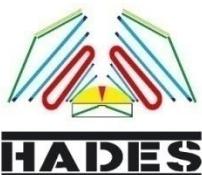


The $\Lambda(1405)$ in p+p collisions at 3.5 GeV

Eliane Epple

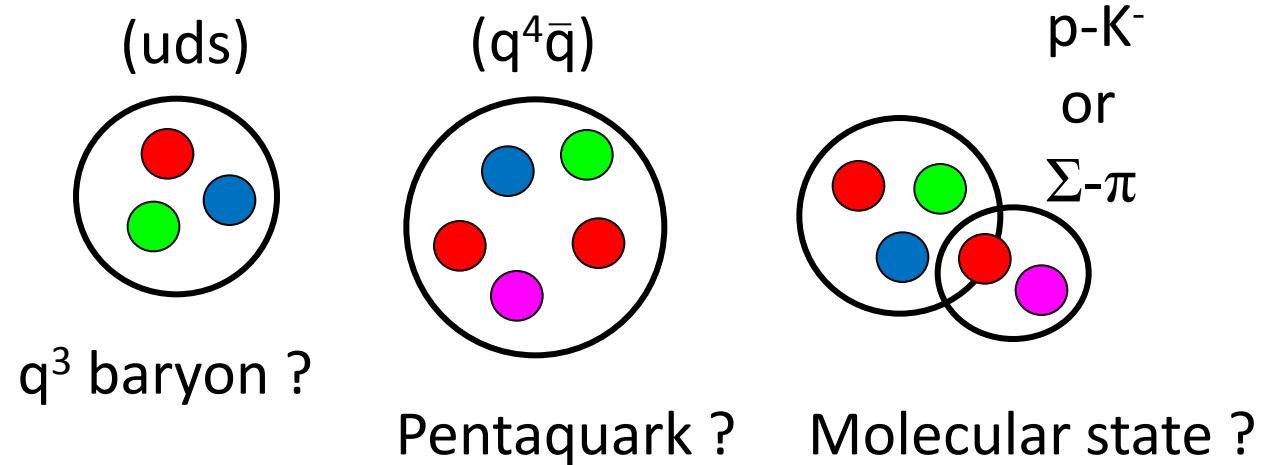
Fr, 11.6.10 - Kraków

- Introduction
- Analysis of the Channel
 $\Lambda(1405) \rightarrow \Sigma^0 \pi^0$
- Analysis of the Channels
 $\Lambda(1405) \rightarrow \Sigma^{-/+} \pi^{+/-}$
- Outlook



Nature of the $\Lambda(1405)$

$m \approx 1406 \text{ MeV}/c^2$
 $\Gamma \approx 50 \text{ MeV}/c^2$



$\Lambda(1405)$ oscillates mainly between a (K^- ,p) - and a (Σ,π) – bound state.

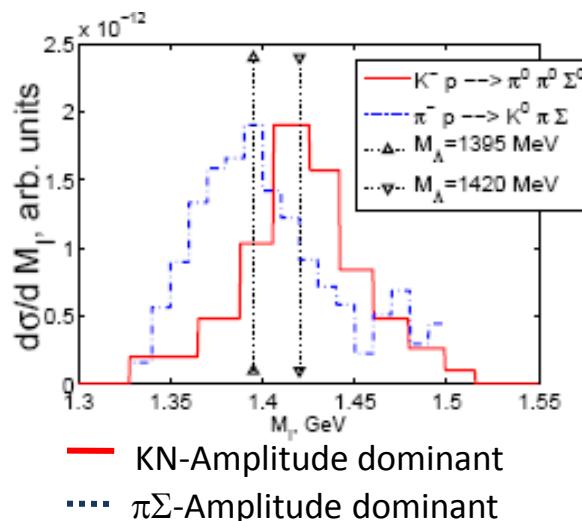
→ Information about the kaon-nucleon interaction.

But not known is the contribution of the two states to the $\Lambda(1405)$.

With this (K^- ,p)- intermediate - bound state, the $\Lambda(1405)$ could be a doorway for the simplest kaonic cluster the ppK-

Properties of the $\Lambda(1405)$

Line shape depends on production Mechanism



V. K. Magas et al.
Phys. Rev. Lett.
95, 052301 (2005)

S. Prakhov et al.
Phys. Rev. C 70
(2004) 034605

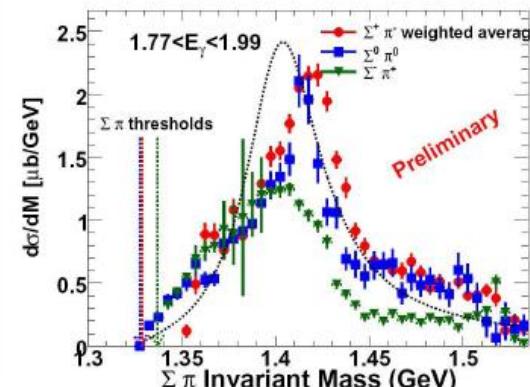
A.W. Thomas et
al., Nucl. Phys.
B56 15 (1973)

— KN-Amplitude dominant
--- πΣ-Amplitude dominant

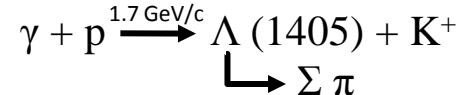
Pole mass seems to be different
when the $\Lambda(1405)$ is produced in different
production mechanisms.

Could be explained by different
coupling strength of the production to the two
poles of the resonance

Line shape depends on decay products



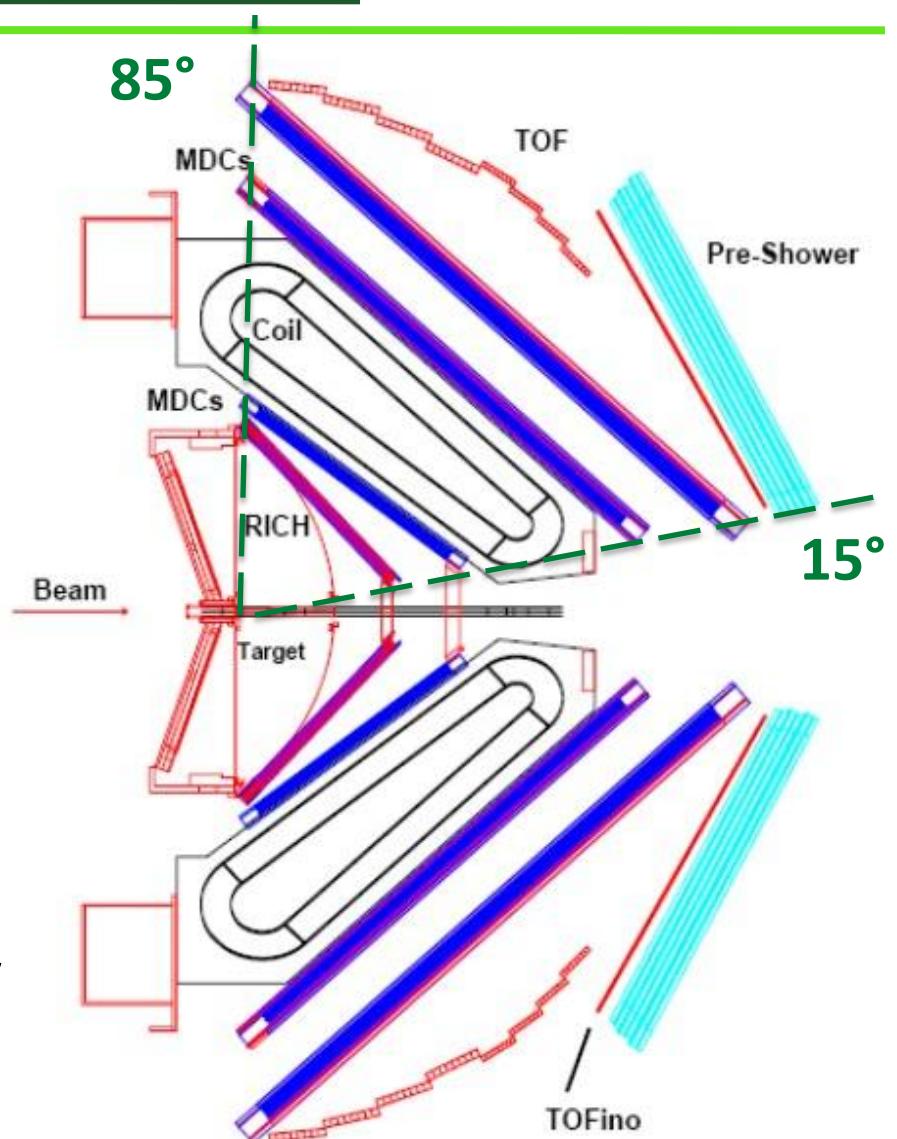
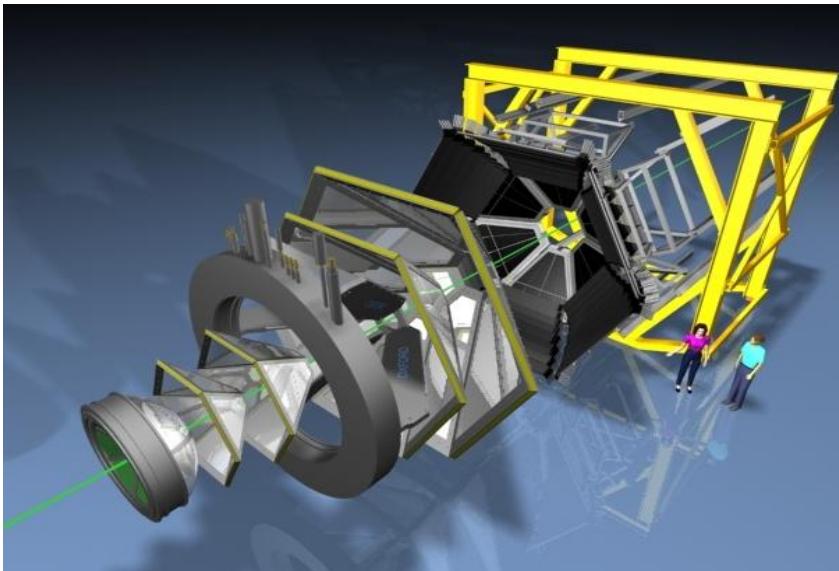
K. Morya et al.,
arXiv:0911.0925



Line shape seems to be different
in different decay channels of the
 $\Lambda(1405)$
→ Interference effects with $I=1$
states.

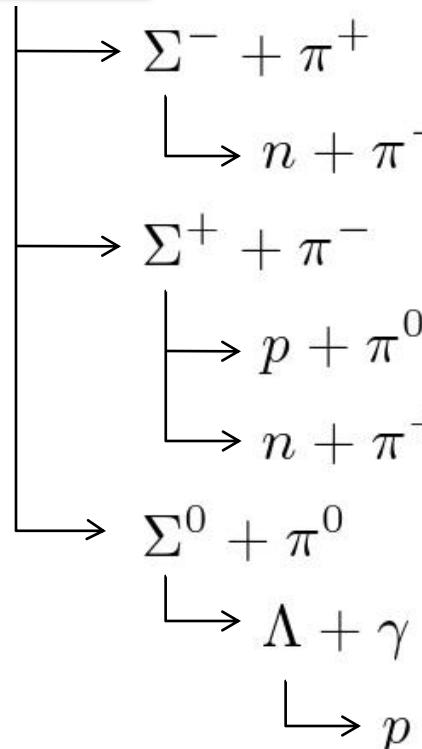
HADES at GSI

High Acceptance Dielectron Spectrometer

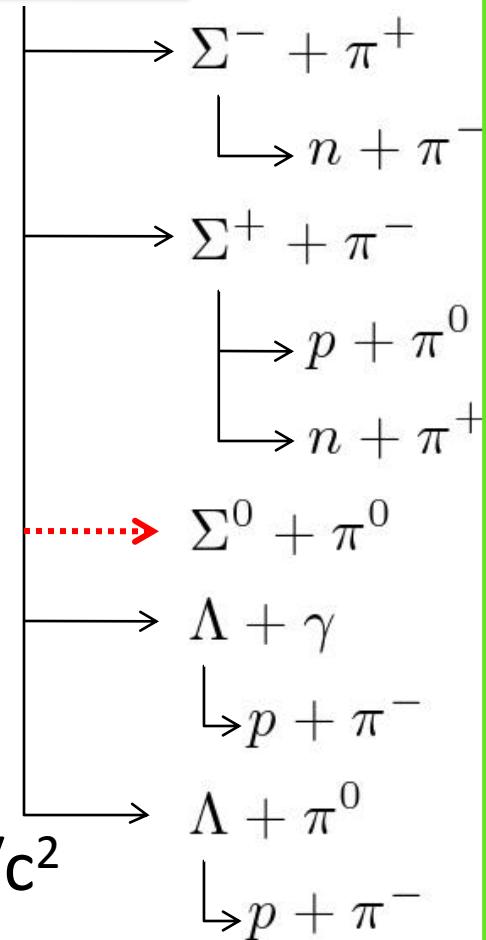
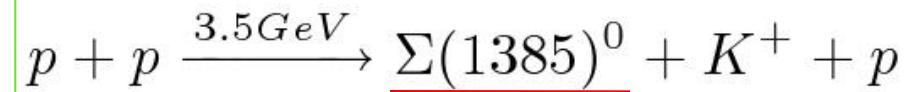


- High acceptance for dilepton pairs
- Momentum resolution $\approx 1\% - 5\%$
- Particle identification via dE/dx & Tof
- $1.2 \cdot 10^9$ Events in p+p at $E_{beam} = 3.5$ GeV

