics (it is quite CPU-demanding)

$\Longrightarrow$ Budapest-Marseille-Wuppertal (BMW) Collaboration
supercomputers: Jugene at Juelich (Idris at CNRS)

try to control all systematics: Science 322:1224-1227,2008
A. Kronfeld, Science 322:1198-1199,2008
F. Wilczek, Nature 456:449-450,2008: Mass by numbers (balance)

http://www.bmw.uni-wuppertal.de

Budapest-Marseille-Wuppertal Collaboration

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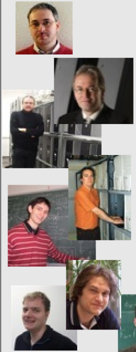

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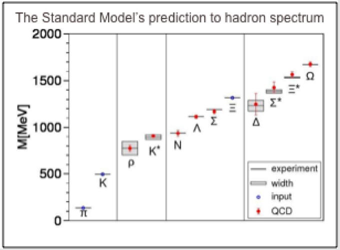
Recent results



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4 CNRS, Centre de Physique Theorique UMR 6207
5 FZ-Jülich Supercomputing Centre

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Ingredients to control systematics

- inclusion of det[M] with an exact n_f=2+1 algorithm
  action: universality class is known to be QCD (Wilson-quarks)
- spectrum: light mesons, octet & decuplet baryons (resonances)
  (three of these fix the averaged m_{ud}, m_s and the cutoff)
- large volumes to guarantee small finite-size effects
  rule of thumb: $M_\pi L \gtrsim 4$ is usually used (correct for that)
- controlled interpolations & extrapolations to physical m_s and m_{ud}
  (or eventually simulating directly at these masses)
  since $M_\pi \simeq 135$ MeV extrapolations for m_{ud} are difficult
  CPU-intensive calculations with M_π reaching down to $\approx$200 MeV
- controlled extrapolations to the continuum limit ($a \to 0$)
  calculations are performed at no less than 3 lattice spacings

Parameters of the Lagrangian

three parameters of the Lagrangian: coupling strength g, m_{ud} and m_s

asymptotic freedom: for large cutoff (small lattice spacing) g is small
in this region the results are already independent of g (scaling)

QCD predicts only dimensionless combinations (e.g. mass ratios)
$\Rightarrow$ we can eliminate g as an input parameter by taking ratios

the pion mass M_π is particularly sensitive to m_{ud}
the kaon mass M_K is particularly sensitive to m_s

relatively easy to set the strange quark mass m_s to its physical value
it is very CPU demanding to approach the physical m_{ud}

Scale setting, dimensionless ratios

QCD predicts only dimensionless combinations (e.g. mass ratios)
set the scale: one dimensionful observable (mass) can be used

practical issues: should be a mass, which can be calculated precisely
weak dependence on m_{ud} (not to strongly alter the chiral behavior)
should not decay under the strong interaction

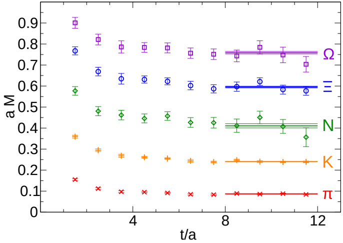
larger the strange content, the more precise the mass determination
these facts support that the Ω baryon is a good choice

baryon decuplet masses are less precise than those of the octet
this observation suggests that the Ξ baryon is a good choice

Scale setting and masses in lattice QCD:

in meteorology, aircraft industry etc. grid spacing is set by hand
in lattice QCD we use g,m_{ud} and m_s in the Lagrangian ('a' not)
measure e.g. the vacuum mass of a hadron in lattice units: $M_\Omega a$
since we know that M_Ω=1672 MeV we obtain 'a'

masses are obtained by correlated fits (choice of fitting ranges)
illustration: mass plateaus at our smallest $M_\pi \approx$190 MeV (noisiest)

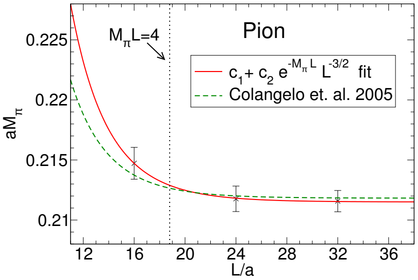


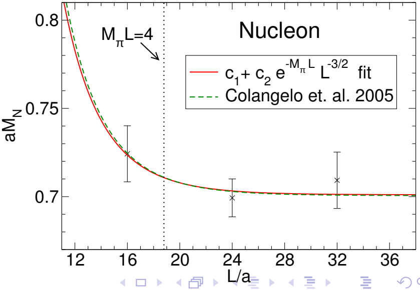
volumes and masses for unstable particles: avoided level crossing
decay phenomena included: in finite V shifts of the energy levels

Two types of finite volume effects: type I

usually $M_\pi L \gtrsim 4$ is assumed as satisfactory: more care is needed
type I: periodic system, virtual π exchange, decreases with $M_\pi L$