Decay channels of $\Lambda(1405)/\Sigma(1385)^0$

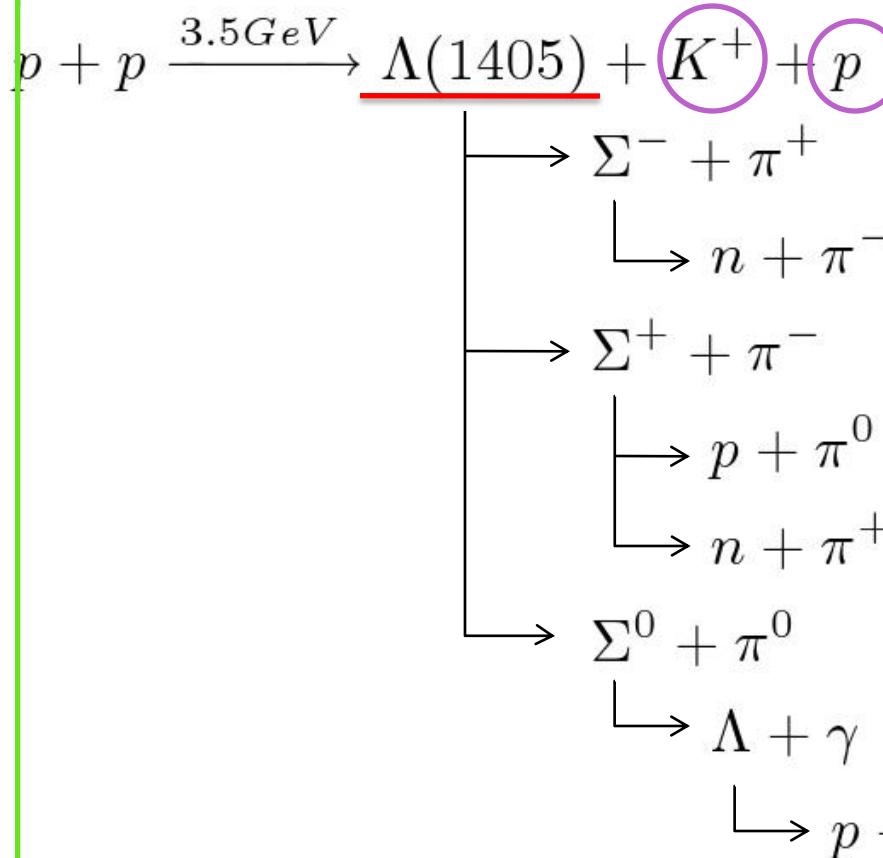


$$\Gamma \approx 50 \text{ MeV}/c^2$$

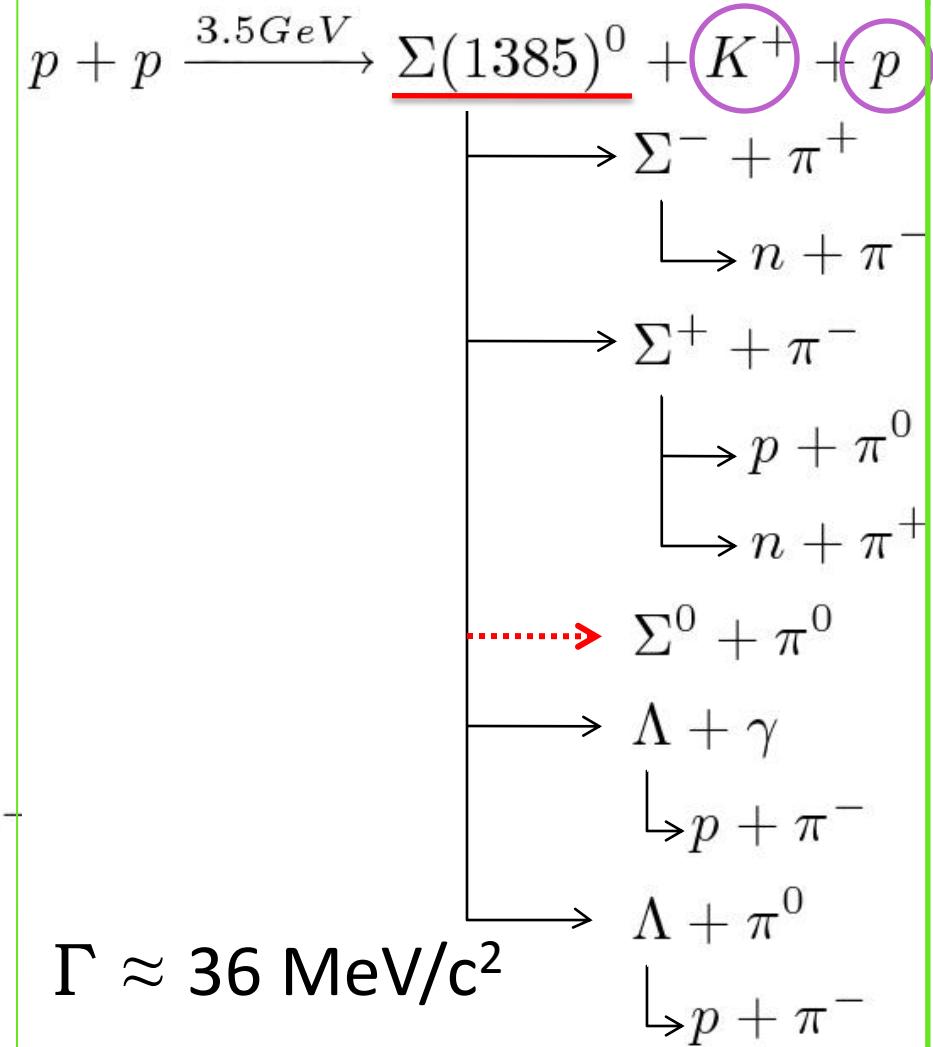


$$\Gamma \approx 36 \text{ MeV}/c^2$$

Decay channels of $\Lambda(1405)/\Sigma(1385)^0$

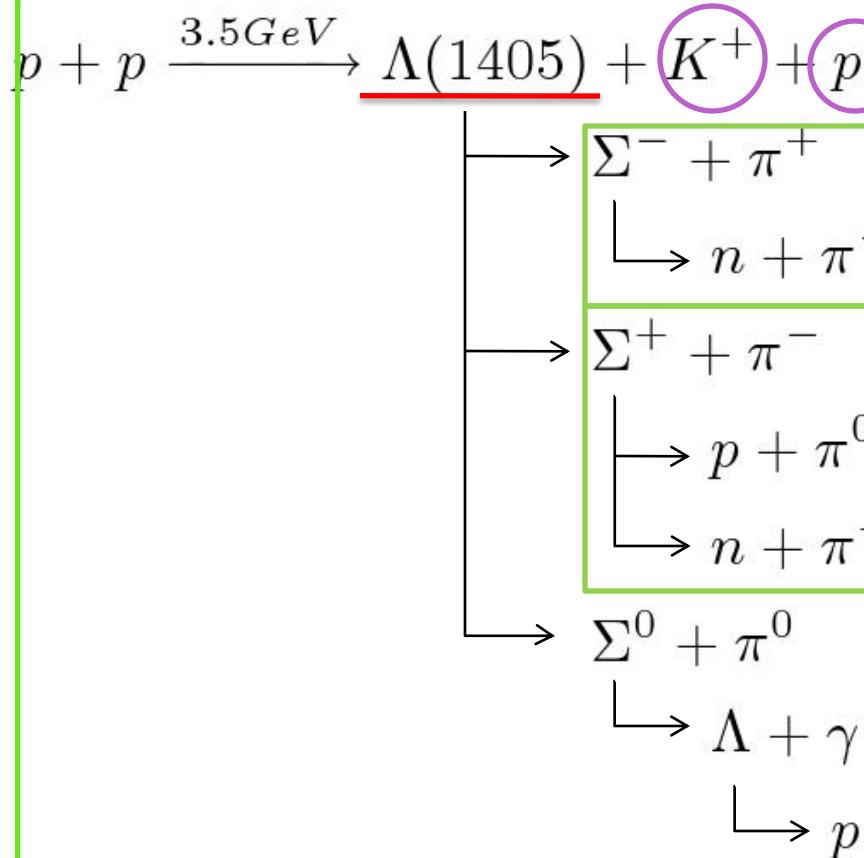


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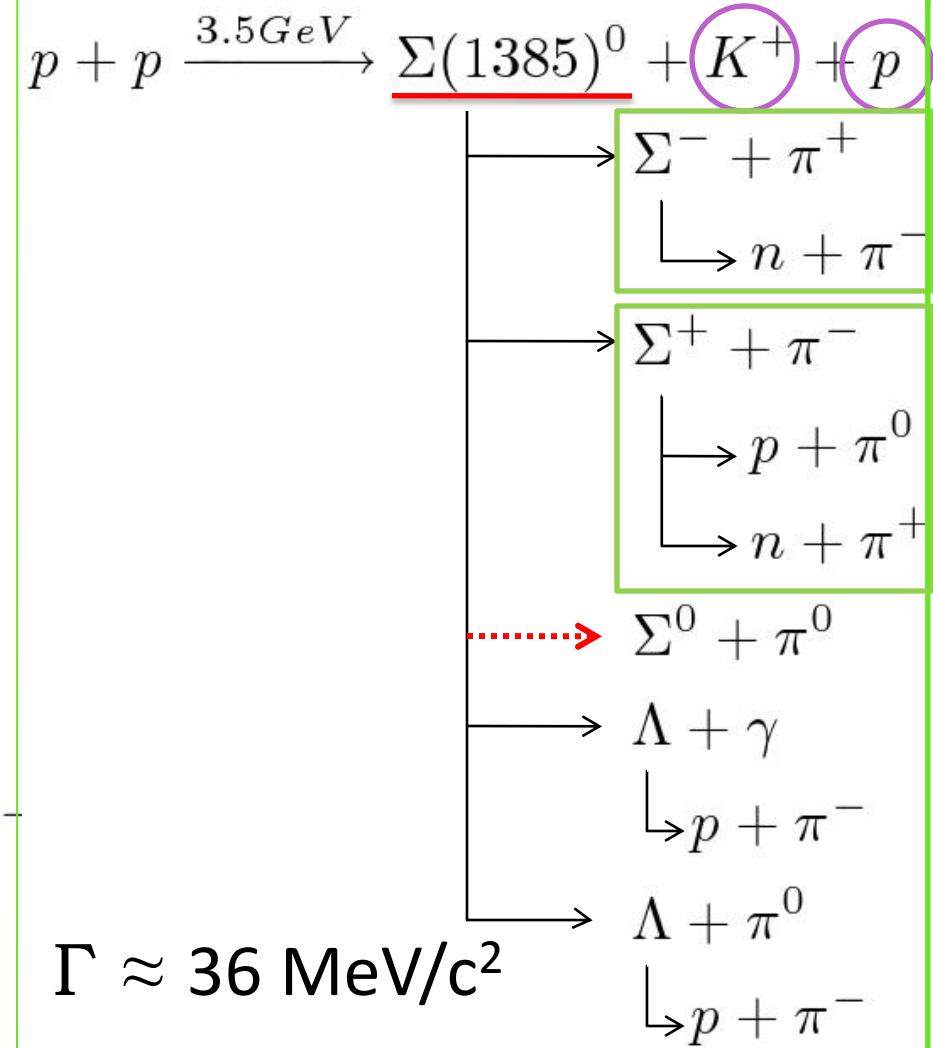


$$\Gamma \approx 36 \text{ MeV}/c^2$$

Decay channels of $\Lambda(1405)/\Sigma(1385)^0$

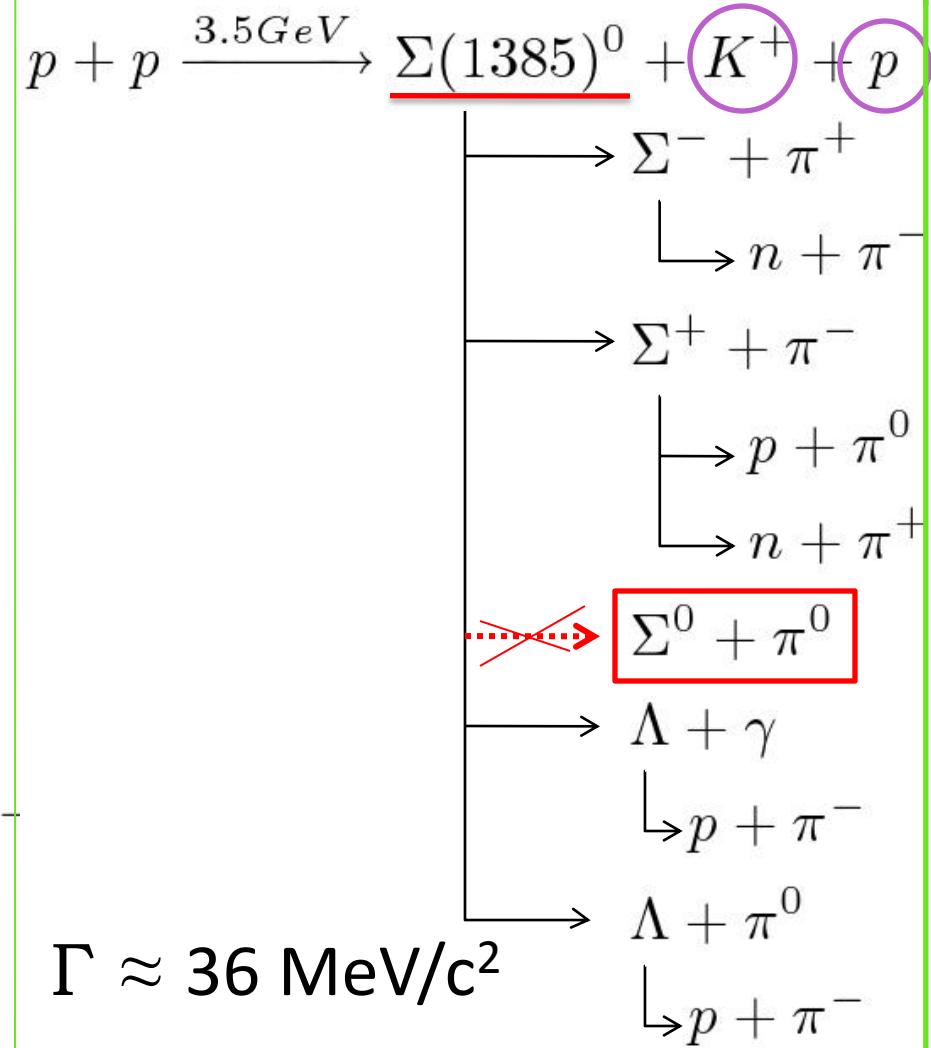
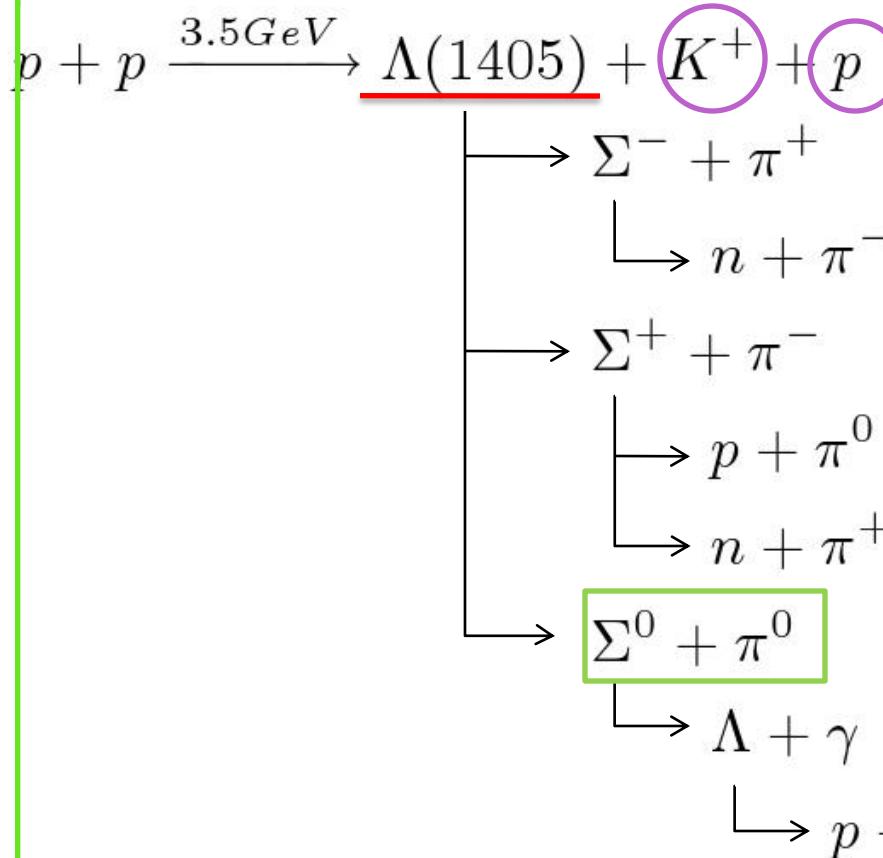


$$\Gamma \approx 50 \text{ MeV}/c^2$$

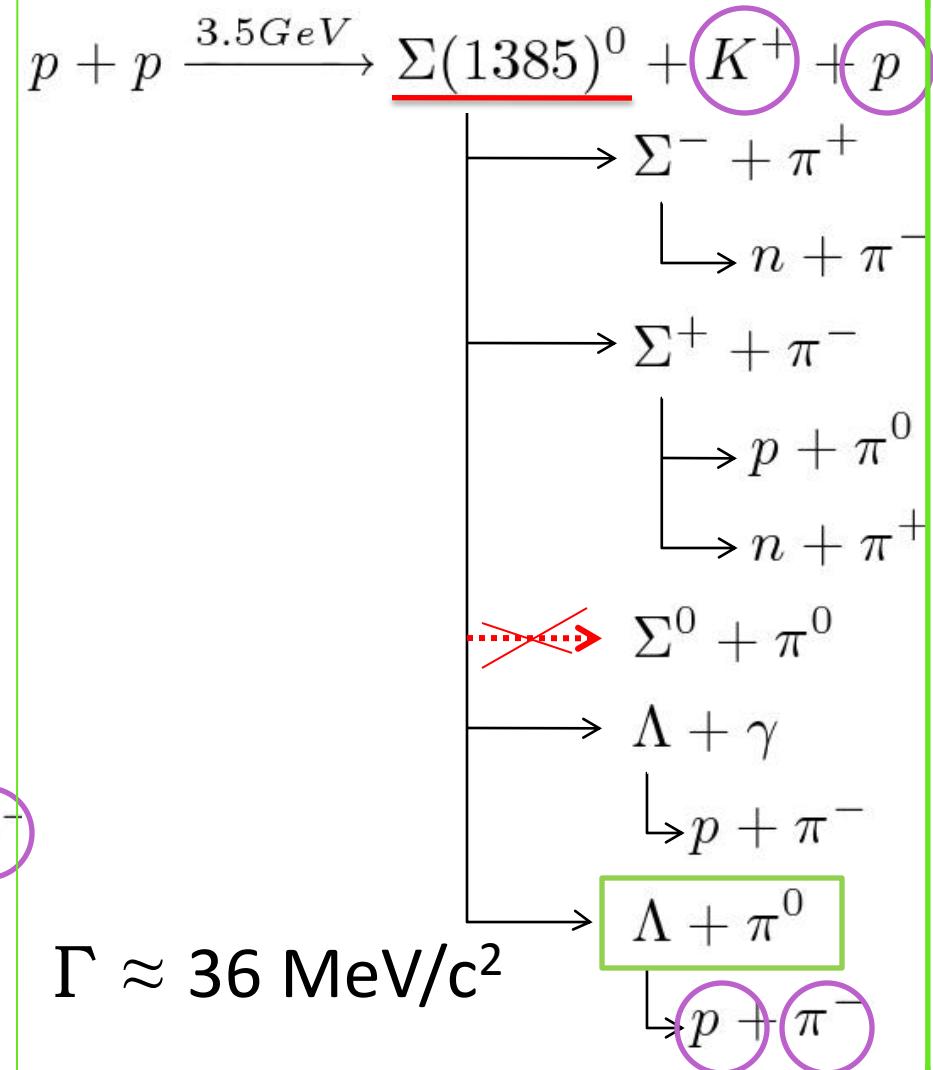
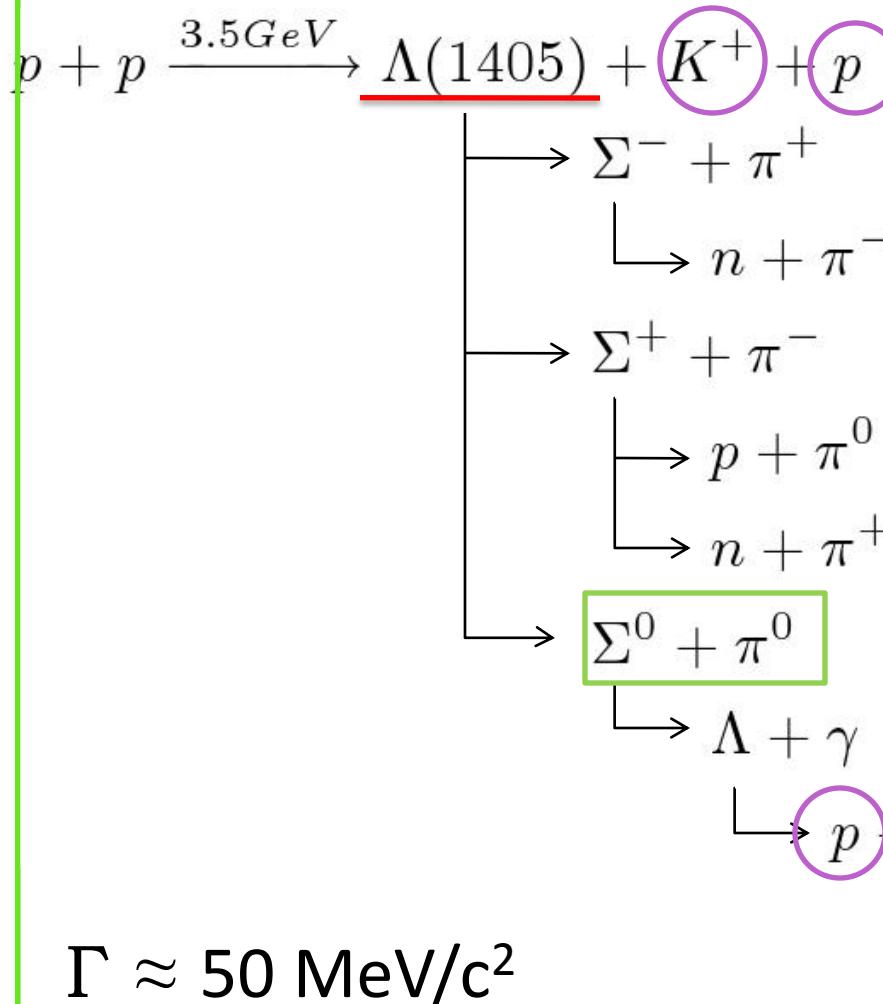


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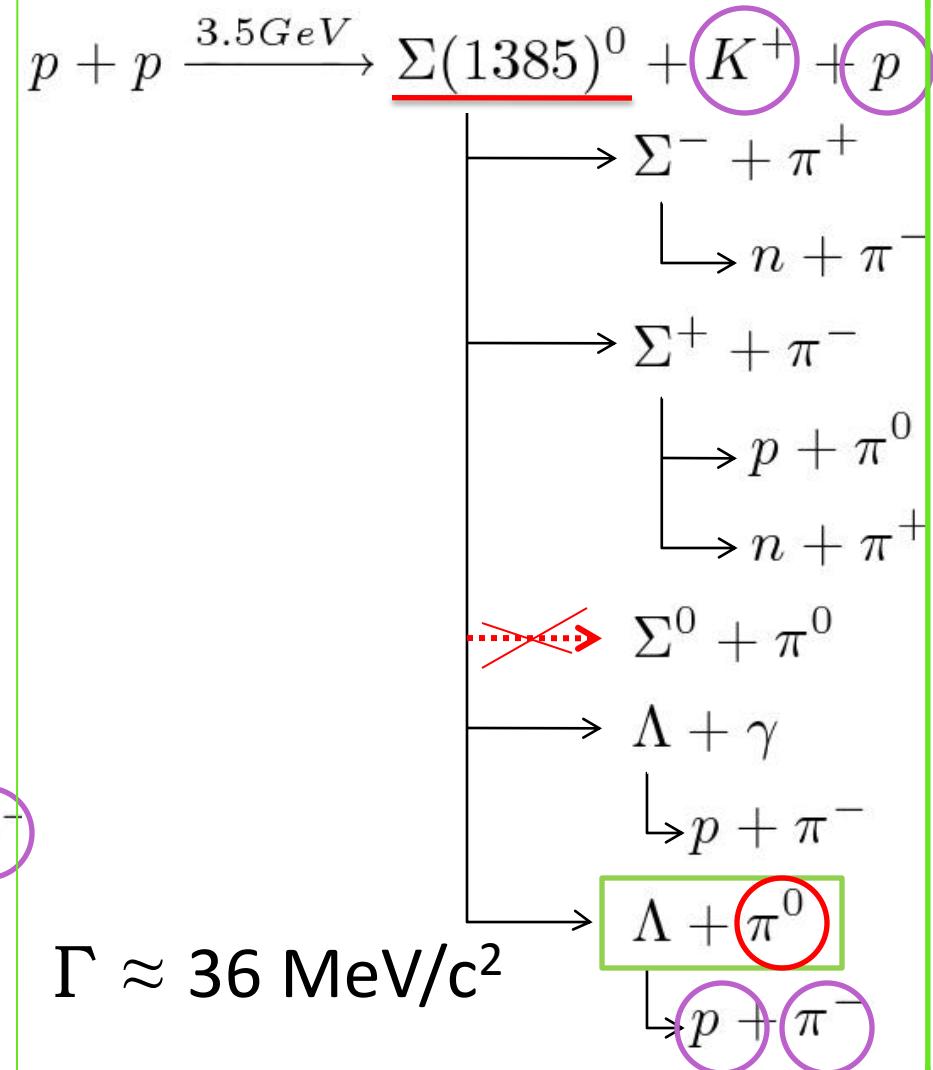
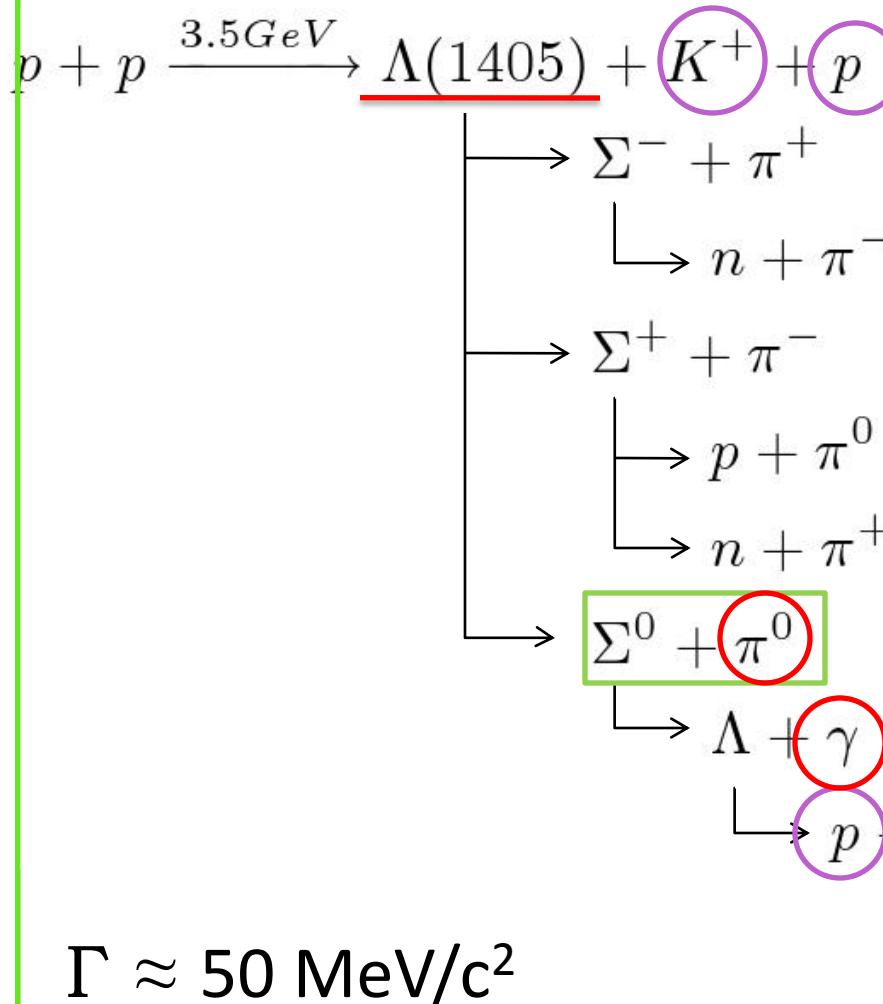
Decay channels of $\Lambda(1405)/\Sigma(1385)^0$



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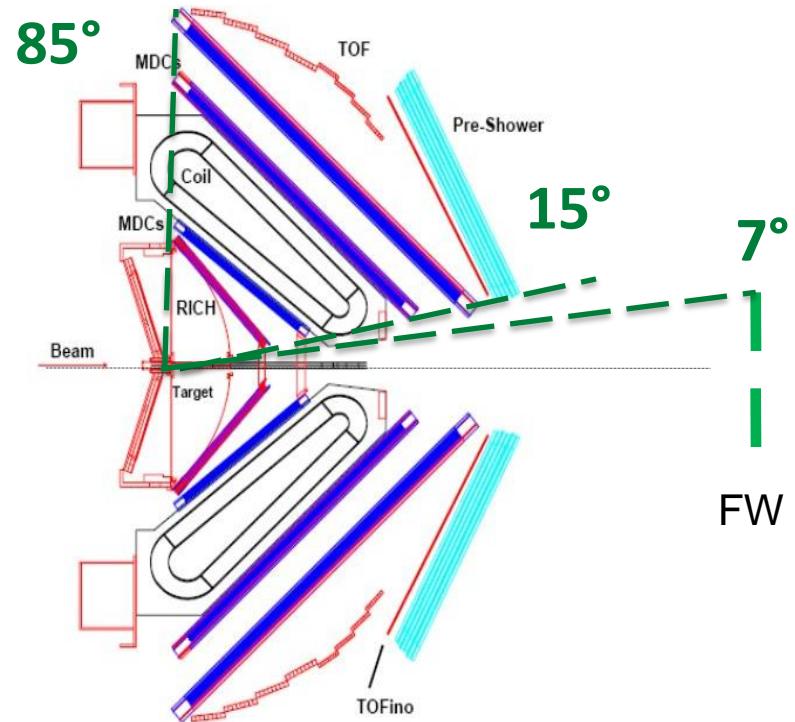
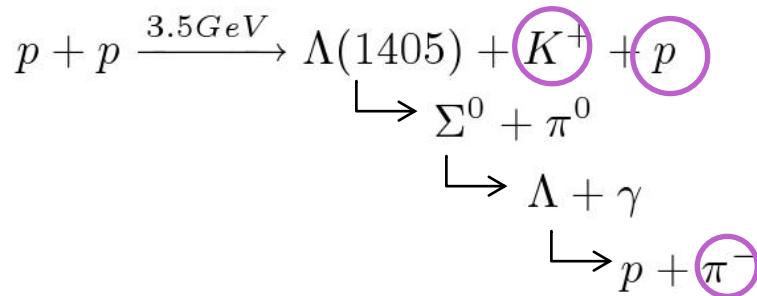


Analysis of the $\Lambda(1405)$

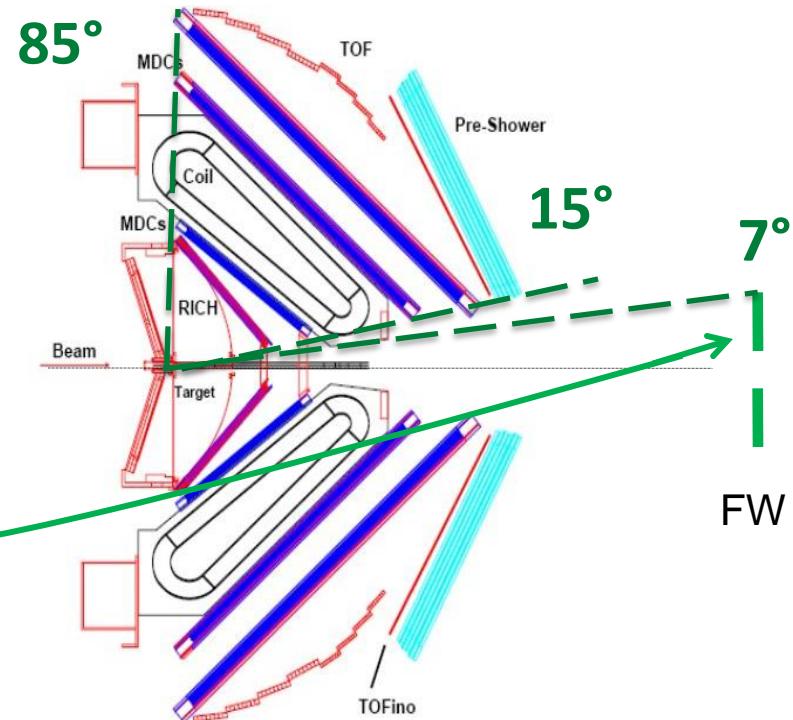
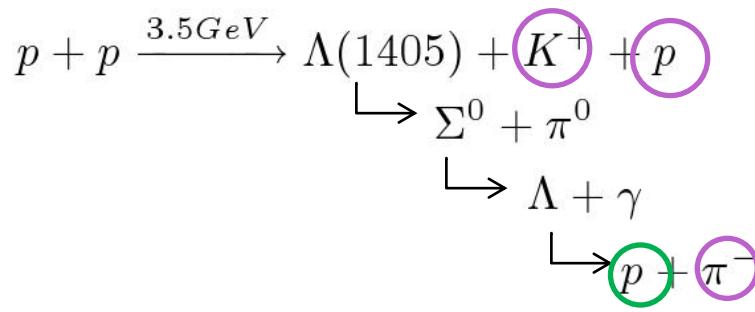
by its decay into

$\Sigma^0\pi^0$

Hades and the Forward Wall



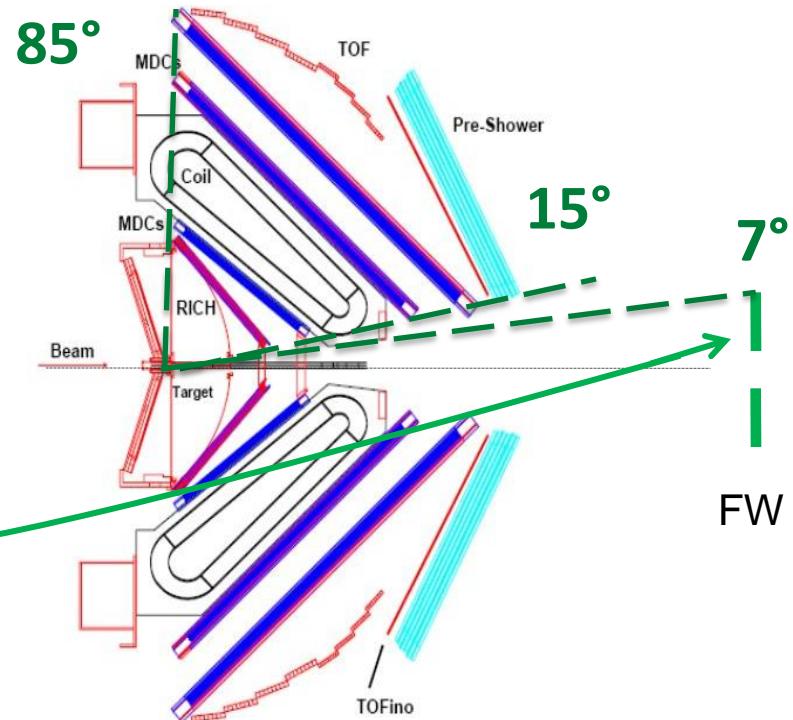
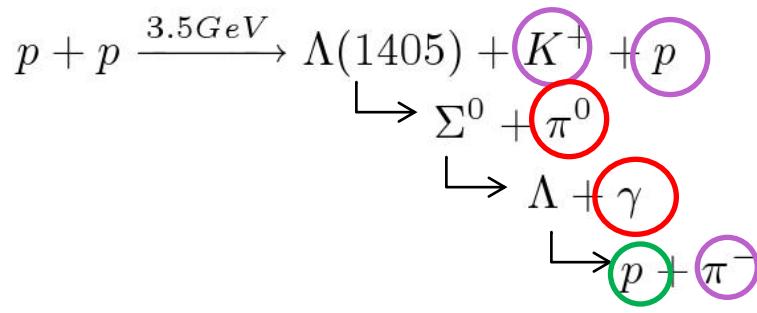
Hades and the Forward Wall



On that sample the analysis will proceed

- cuts on $\Lambda(1116)$ [mass and track cut]
- cuts on the K^+ mass
- cuts on missing mass of all charged particles [$> \pi^0$]

Hades and the Forward Wall

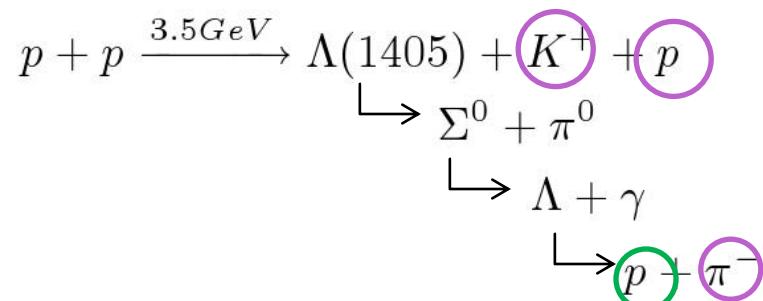
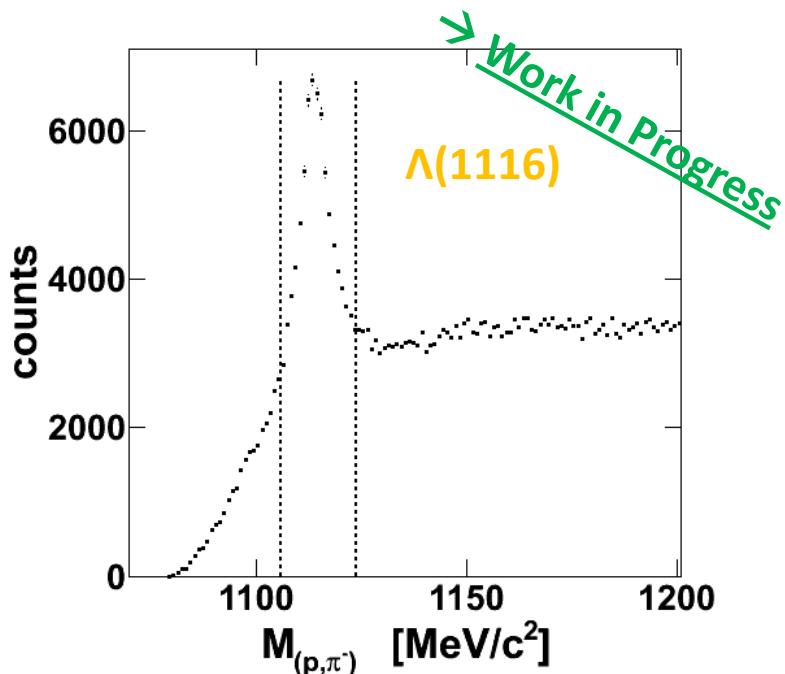