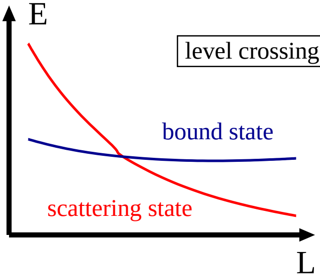
map the volume dependence at the $M_\pi \approx 320$ MeV point
self-consistent analysis with volumes $M_\pi L \approx 3.5$ to 7
$M_X(L) = M_X + c_X(M_\pi) \cdot \exp(-M_\pi L)/(M_\pi L)^{3/2}$ describes the data

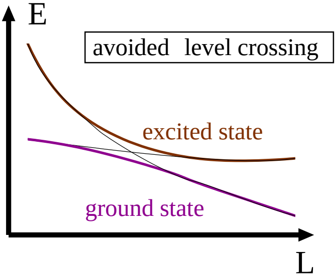




Type II finite volume effect: resonance states

parameters, for which resonances would decay at V=∞
at V=∞ the lowest energy state is a two-particle scattering state
hypothetical case with no coupling $\Rightarrow$ level crossing as V increases
realistic case: non-vanishing decay width $\Rightarrow$ avoided level crossing





M. Luscher, Nucl. Phys. B364 (1991) 237

self-consistent analysis: width is an unknown quantity and we fit it

Approach physical masses & the continuum limit

systematic analyses both for the Ξ and for the Ω sets

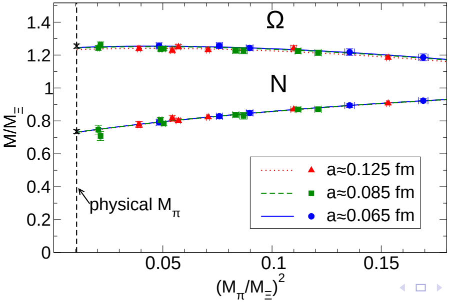
- two ways of normalizing the hadron masses (set the scale):

a. ratio method: QCD predicts only dimensionless quantities

use $r_X=M_X/M_\Xi$ and parameterize it by $r_\pi=M_\pi/M_\Xi$ and $r_K=M_K/M_\Xi$

$\Rightarrow r_X=r_X(r_\pi,r_K)$ surface is an unambiguous prediction of QCD

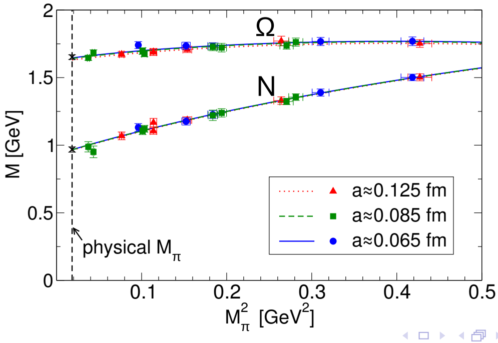
one-dimensional slice (set $2r_K^2 - r_\pi^2 \propto m_s$ to its physical value: 0.27)



Approach physical masses & the continuum limit

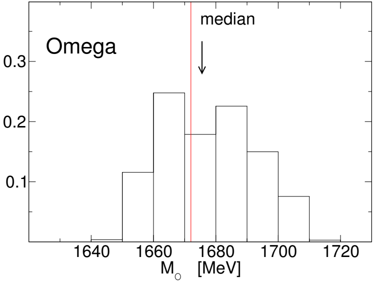
- two ways of normalizing the hadron masses (set the scale):

b. mass independent scale setting: more conventional way
set the lattice spacing by extrapolating M_Ξ to the physical point
(given by the physical ratios of M_π/M_Ξ and M_K/M_Ξ)



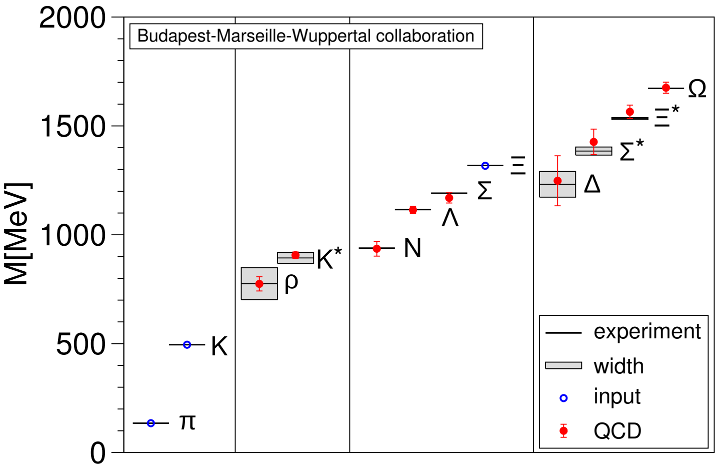
Blind data analysis: avoid any arbitrariness

extended frequentist's method:
2 ways of scale setting, 2 strategies to extrapolate to $M_\pi(phys)$
3 pion mass ranges, 2 different continuum extrapolations
18 time intervals for the fits of two point functions
2·2·3·2·18=432 different results for the mass of each hadron



central value and systematic error is given by the mean and the width
statistical error: distribution of the means for 2000 bootstrap samples