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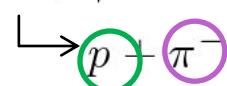
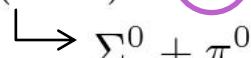
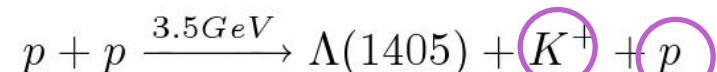
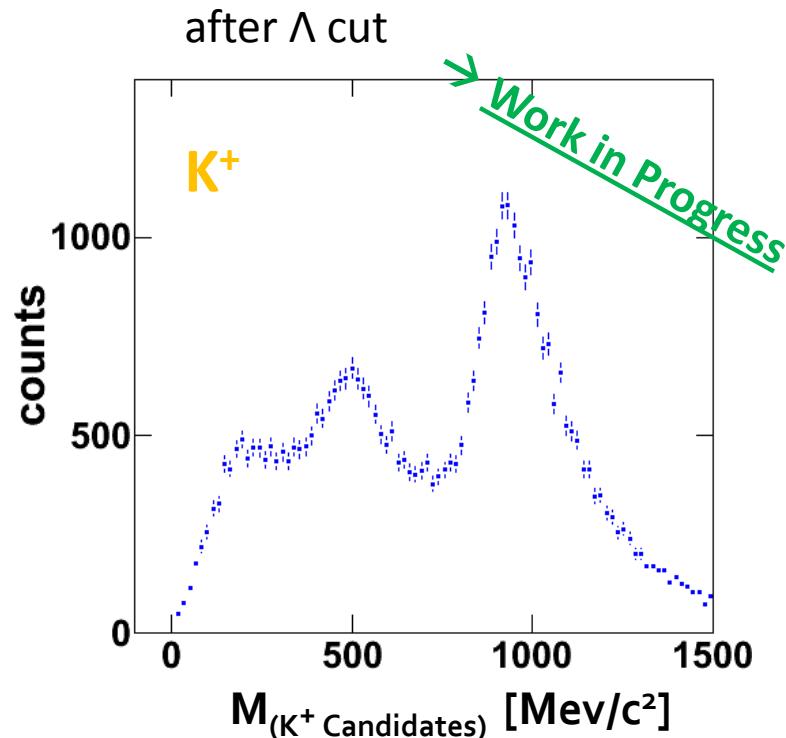
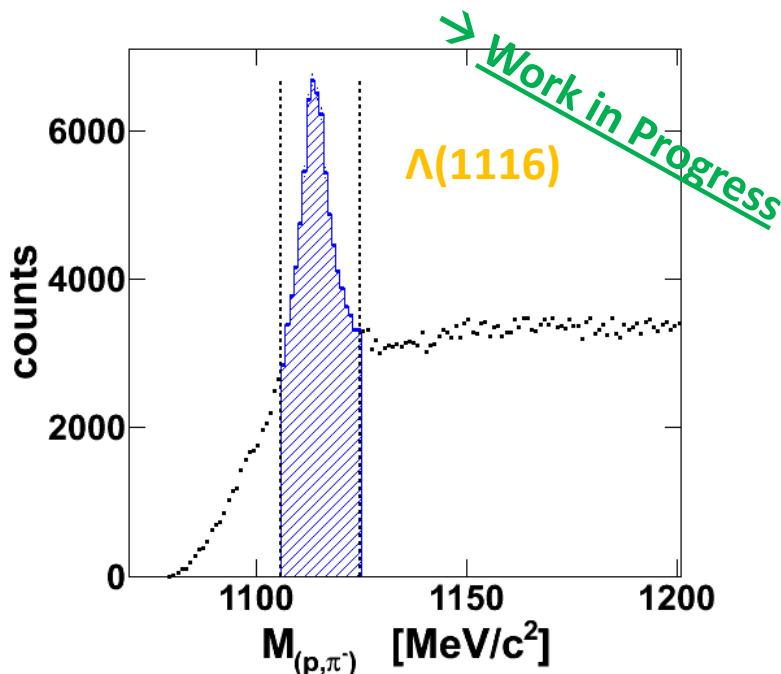
The analysis of the $\Lambda(1405)$

HADES + FW DATA



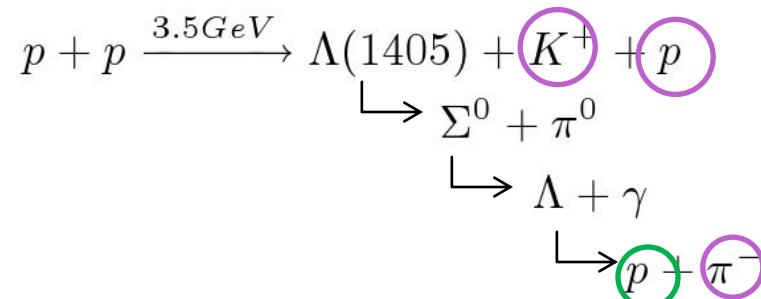
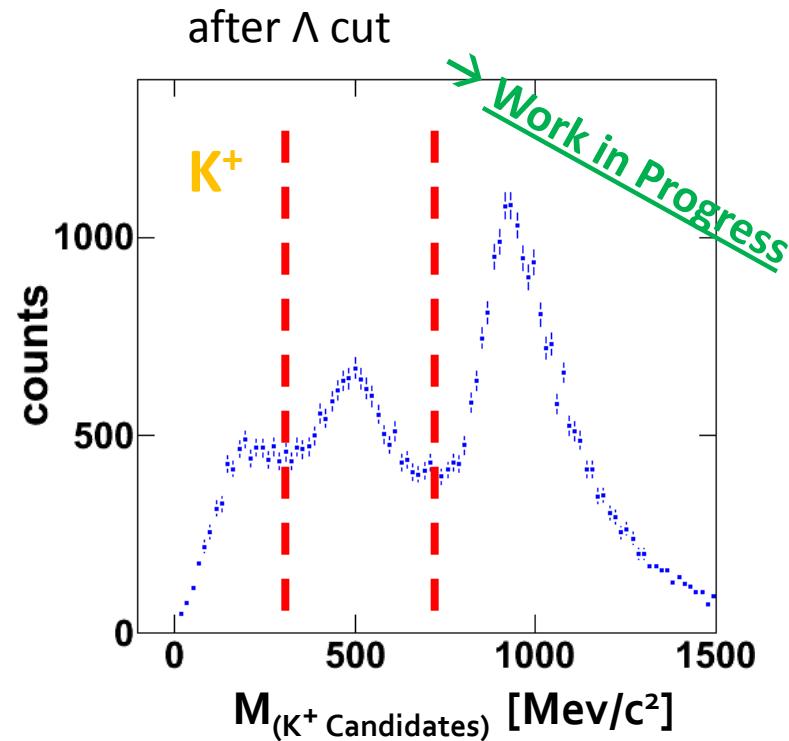
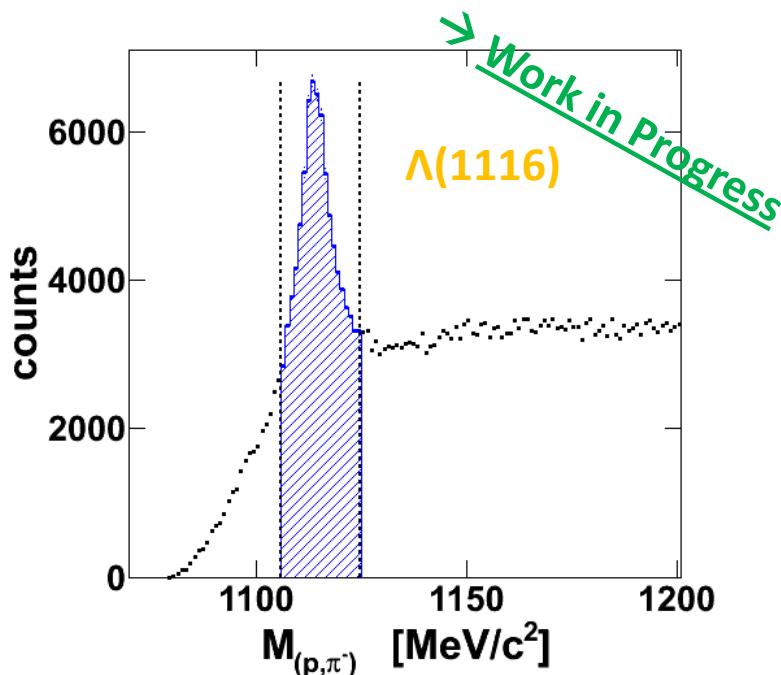
The analysis of the $\Lambda(1405)$

HADES + FW DATA

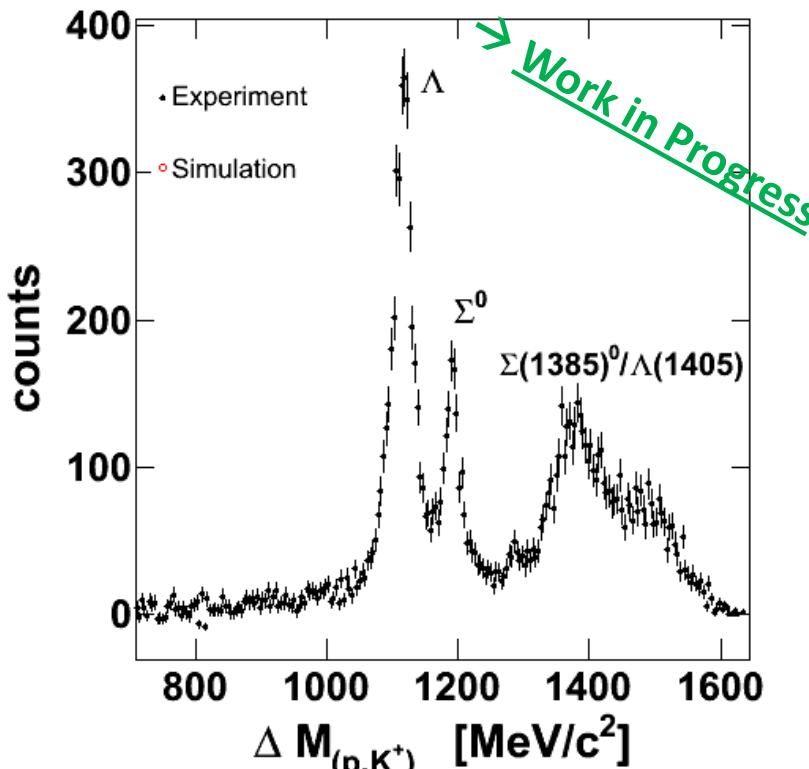


The analysis of the $\Lambda(1405)$

HADES + FW DATA



Understanding the data



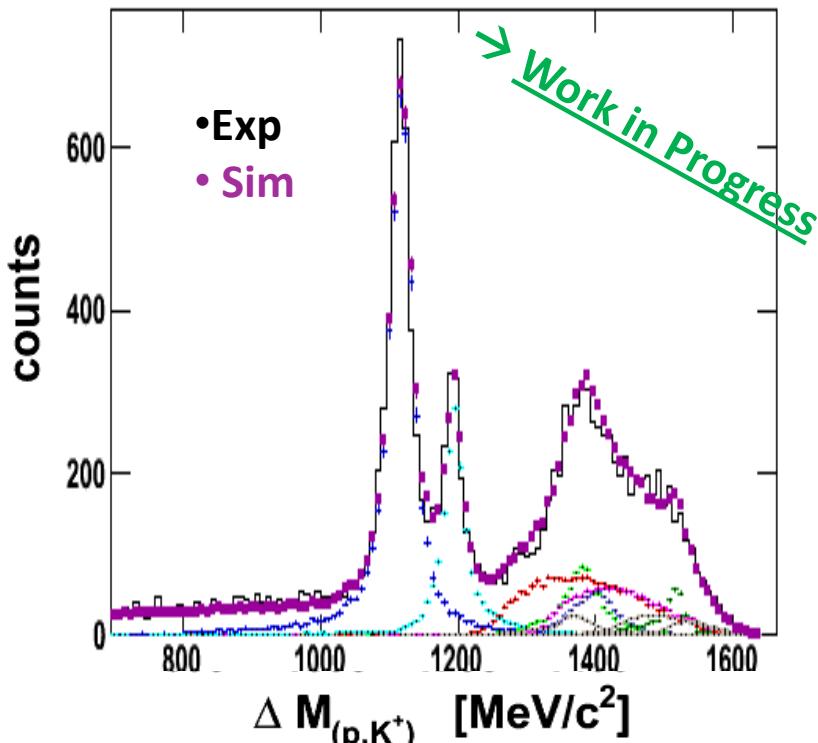
Contributing channels

- $\Lambda + p + K^+$
- $\Lambda + p + K^+ + \pi^0$
- $\Sigma + p + K^+$
- $\Sigma + p + K^+ + \pi^0$
- $\Lambda(1405) + p + K^+$
- $\Lambda(1520) + p + K^+$
- $\Sigma(1385) + p + K^+$
- + K^+ misidentification

Try to describe experimental data:

- Simulate all physical channels containing a real K^+ .
- Describe misidentification background with sideband analysis.
- Determine the strengths off all contributions via a multi-parameter fit to the experimental data.

Understanding the data



Contributing channels

- $\Lambda + p + K^+$
- $\Lambda + p + K^+ + \pi^0$
- $\Sigma + p + K^+$
- $\Sigma + p + K^+ + \pi^0$
- $\Lambda(1405) + p + K^+$
- $\Lambda(1520) + p + K^+$
- $\Sigma(1385) + p + K^+$

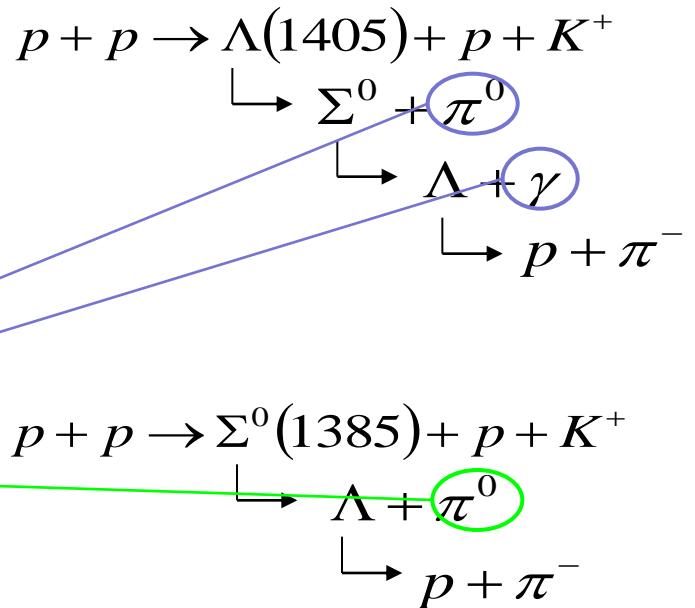
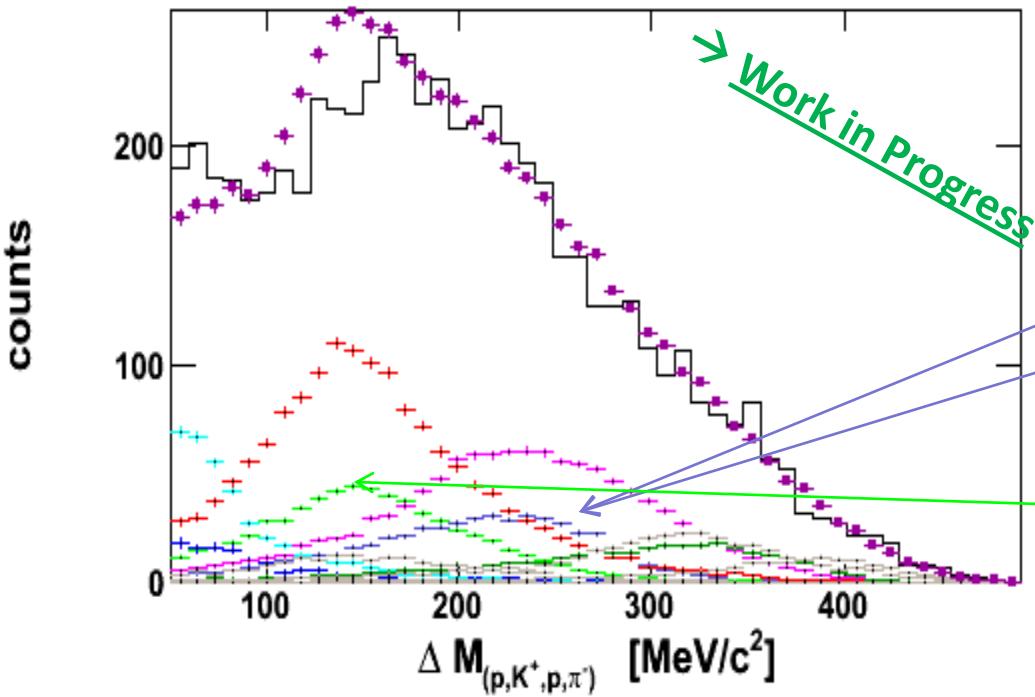
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Try to describe experimental data:

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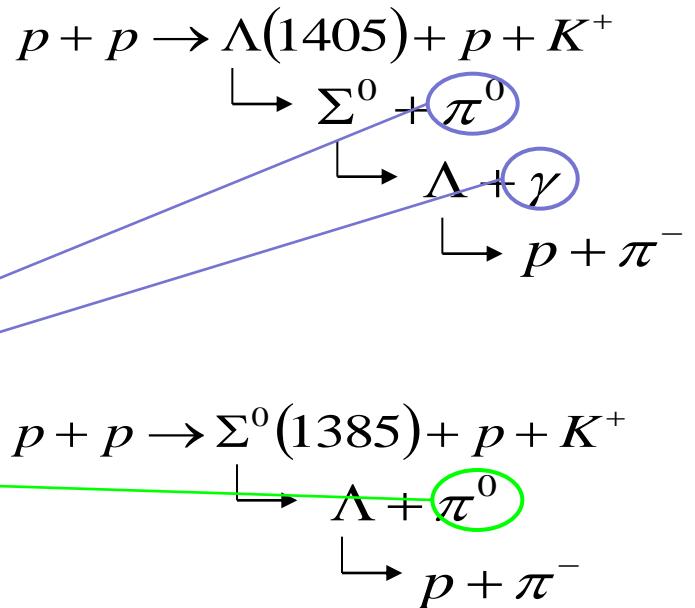
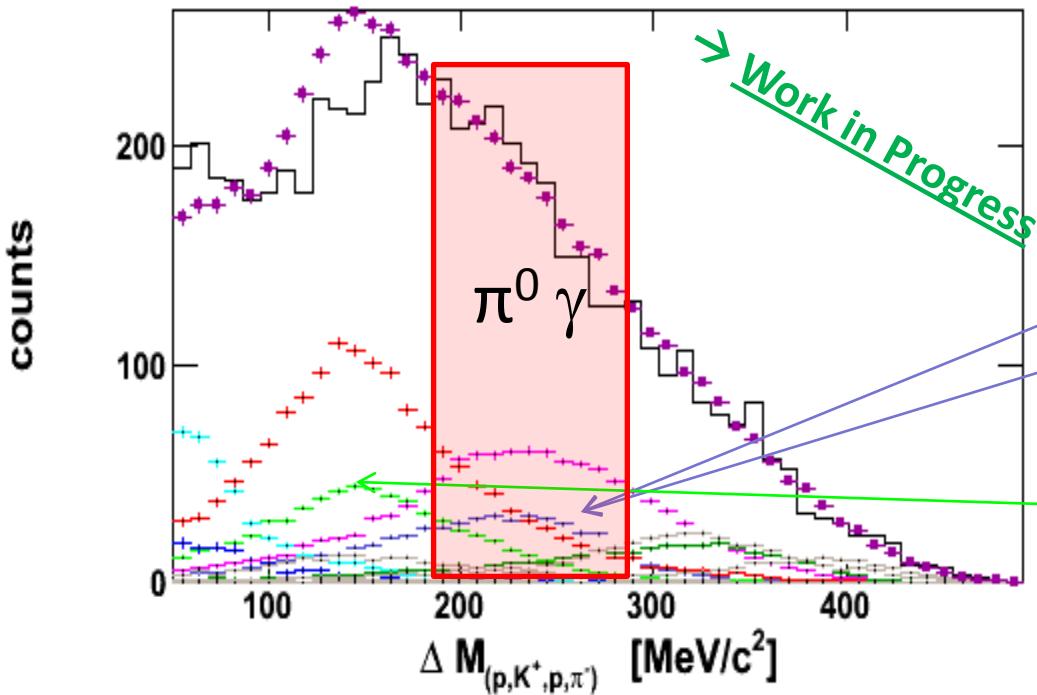
Final $\Lambda(1405)$ selection

The $\Lambda(1405)$ and $\Sigma(1385)$ can be distinguished by investigating the missing mass of all particles:

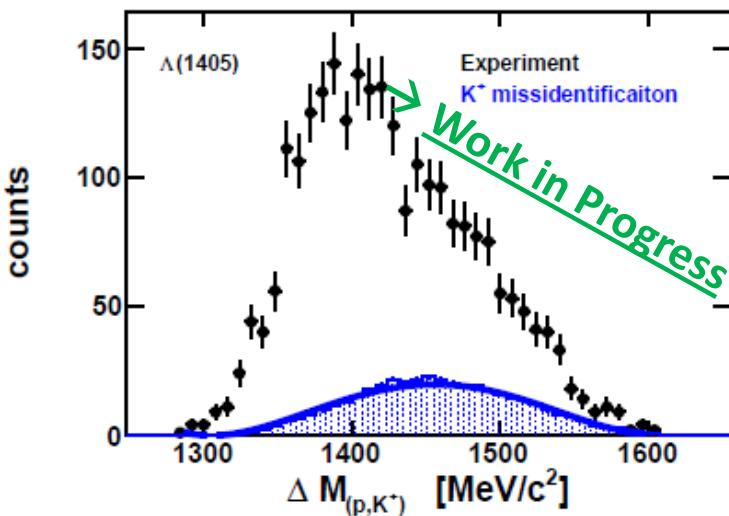


Final $\Lambda(1405)$ selection

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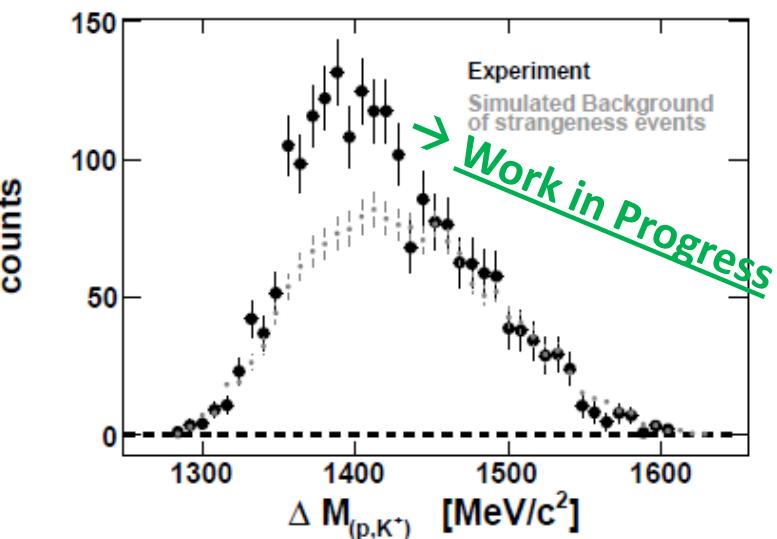
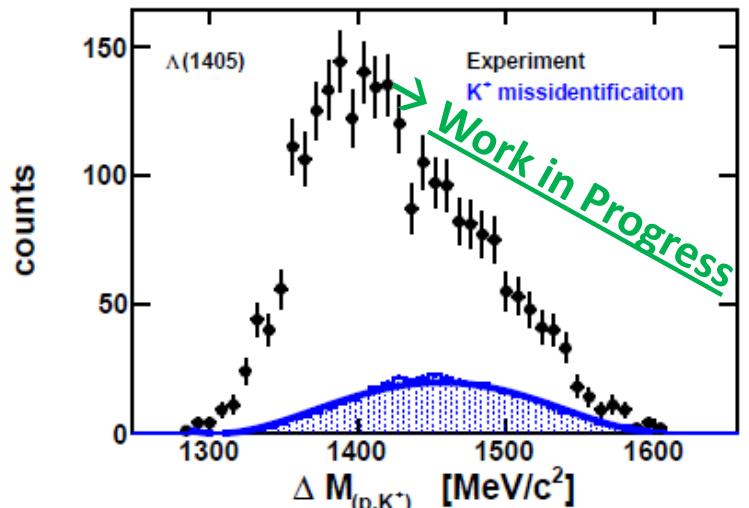
Final $\Lambda(1405)$ selection



Spectrum still contains other contributions:

- Subtract misidentification background
(obtained from sideband analysis)

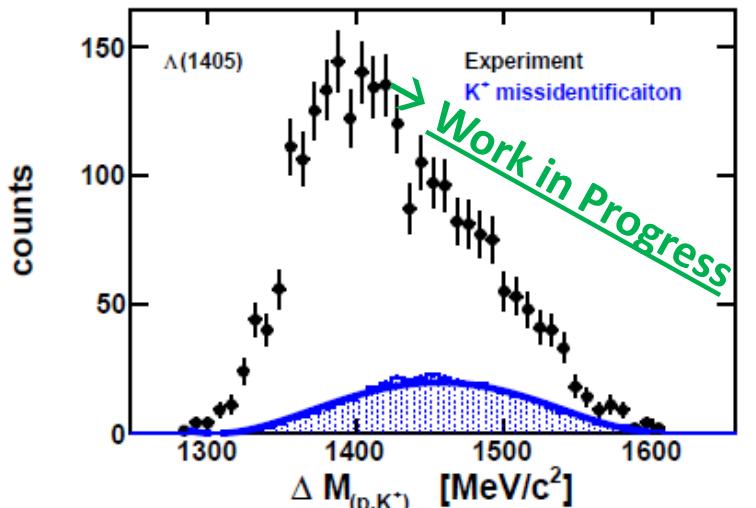
Final $\Lambda(1405)$ selection



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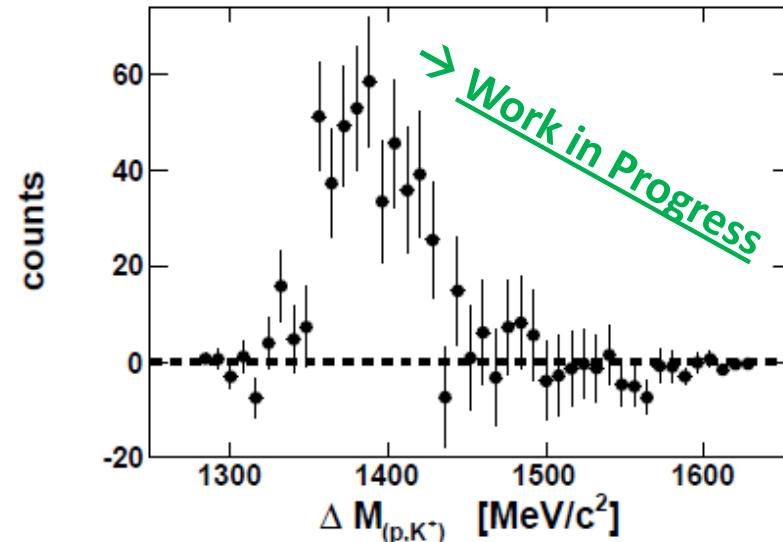
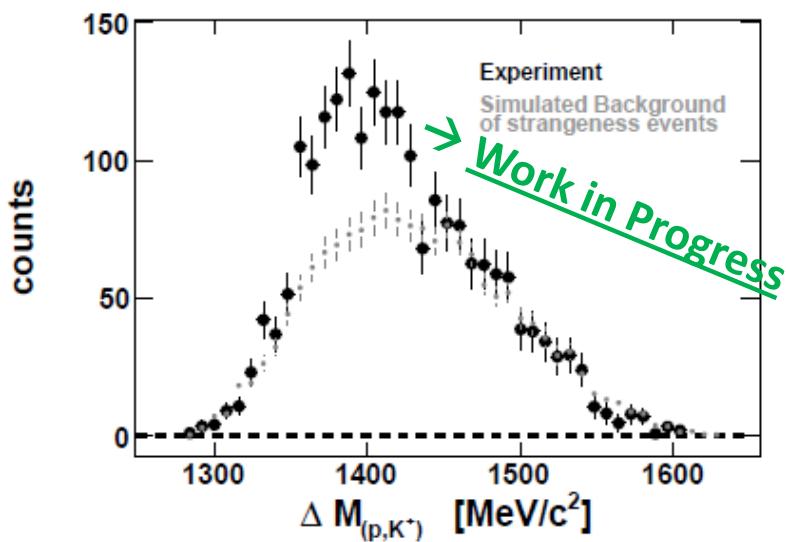
- Subtract misidentification background (obtained from sideband analysis).
- Subtract all other contributing channels (obtained from simulations).

Final $\Lambda(1405)$ selection



Spectrum still contains other contributions:

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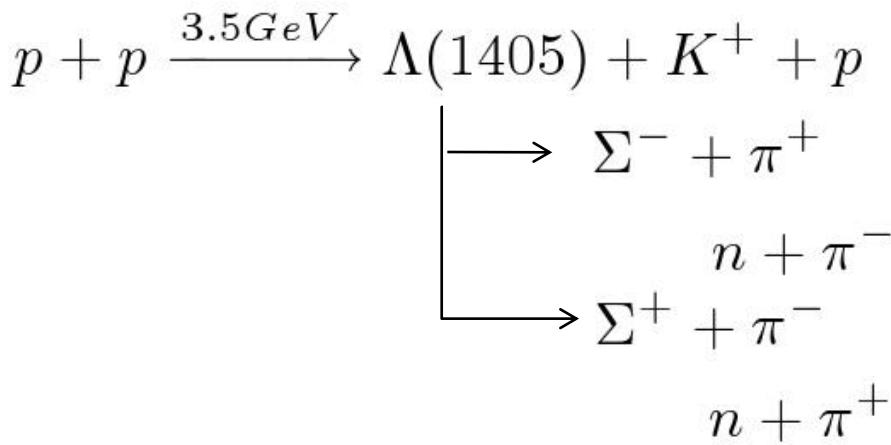


Analysis of the $\Lambda(1405)$

by its decay into

$\Sigma^+ \pi^- / \Sigma^- \pi^+$

$\Lambda(1405)$ in the charged decay channels:



Steps of the Analysis:

Identify the four charged particles (p , K^+ , π^+ , π^-)

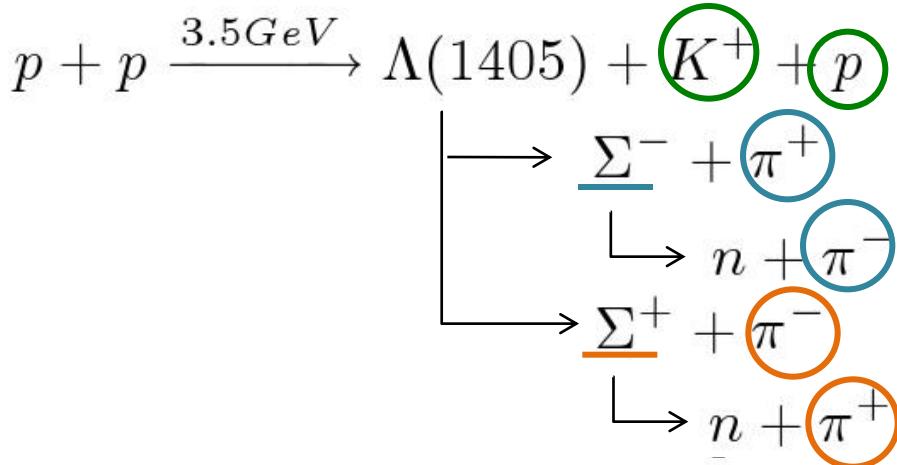
Use kinematic refit on the neutron

Apply different cuts to optimize signal to background ratio

Cut on the neutron and Σ^+ or Σ^- [mass cuts]

Modeling of the remaining background

$\Lambda(1405)$ in the charged decay channels:



Steps of the Analysis:

Identify the four charged particles (p , K^+ , π^+ , π^-)

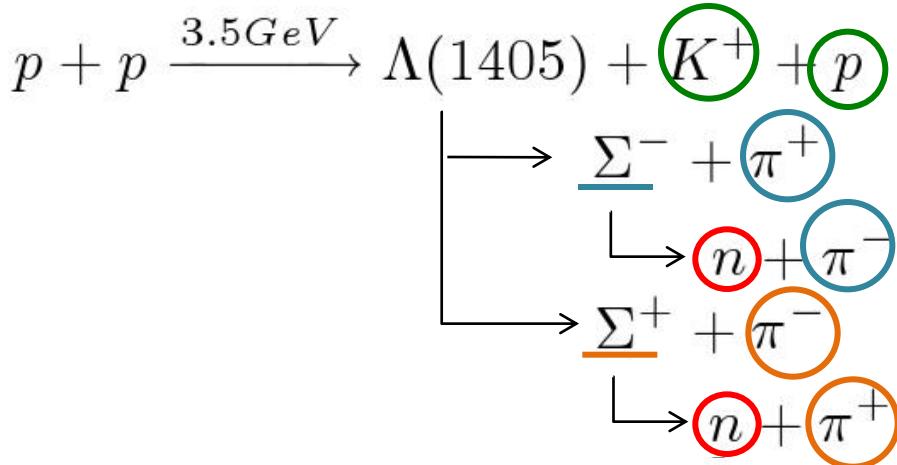
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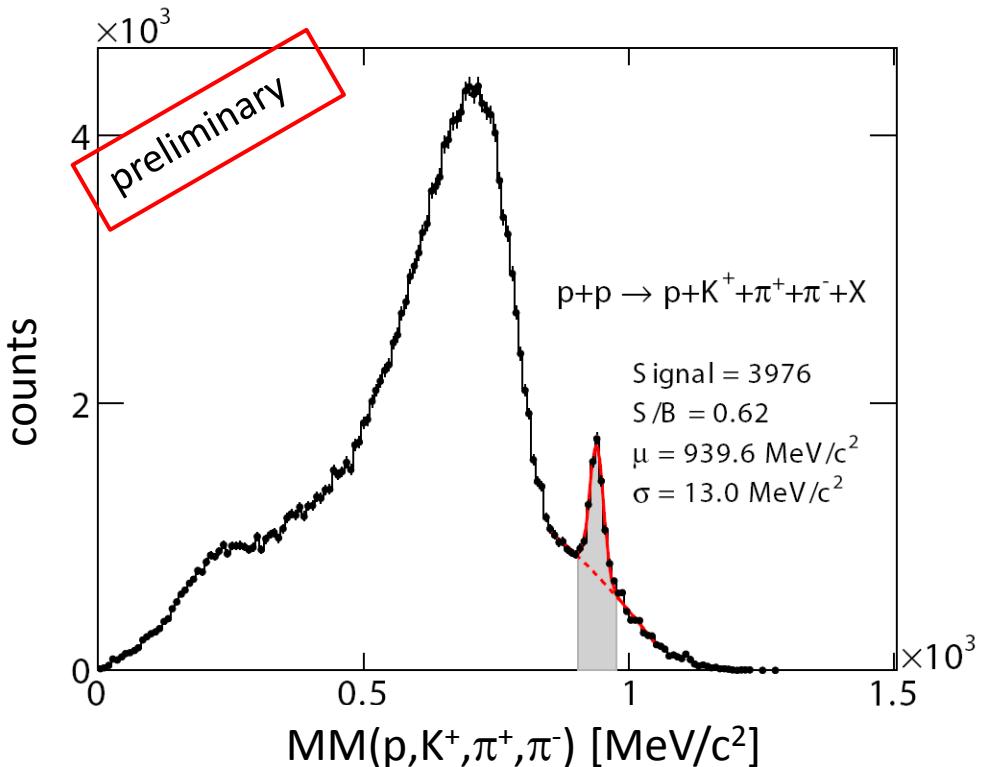
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Modeling of the remaining background