Final result for the hadron spectrum



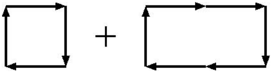
Summary

- understanding the source and the course of the mass generation of ordinary matter is of fundamental importance
- after 35 years of work these questions can be answered (cumulative improvements of algorithms and machines are huge)
- they belong to the largest computational projects on record
- perfect tool to understand hadronic processes (strong interaction)

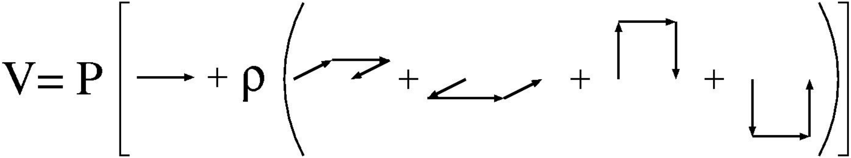
Choice of the action

no consensus: which action offers the most cost effective approach

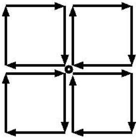
our choice: tree-level $O(a^2)$-improved Symanzik gauge action



6-level (stout) or 2-level (HYP) smeared improved Wilson fermions



with tree-level $O(a)$ clover improved fermions:



Action and algorithms

action:
good balance between gauge (Symanzik improvement) and fermionic improvements (clover and stout smearing) and CPU gauge and fermion improvement with terms of $O(a^4)$ and $O(a^2)$

algorithm:
rational hybrid Monte-Carlo algorithm,
Hasenbusch mass preconditioning, mixed precision techniques,
multiple time-scale integration, Omelyan integrator

parameter space:
series of n_f=2+1 simulations (degenerate u and d sea quarks)
we vary m_{ud} in a range which corresponds to $M_\pi \approx$190—580 MeV
separate s sea quark, with m_s at its approximate physical value
repeat some simulations with a slightly different m_s and interpolate
three different β-s, which give $a \approx$0.125 fm, 0.085 fm and 0.065 fm

Further advantages of the action

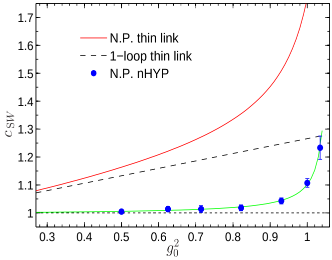
smallest eigenvalue of M: small fluctuations
$\Rightarrow$ simulations are stable (major issue of Wilson fermions)

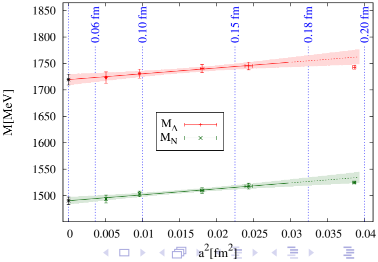
non-perturbative improvement coefficient: $\approx$ tree-level (smearing)

R. Hoffmann, A. Hasenfratz, S. Schaefer, PoS **LAT2007** (2007) 1 04

good a^2 scaling of hadron masses (M_π/M_ρ=2/3) up to $a\approx$0.2 fm

S. Dürr et al. [Budapest-Marseille-Wuppertal Collaboration] arXiv :0802.2706

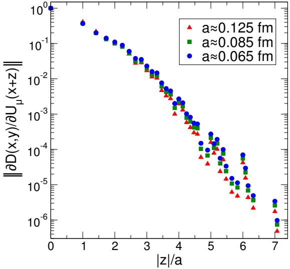




Locality properties of the action

stout smearing 6 times: should we worry about locality (2 types)?
– in continuum the proper QCD action is recovered (ultra-local)
– does one receive at $a \neq 0$ unwanted contributions?

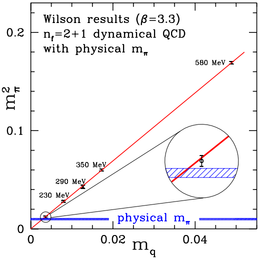
type A: $D(x,y)$=0 for all (x,y) except for nearest neighbors
type B: dependence of $D(x,y)$ on U_μ at distance z

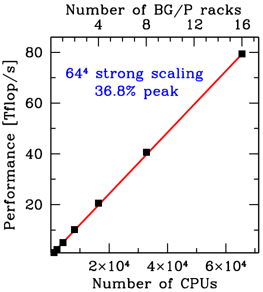


drops exponentially to 10^{-6} within the ultra-locality region: OK

Simulation at physical quark masses

M's eigenvalues close to 0: CPU demanding (large condition number)
our choice of action and large volumes (6 fm):
the spread of the smallest eigenvalue decreases $\Rightarrow$ away from zero





we can go down to physical pion masses $\Rightarrow$ algorithmically safe

Blue Gene shows perfect strong scaling from 1 midplane to 16 racks
our sustained performance is as high as 37% of the peak 0.2 Pflops

carry out two analyses
one with M_Ω (Ω set) and one with M_Ξ (Ξ set)

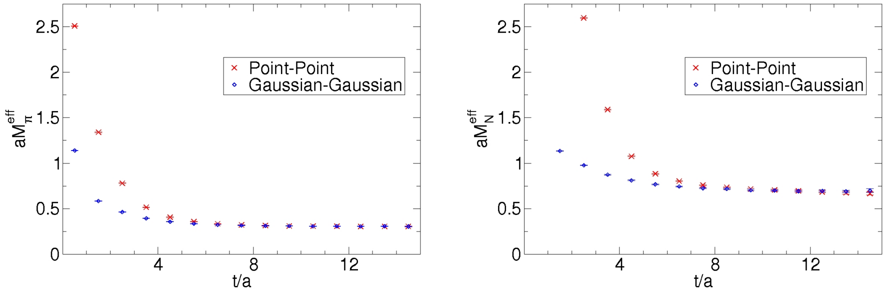
fix the bare quark masses:
use the mass ratio pairs (M_π/M_Ω,M_K/M_Ω) or (M_π/M_Ξ,M_K/M_Ξ)

we calculate the hadron masses of the

baryon octet (N, Σ, Λ, Ξ)
baryon decuplet (Δ, Σ^*, Ξ^*, Ω)
light pseudoscalar mesons (π, K)
vector meson (ρ, K^*) octets

Suppression of excited states

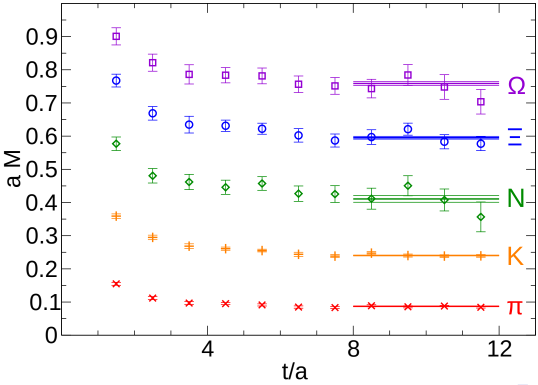
effective masses for different source types
point sources have vanishing extents
Gaussian sources have radii of approximately 0.3 fm



every tenth trajectory is used in the analysis
upto eight timeslices as sources for the correlation functions
integrated autocorrelation times for hadron propagator: $\approx$0.5

Correlation functions, mass fits

masses are obtained by correlated fits
several fitting ranges were chosen (see later our error analysis)
illustration: mass plateaus at our smallest $M_\pi \approx 190$ MeV (noisiest)



Approach physical masses & the continuum limit

- two strategies to extrapolate to the physical pion mass:

form of the function: given by an expansion around a reference point
$r_X = r_X(ref) + \alpha_X[r_\pi^2 - r_\pi^2(ref)] + \beta_X[r_K^2 - r_K^2(ref)] + hoc$
(with higher order contributions)

a. chiral fit: conventional strategy:
reference point: $r_\pi^2(ref)$=0 and $r_K^2(ref)$ is in the middle of the r_K^2 range
this choice corresponds to chiral perturbation theory: $hoc \propto r_\pi^3$
(all coefficients left free for the analysis)

b. Taylor fit: $r_\pi^2(ref)$: non-singular point in the middle of the r_π^2 range
all points are well within the radius of convergence of the expansion
$hoc \propto r_\pi^4$ turned out to be sufficient

Approach physical masses & the continuum limit

applicability of the chiral/Taylor expansions is not known a priori

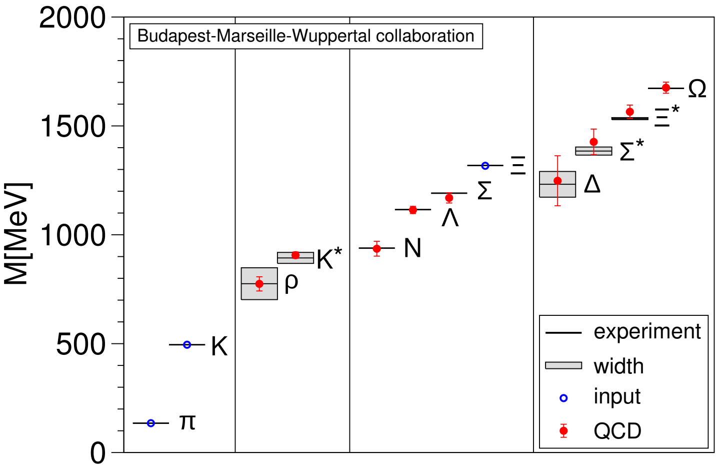
vector mesons: *hoc*-s consistent with zero (include)
baryons: *hoc*-s are significant
two strategies $\Rightarrow$ differences between $M_X(phys)$
possible contributions of yet *hoc*-s, not included in our fits

quantify these contributions further: take different ranges of M_π
three ranges: all points/upto M_π=560 MeV/upto M_π=450 MeV

• three lattice spacings are used for continuum extrapolation
our scaling analysis showed that cutoff effects are linear in a^2
however, one cannot exclude a priori a leading term linear in a
$\Rightarrow$ we allow for the masses both a or a^2 type cutoff effects

two ways for scale setting & two strategies for mass extrapolation
three M_π ranges & two types of cutoff effects $\Rightarrow$ error analysis