Neutron

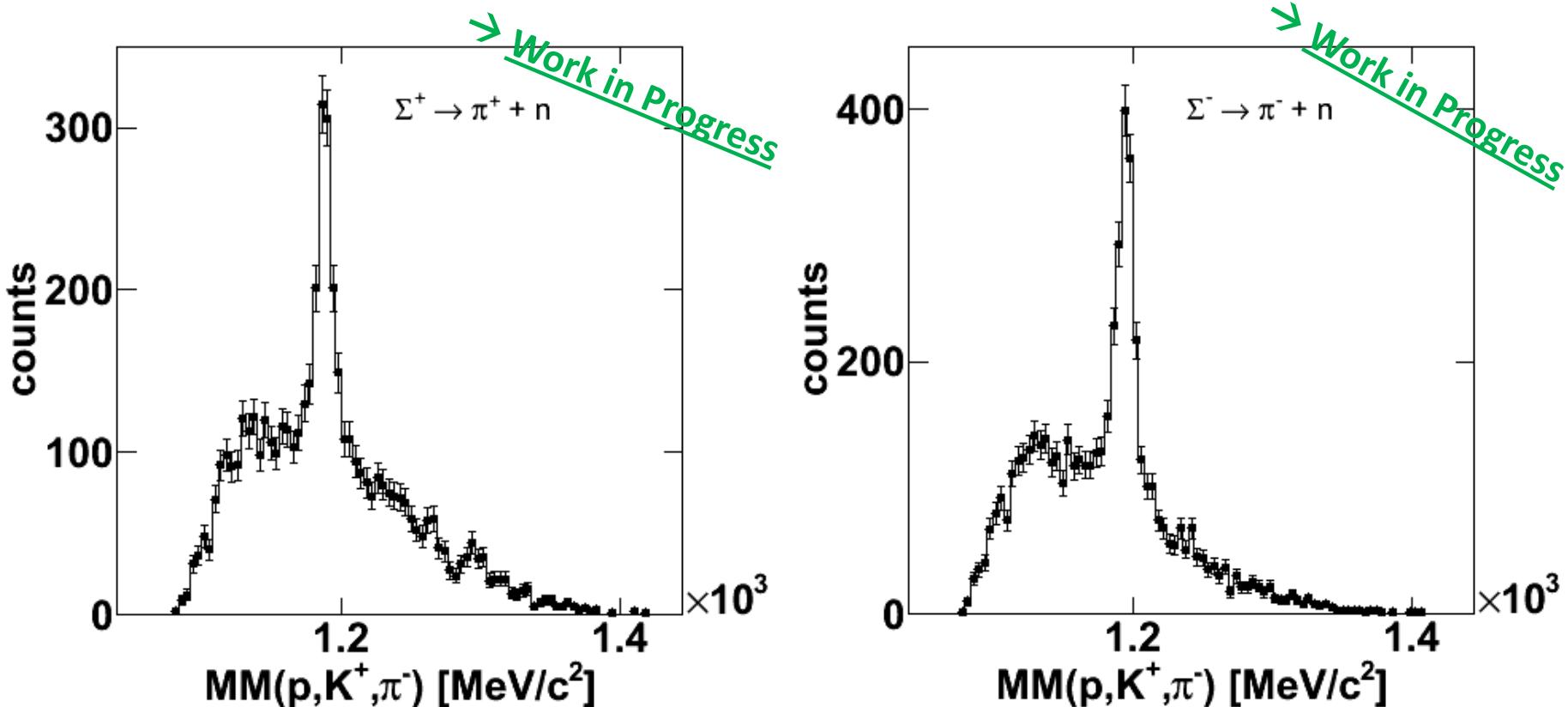
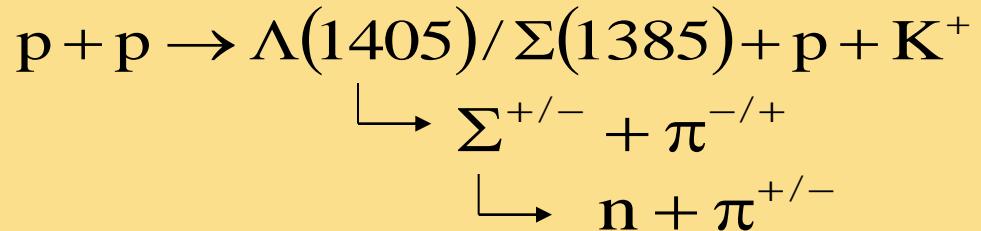
After mass cut on the K^+ ($350 \text{ MeV}/c^2 < m_{K^+} < 650 \text{ MeV}/c^2$):



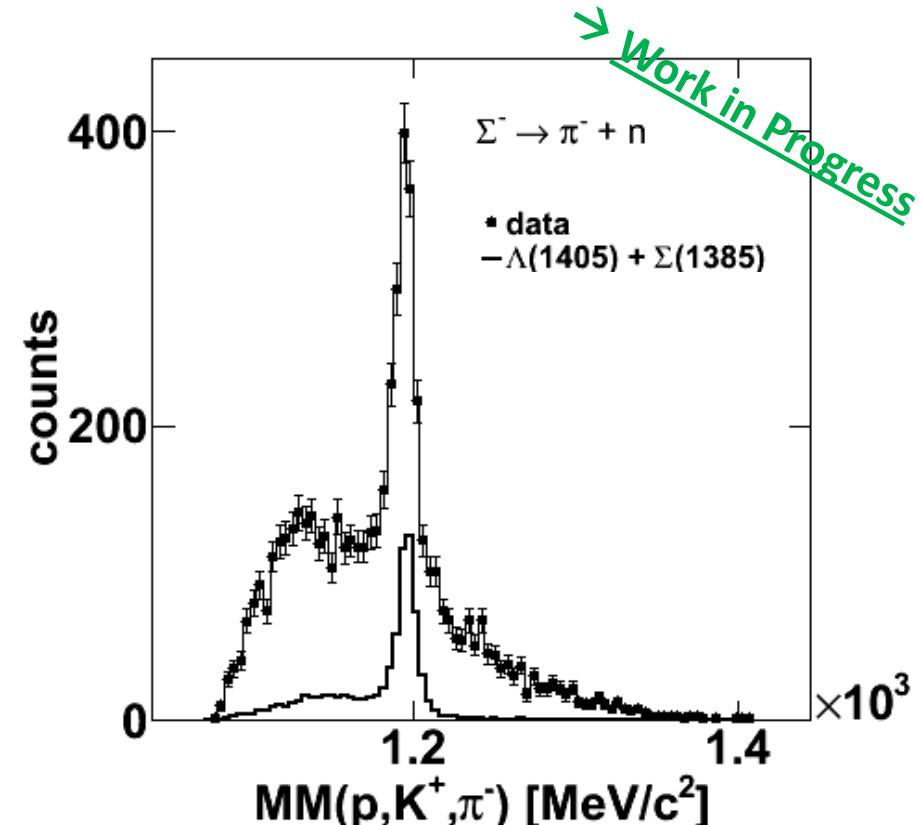
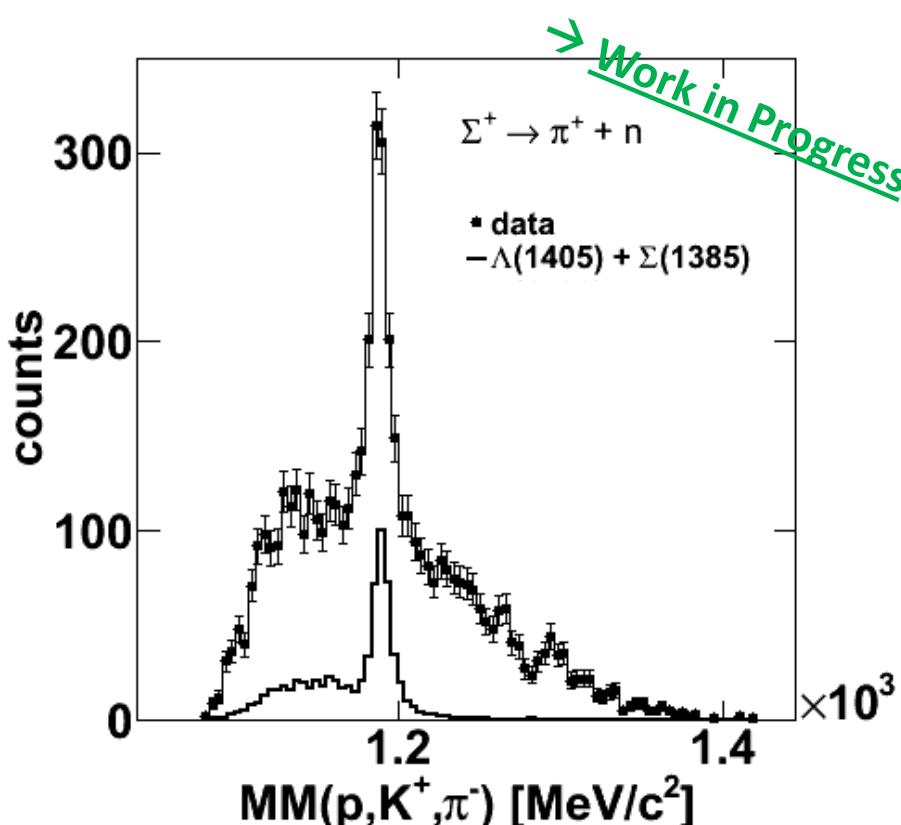
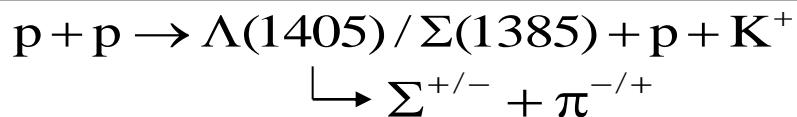
Clear neutron peak on top of large background. This is **only** misidentification of a proton or a pion as a K^+ .

Cut on the gray shaded area to extract $\Lambda(1405)$ and apply kinematic refit.

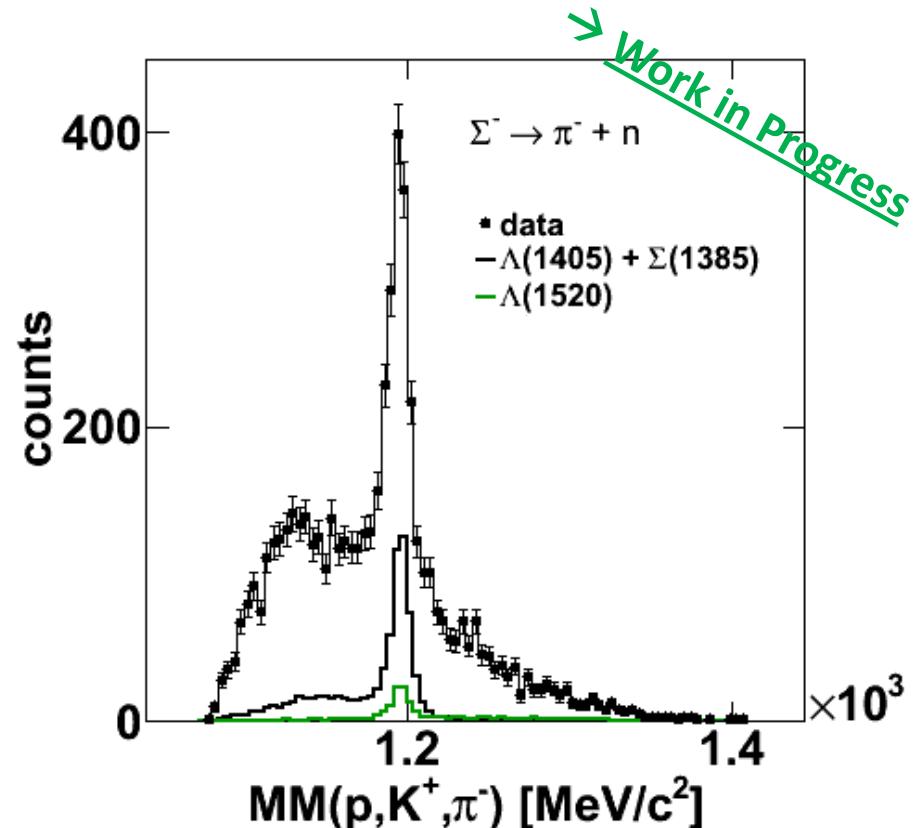
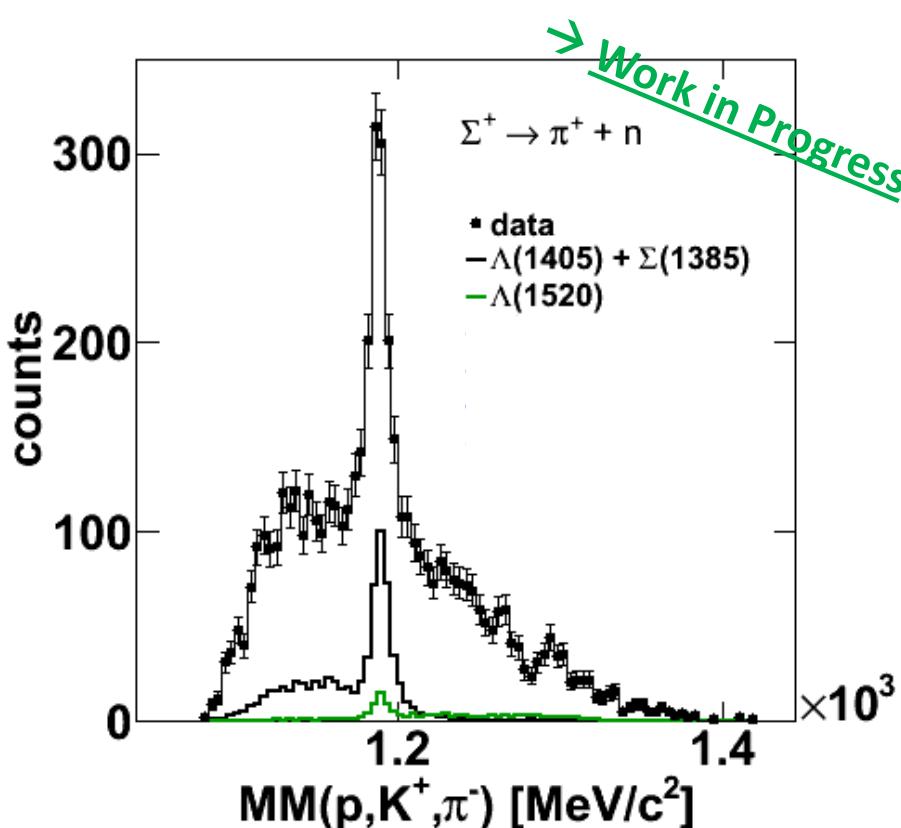
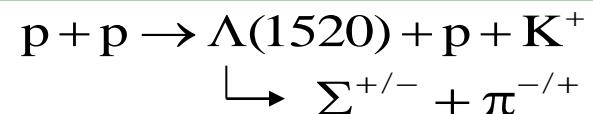
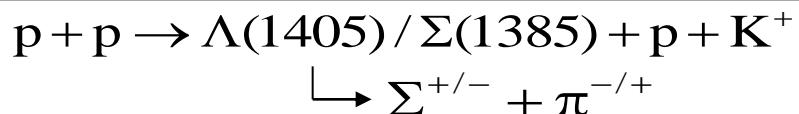
Σ^+ and Σ^-



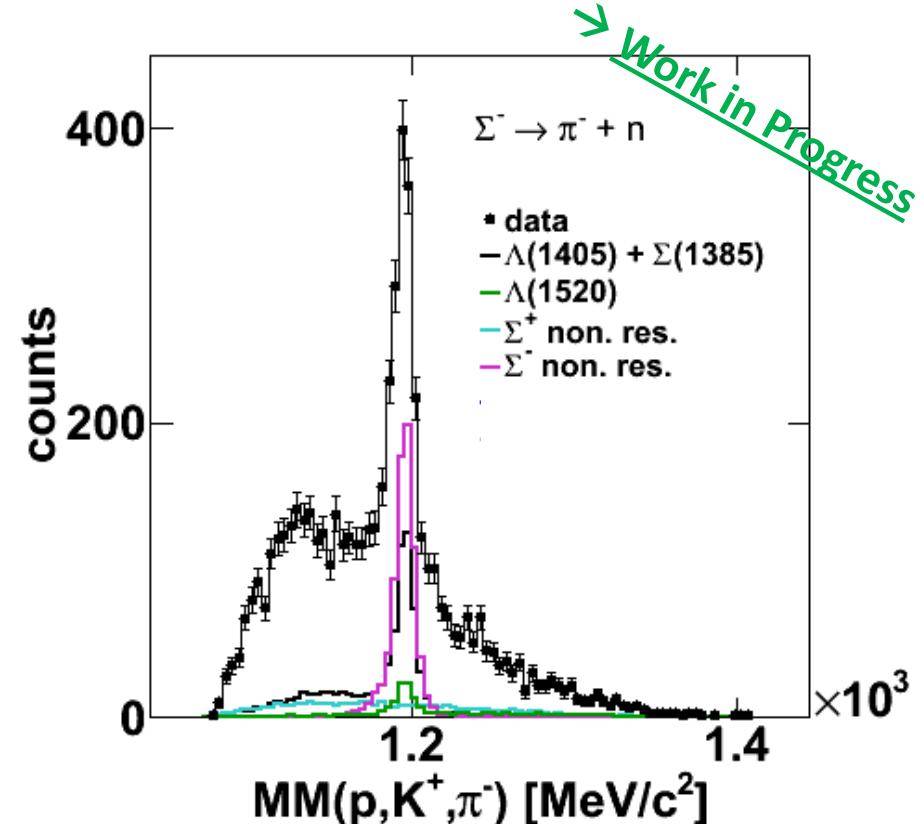
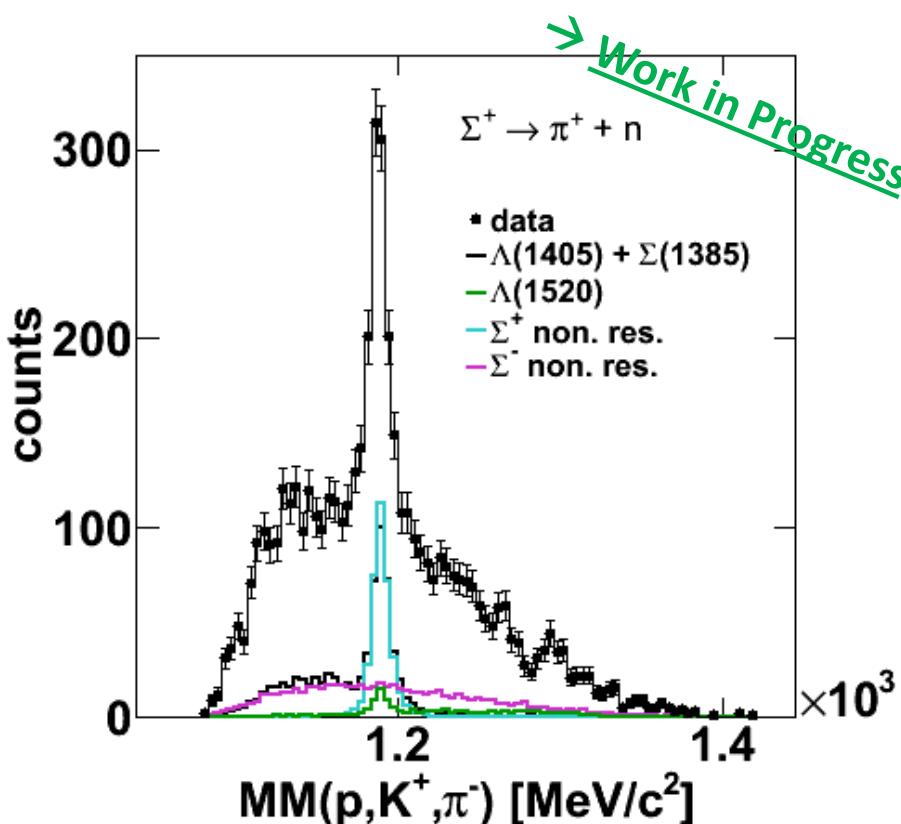
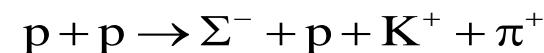
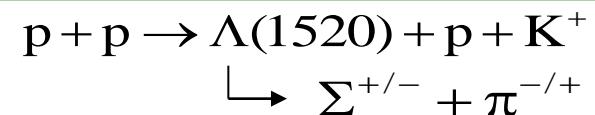
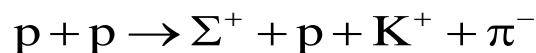
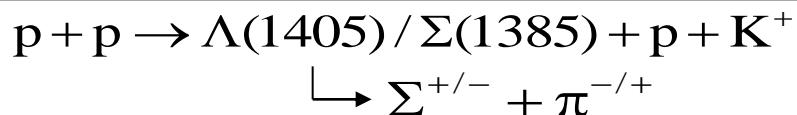
Σ^+ and Σ^-