Final result for the hadron spectrum



Final results in GeV

X	Exp. [PDG]	M_X (Ξ set)	M_X (Ω set)
ρ	0.775	0.775(29)(13)	0.778(30)(33)
K^*	0.894	0.906(14)(4)	0.907(15)(8)
N	0.939	0.936(25)(22)	0.953(29)(19)
Λ	1.116	1.114(15)(5)	1.103(23)(10)
Σ	1.191	1.169(18)(15)	1.157(25)(15)
Ξ	1.318	1.318	1.317(16)(13)
Δ	1.232	1.248(97)(61)	1.234(82)(81)
Σ^*	1.385	1.427(46)(35)	1.404(38)(27)
Ξ^*	1.533	1.565(26)(15)	1.561(15)(15)
Ω	1.672	1.676(20)(15)	1.672

isospin averaged experimental masses: members within 3.5 MeV
statistical/systematic errors in the first/second parentheses

Error budget as fractions of the total systematic error

	$a \longrightarrow 0$	chiral/normalization	excited states	finite V
ρ	0.20	0.55	0.45	0.20
K^*	0.40	0.30	0.65	0.20
N	0.15	0.90	0.25	0.05
Λ	0.55	0.60	0.40	0.10
Σ	0.15	0.85	0.25	0.05
Ξ	0.60	0.40	0.60	0.10
Δ	0.35	0.65	0.95	0.05
Σ^*	0.20	0.65	0.75	0.10
Ξ^*	0.35	0.75	0.75	0.30
Ω	0.45	0.55	0.60	0.05

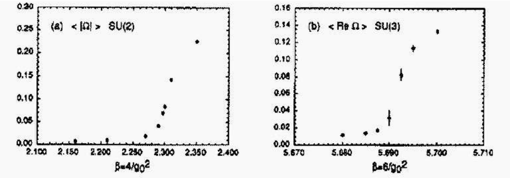
Finite-size scaling theory

problem with phase transitions in Monte-Carlo studies

Monte-Carlo applications for pure gauge theories ($V = 24^3 \cdot 4$)

existence of a transition between confining and deconfining phases:

Polyakov loop exhibits rapid variation in a narrow range of β



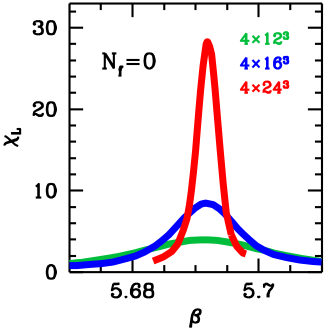
• theoretical prediction: SU(2) second order, SU(3) first order

$\Longrightarrow$ Polyakov loop behavior: SU(2) singular power, SU(3) jump

data do not show such characteristics!

Finite size scaling in the quenched theory

look at the susceptibility of the Polyakov-line
first order transition (Binder) $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V



finite size scaling shows: the transition is of first order

The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

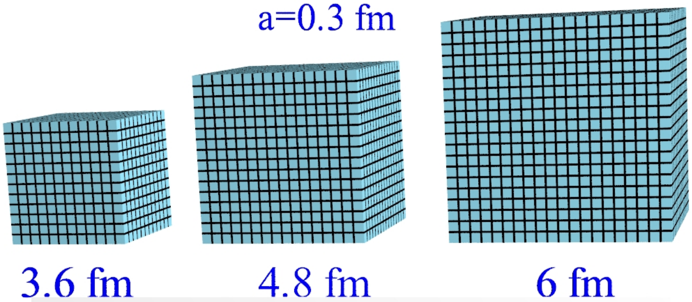
finite size scaling study of the chiral condensate (susceptibility)

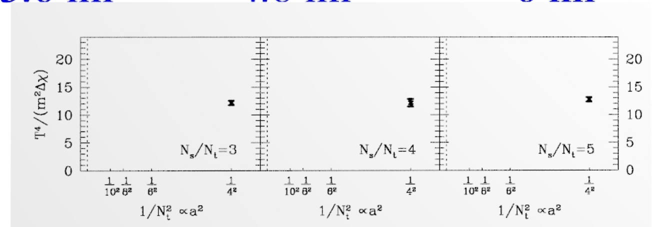
$$\chi = (T/V)\partial^2 \log Z/\partial m^2$$

phase transition: finite V analyticity $V \to \infty$ increasingly singular
(e.g. first order phase transition: height $\propto$ V, width $\propto$ 1/V)
for an analytic cross-over χ does not grow with V

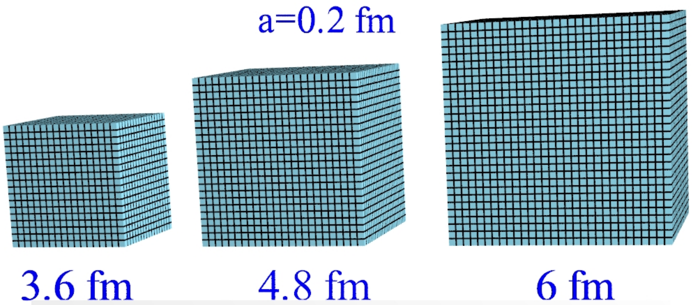
two steps (three volumes, four lattice spacings):
a. fix V and determine χ in the continuum limit: a=0.3,0.2,0.15,0.1fm
b. using the continuum extrapolated χ_{max}: finite size scaling