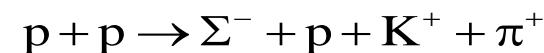
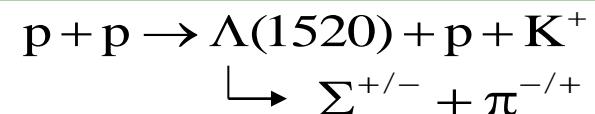
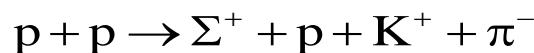
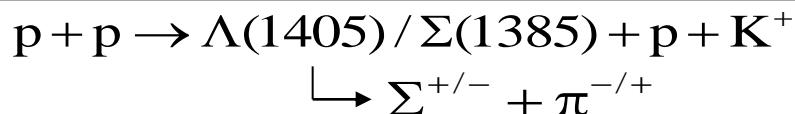
Σ^+ and Σ^-



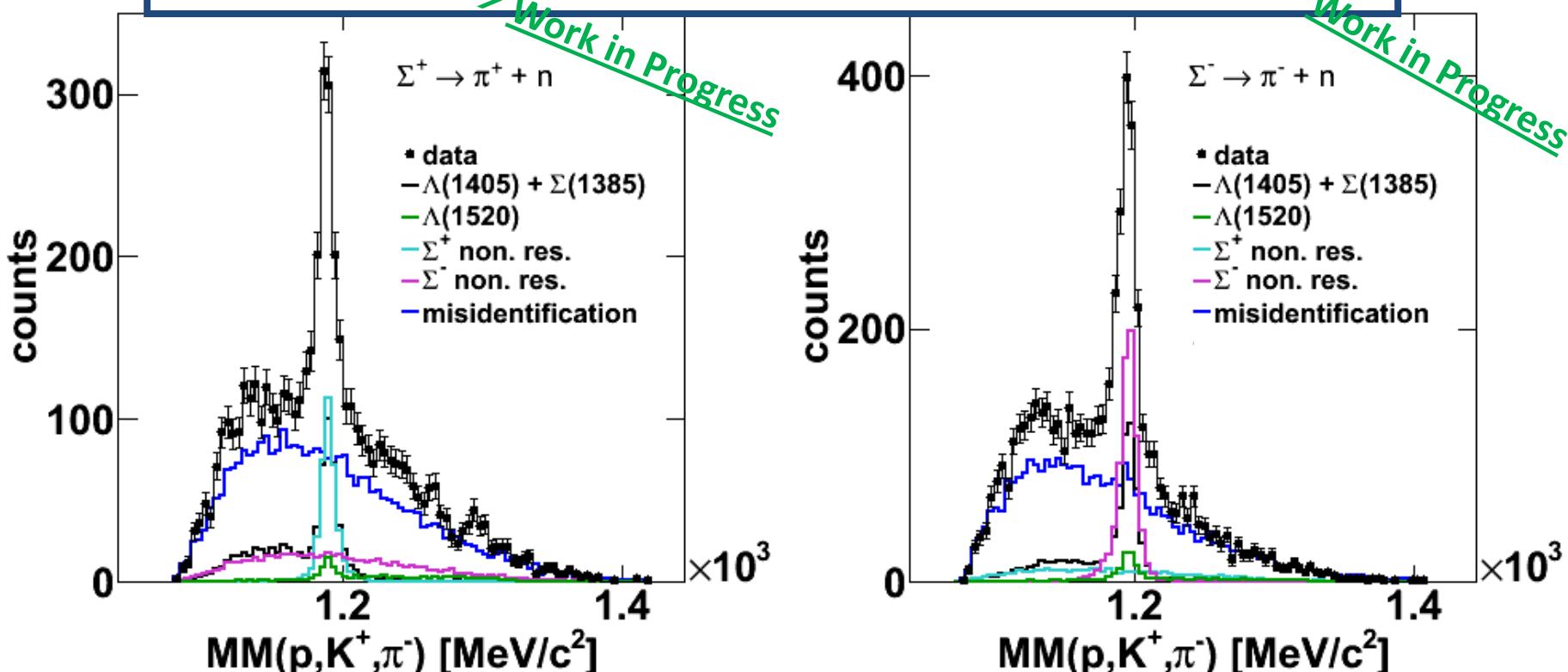
Σ^+ and Σ^-



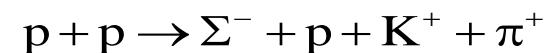
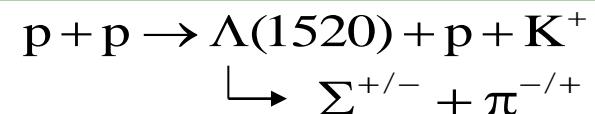
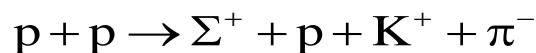
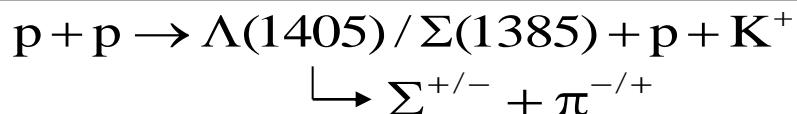
Σ^+ and Σ^-



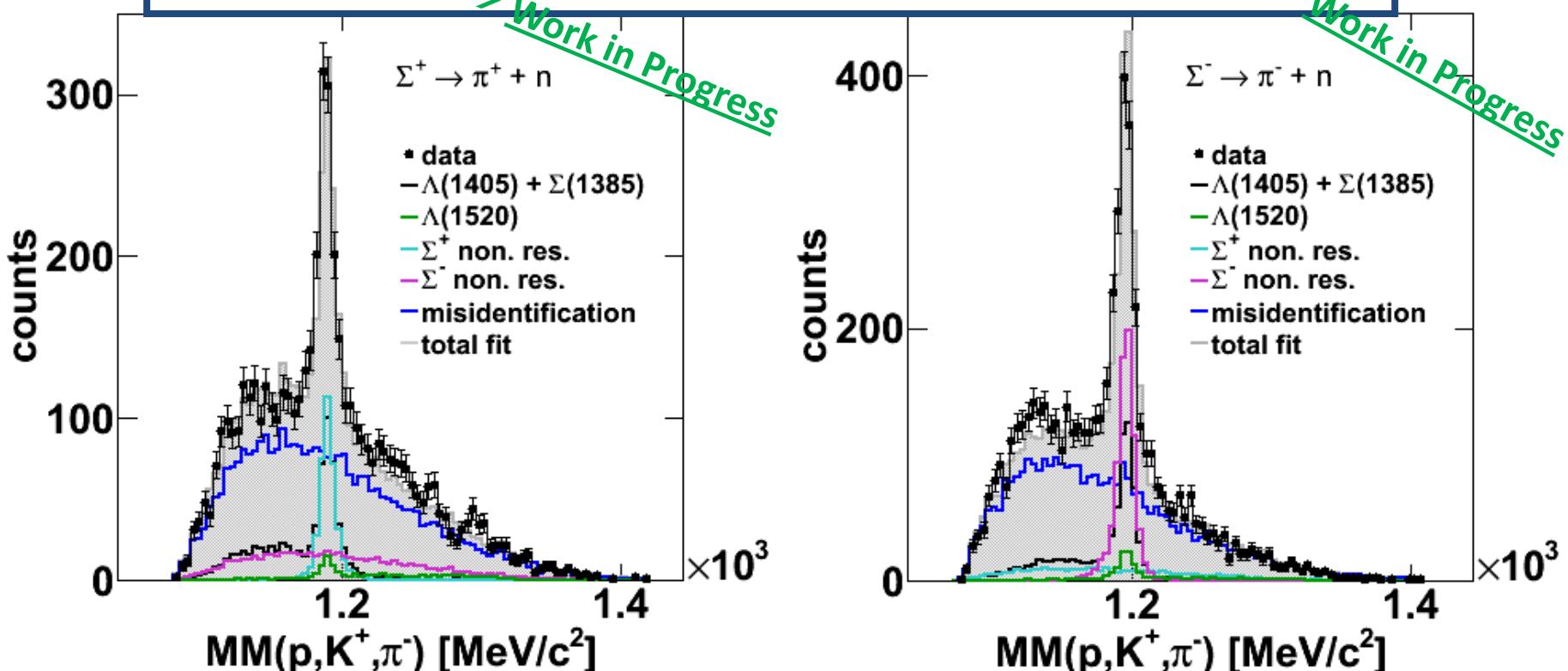
misidentification of p or π^+ as K^+ obtained with sideband analysis



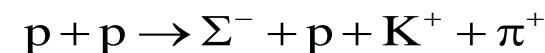
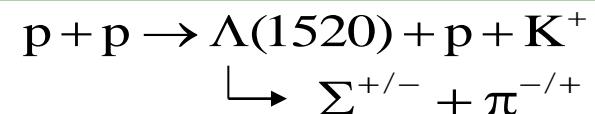
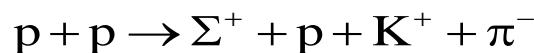
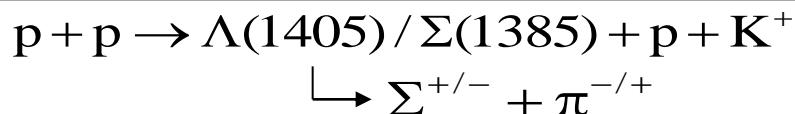
Σ^+ and Σ^-



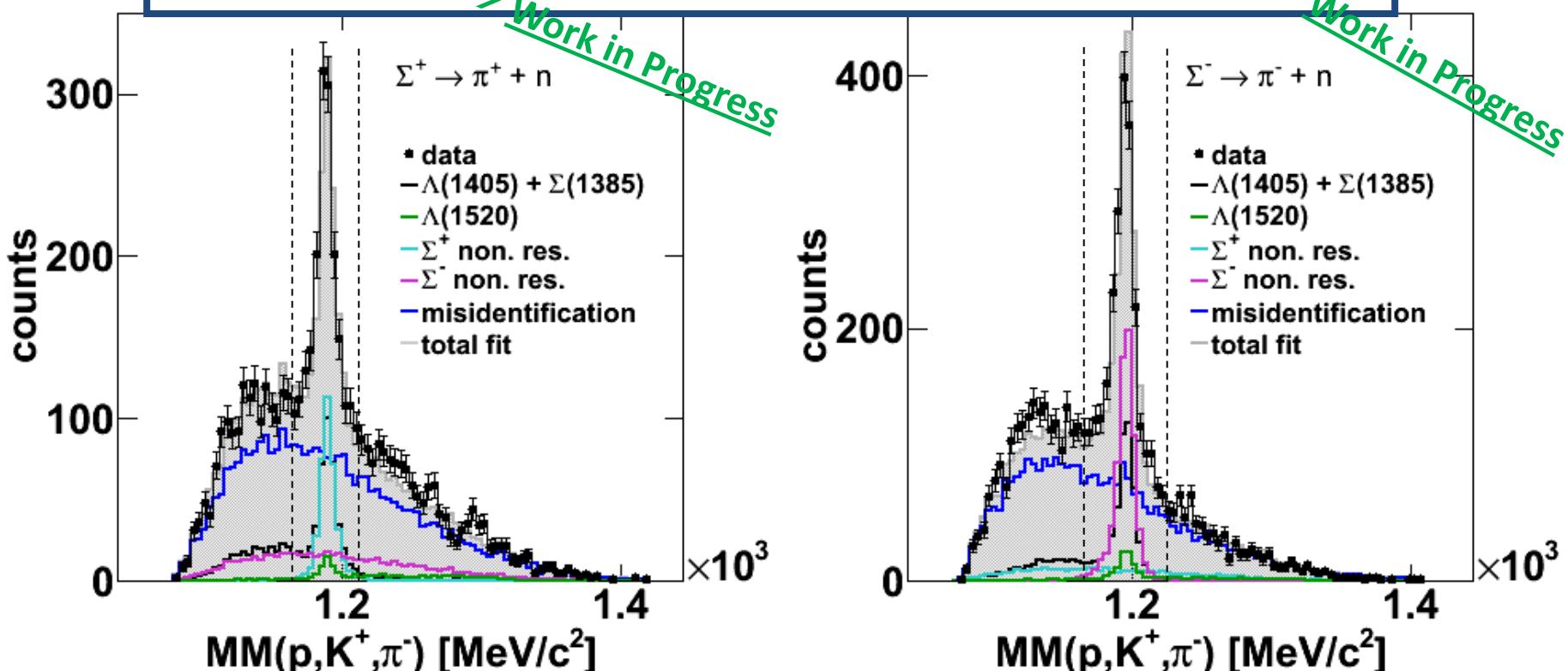
misidentification of p or π^+ as K^+ obtained with sideband analysis



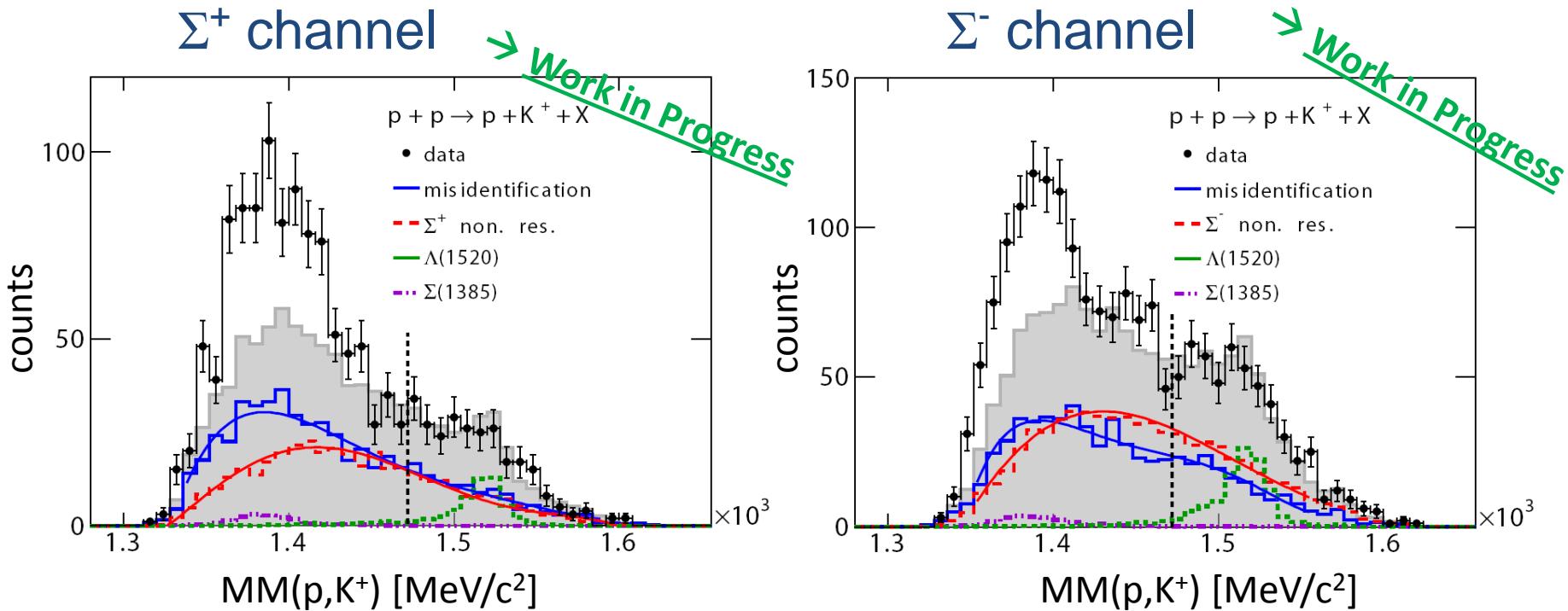
Σ^+ and Σ^-



misidentification of p or π^+ as K^+ obtained with sideband analysis



$\Lambda(1405)$ in Σ^+ and Σ^- channel



Besides the fit to the Σ^+ and Σ^- resonances, also the mass range above the vertical dashed line is simultaneously fitted with the different contributions.

Summary

Reconstruction of the $\Lambda(1405)$ in the neutral decay channel was shown.

- The intermediate $\Lambda(1116)$ can be reconstructed and improves the K^+ purity remarkably
- The contribution of the background can be extracted from multi parameter fit
- The FW hodoscope increases our acceptance for the investigation of the $\Lambda(1405)$ decaying into $\Sigma^0\pi^0$

Reconstruction of the $\Lambda(1405)$ in the charged decay channels was shown.

- The kinematic refit is essential for an optimized background suppression.
- The total background below the $\Lambda(1405)$ is well understood.
- The subtraction of the different background sources requires more systematic studies and careful investigations.