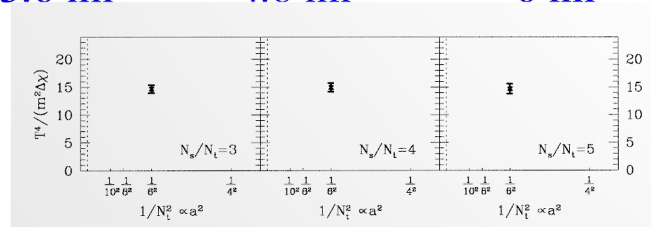
Approaching the continuum limuit





Approaching the continuum limuit





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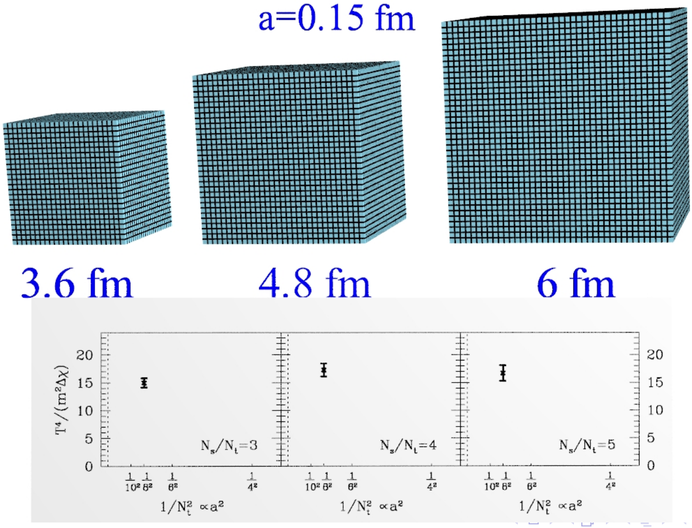
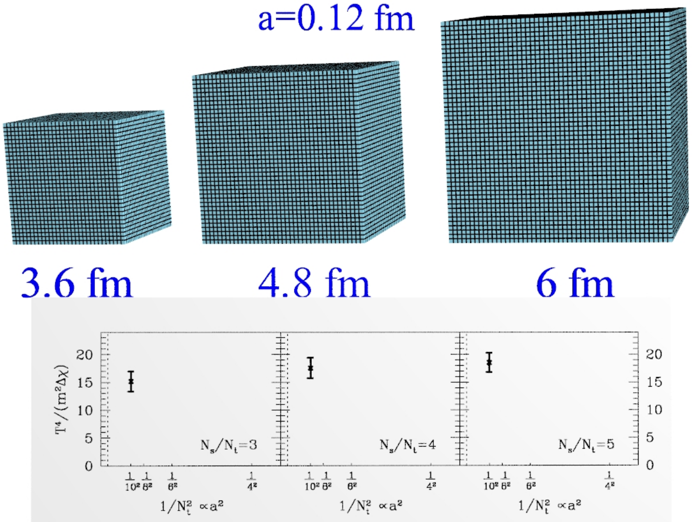
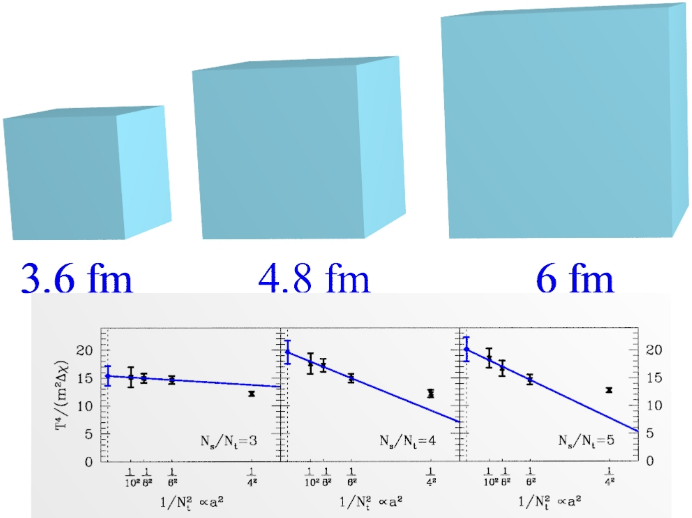


Approaching the continuum limuit

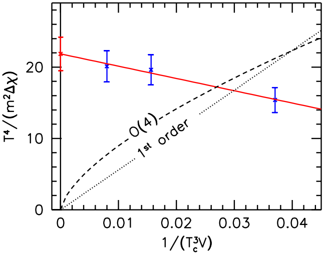


Approaching the continuum limuit



The nature of the QCD transition: result

- finite size scaling analysis with continuum extrapolated $T^4/m^2\Delta\chi$



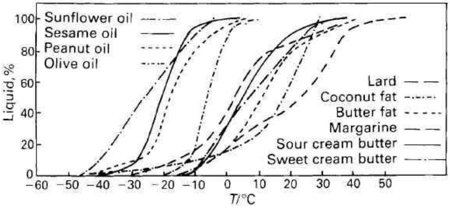
the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range
chance probability for 1/V is 10^{-19} for O(4) is $7 \cdot 10^{-13}$
continuum result with physical quark masses in staggered QCD:

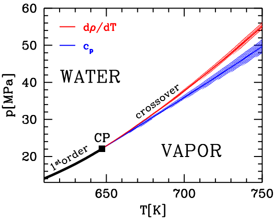
the QCD transition is a cross-over

The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

analytic transition (cross-over) $\Rightarrow$ it has no unique T_c:
examples: melting of butter (not ice) & water-steam transition

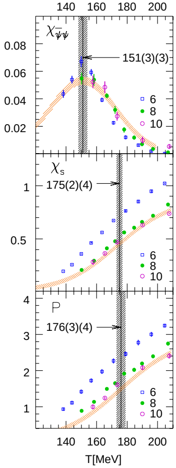




above the critical point c_p and $d\rho/dT$ give different T_cs.
QCD: chiral & quark number susceptibilities or Polyakov loop
they result in different T_c values $\Rightarrow$ physical difference

The transition temperature: results and scaling

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46



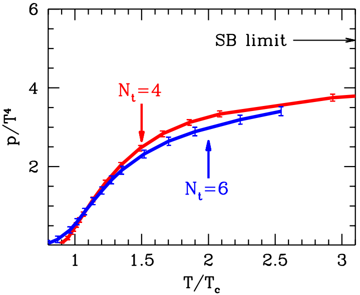
Chiral susceptibility
T_c=151(3)(3) MeV
ΔT_c=28(5)(1) MeV

Quark number susceptibility
T_c=175(2)(4) MeV
ΔT_c=42(4)(1) MeV

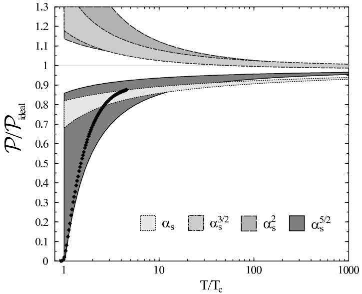
Polyakov loop
T_c=176(2)(4) MeV
ΔT_c=38(5)(1) MeV

Equation of state: difficulties at high temperatures

lattice results for the EoS extend upto a few times T_c



perturbative series "converges" only at asymptotically high T



applicability ranges of perturbation theory and lattice don't overlap
it was believed to be "impossible" to extend the range for lattice QCD

The standard technique is the integral method

$\bar{p}$=T/V·log(Z), but Z is difficult
$\Rightarrow$ $\bar{p}$ integral of (∂log(Z)/$\partial\beta$,∂log(Z)/∂m)

substract the T=0 term, the pressure is given by:
$p(T)$=$\bar{p}(T)$-$\bar{p}(T=0)$

back of an envelope estimate:

T_c≈150–200 MeV, m_π=135 MeV
try to reach T=20·T_c for N_t=8 (a=0.0075 fm)

$\Rightarrow N_s > 4/m_\pi \approx 6/T_c$ = 6·20/T = 6·20·$N_t \approx$ 1000

$\Rightarrow$ completely out of reach

Practical solution for the problem

G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, arXiv:0710.4197

a. substract successively:

$p(\mathrm{T})=\bar{p}(\mathrm{T})-\bar{p}(\mathrm{T}=0)=[\bar{p}(\mathrm{T})-\bar{p}(\mathrm{T}/2)]+[\bar{p}(\mathrm{T}/2)-\bar{p}(\mathrm{T}/4)]+\ldots$

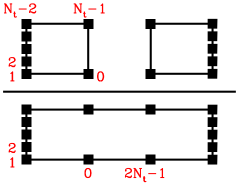
$\Longrightarrow$ for substractions at most twice as large lattices are needed (physical reason: there are no new UV divergencies at finite T)

b. instead of the integral method calculate:

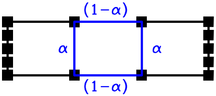
$\bar{p}(\mathrm{T})-\bar{p}(\mathrm{T}/2)=\mathrm{T}/(2\mathrm{V})\cdot\log[\mathrm{Z}^2(N_t)/\mathrm{Z}(2N_t)]$

and introduce an interpolating partition function $Z(\alpha)$

$$\frac{Z^2(N_t)}{Z(2N_t)} =$$



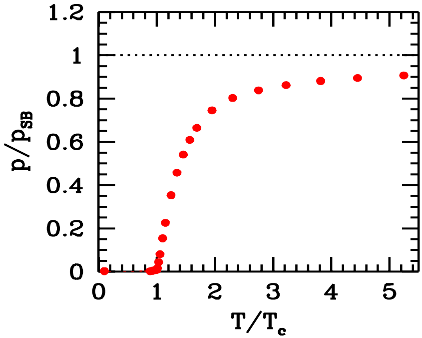
$$\overline{Z}(\alpha) =$$



define $\bar{Z}(\alpha)=\int\mathcal{D}U\exp[-\alpha S_{1b}-(1-\alpha)S_{2b}] \Rightarrow Z^2(N_t)=\bar{Z}(0), \quad Z(2N_t)=\bar{Z}(1)$

one gets directly for $\bar{p}(T)-\bar{p}(T/2)=T/(2V)\cdot\log[Z^2(N_t)/Z(2N_t)]$

$T/(2V)\int_0^1 d\log[\bar{Z}(\alpha)]/d\alpha\cdot d\alpha=T/(2V)\int_0^1\langle S_{1b}-S_{2b}\rangle\alpha\cdot d\alpha$



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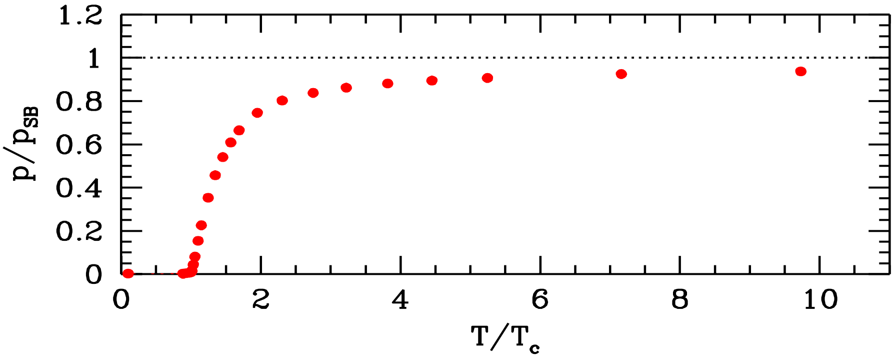

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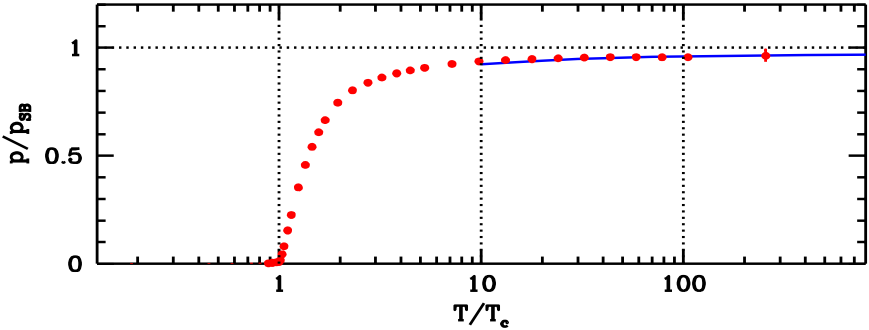

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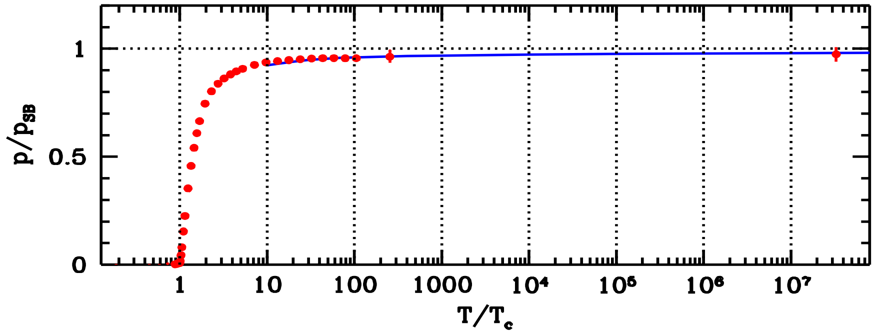

long awaited link between lattice thermodynamics and pert. theory define $\bar{Z}(\alpha)=\int \mathcal{D}U \exp[-\alpha S_{1b}-(1-\alpha)S_{2b}] \Rightarrow Z^2(N_t)=\bar{Z}(0), \quad Z(2N_t)=\bar{Z}\,(1)$

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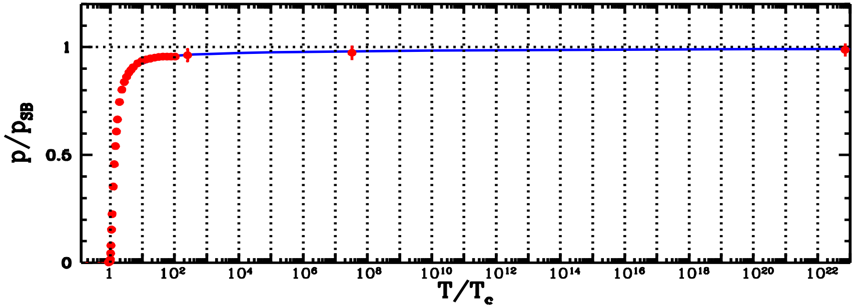
$T/(2V)\int_0^1 d\log[\bar{Z}(\alpha)]/d\alpha\cdot d\alpha=T/(2V)\int_0^1\langle S_{1b}-S_{2b}\rangle\alpha\cdot d\alpha$



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