Outlook

Improve the quality of the multi parameter fit

The final aim is to compare the $\Lambda(1405)$ line shapes of all three decay channels to compare with various theories

Search for Kaonic Bound states in the HADES data

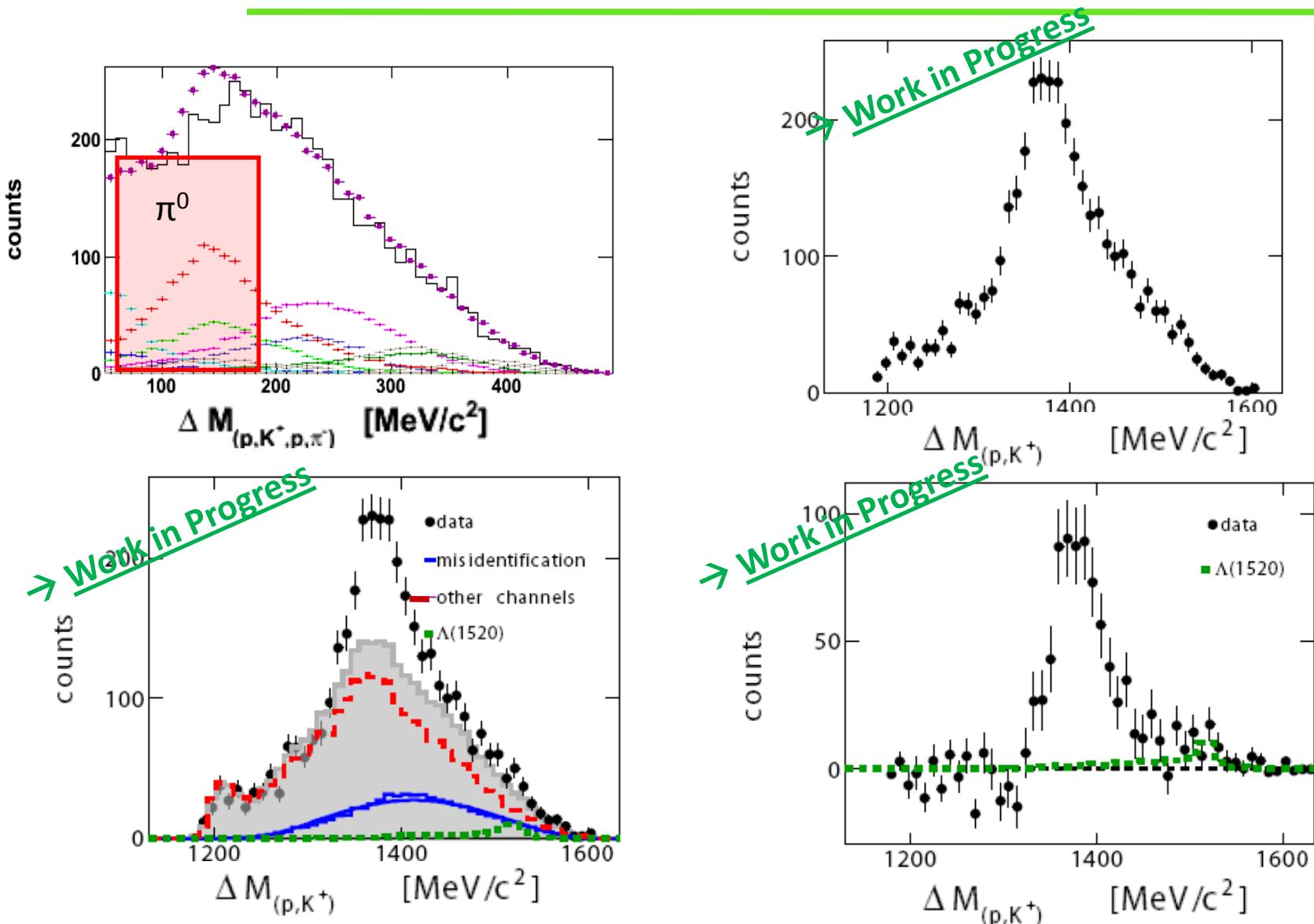
Search for the $\Lambda(1405)$ in pion induced reactions → planned with HADES

HADES Collaboration

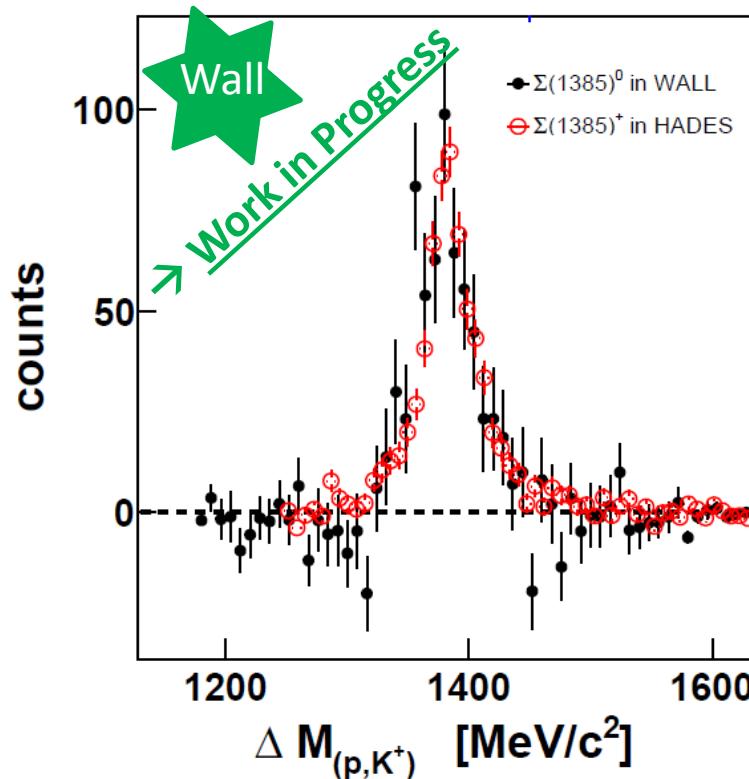
G. Agakishiev⁸, C. Agodi¹, A. Balandá^{3,e}, G. Bellia^{1,a}, D. Belver¹⁵, A. Belyaev⁶, A. Blanco², M. Böhmer¹¹, J. L. Boyard¹³, P. Braun-Munzinger⁴, P. Cabanelas¹⁵, E. Castro¹⁵, S. Chernenko⁶, T. Christ¹¹, M. Destefanis⁸, J. Díaz¹⁶, F. Dohrmann⁵, A. Dybczak³, T. Eberl¹¹, **E. Epple¹¹**, **L. Fabbietti¹¹**, O. Fateev⁶, P. Finocchiaro¹, P. Fonte^{2,b}, J. Friese¹¹, I. Fröhlich⁷, T. Galatyuk⁴, J. A. Garzón¹⁵, R. Gernhäuser¹¹, C. Gilardi⁸, M. Golubeva¹⁰, D. González-Díaz⁴, E. Grosse^{5,c}, F. Guber¹⁰, M. Heilmann⁷, T. Hennino¹³, R. Holzmann⁴, A. Ierusalimov⁶, I. Iori^{9,d}, A. Ivashkin¹⁰, M. Jurkovic¹¹, B. Kämpfer⁵, K. Kanaki⁵, T. Karavicheva¹⁰, D. Kirschner⁸, I. Koenig⁴, W. Koenig⁴, B. W. Kolb⁴, R. Kotte⁵, A. Kozuch^{3,e}, F. Krizek¹⁴, R. Krücken¹¹, W. Kühn⁸, A. Kugler¹⁴, A. Kurepin¹⁰, J. Lamas-Valverde¹⁵, S. Lang⁴, J. S. Lange⁸, K. Lapidus¹⁰, L. Lopes², M. Lorenz⁴, L. Maier¹¹, A. Mangiarotti², J. Marín¹⁵, J. Markert⁷, V. Metag⁸, B. Michalska³, D. Mishra⁸, E. Morinière¹³, J. Mousa¹², C. Müntz⁷, L. Naumann⁵, R. Novotny⁸, J. Otwinowski³, Y. C. Pachmayer⁷, M. Palka⁴, Y. Parpottas¹², V. Pechenov⁸, O. Pechenova⁸, T. Pérez Cavalcanti⁸, J. Pietraszko⁴, W. Przygoda^{3,e}, B. Ramstein¹³, A. Reshetin¹⁰, M. Roy-Stephan¹³, A. Rustamov⁴, A. Sadovsky¹⁰, B. Sailer¹¹, P. Salabura³, **A. Schmah⁴**, **J. Siebenson¹¹**, R. Simon⁴, S. Spataro⁸, B. Spruck⁸, H. Ströbele⁷, J. Stroth^{7,4}, C. Sturm⁷, M. Sudol⁴, A. Tarantola⁷, K. Teilab⁷, P. Tlusty¹⁴, M. Traxler⁴, R. Trebacz³, H. Tsertos¹², I. Veretenkin¹⁰, V. Wagner¹⁴, H. Wen⁸, M. Wisniowski³, T. Wojcik³, J. Wüstenfeld⁵, S. Yurevich⁴, Y. Zanevsky⁶, P. Zumbruch⁴

$\Sigma(1385)^0$

Final $\Sigma(1385)$ selection



$\Sigma(1385)$ candidate in comparison



Reconstructed $\Sigma(1385)^0$ max. 350 counts.
With a first estimation this results to
538 600 produced $\Sigma(1385)^0$

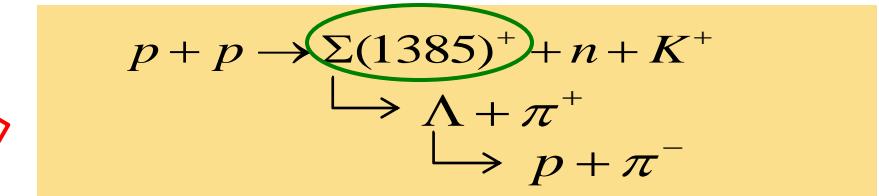
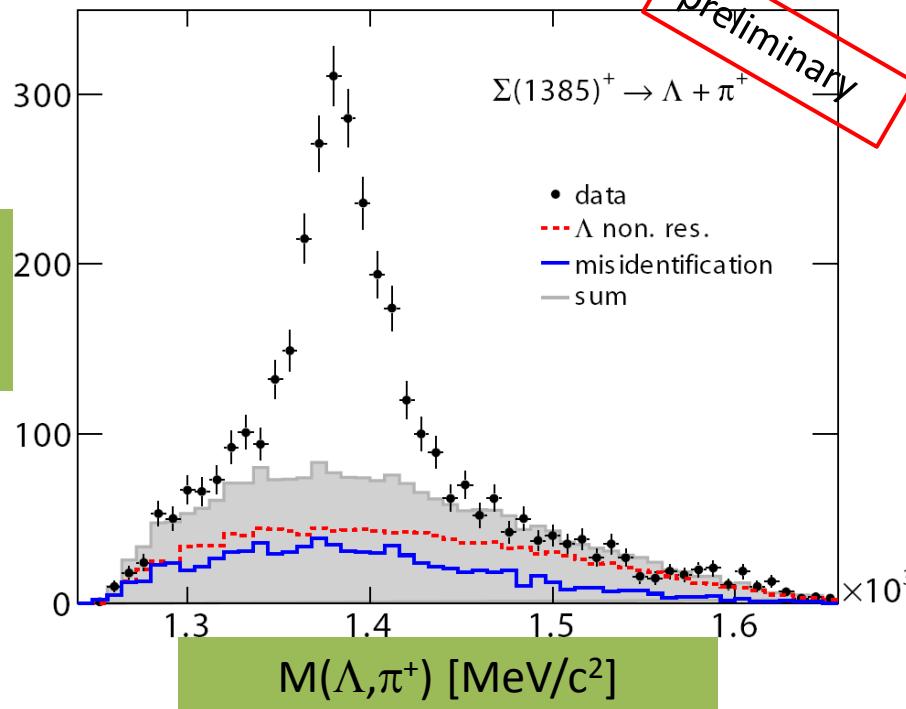
→ $\Sigma(1385)^0$ decay into $\Sigma^+\pi^- / \Sigma^-\pi^+$ with

- Branching ratio 3 % / 6 %
- Eff and acc $1.035 \cdot 10^{-3}$ %

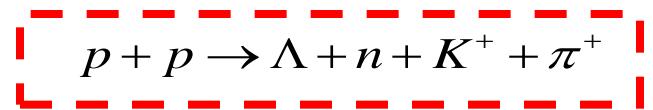
 there are expected max. **17 / 33** counts in the $\Lambda(1405)$ spectrum

$\Sigma(1385)^+$

$\Sigma(1385)^+$



Background due to:

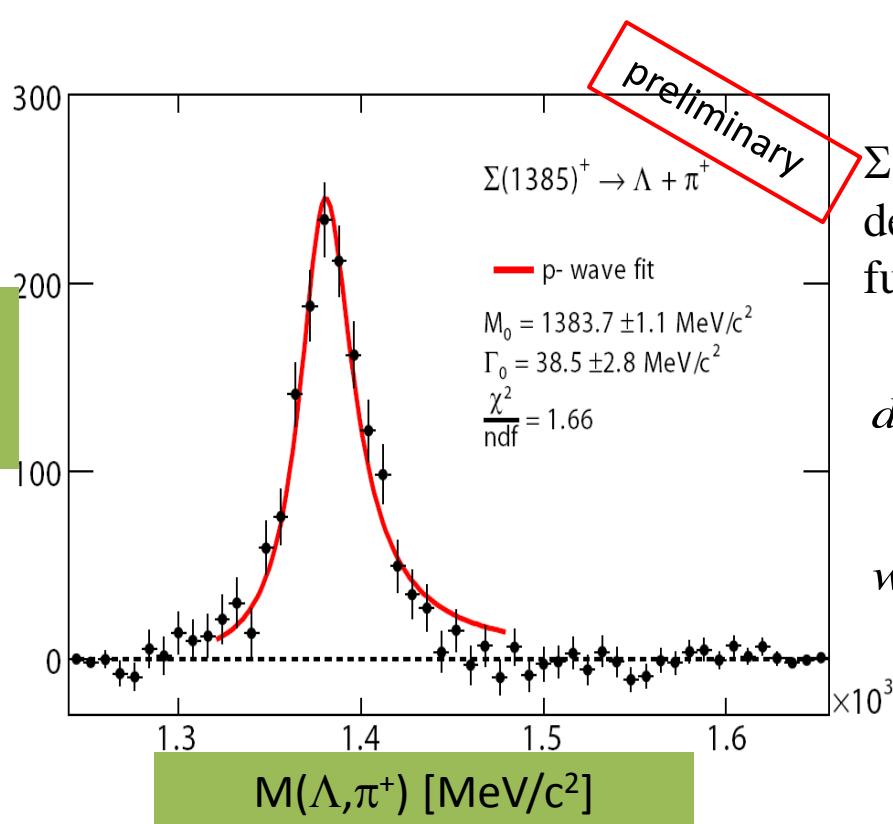


and due to:

misidentification of p or π^+ as K^+

Subtract gray histogram in order to obtain a pure $\Sigma(1385)^+$.

pure $\Sigma(1385)^+$



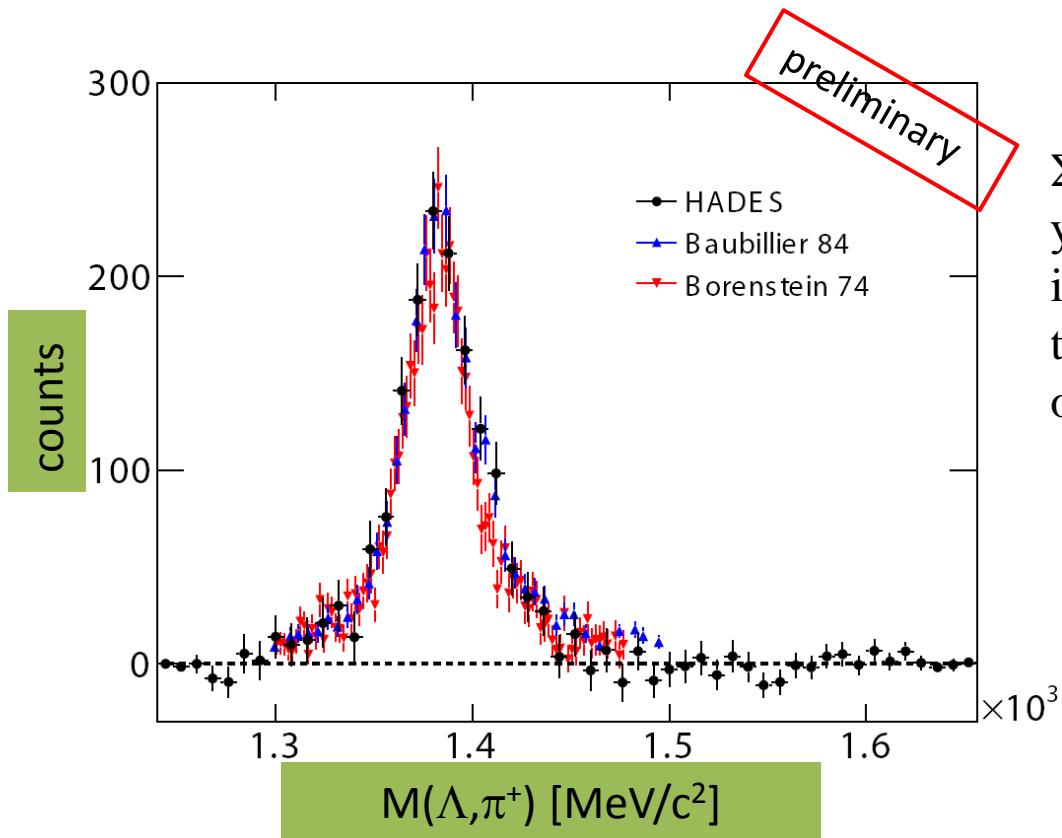
$\Sigma(1385)^+$ is a P_{13} state, therefore it should be described by a p-wave relativistic Breit – Wigner function:

$$\text{data} \pm \text{error} = A \left(\frac{q}{q_0} \right)^2 \frac{M_0^2 \Gamma_0^2}{(M^2 - M_0^2)^2 + M_0^2 \Gamma^2}$$

$$\text{with } \Gamma = \Gamma_0 \left(\frac{M_0}{M} \right) \left(\frac{q}{q_0} \right)^{2l+1} \quad l = 1$$

Comparison to other spectra

$\Sigma(1385)^+$ not corrected for efficiency and acceptance.



$\Sigma(1385)^+$ of HADES (with a total yield of ≈ 1500 entries) fits quite well into other spectra, which were used to determine the nominal quantities of this resonance.

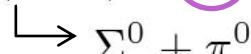
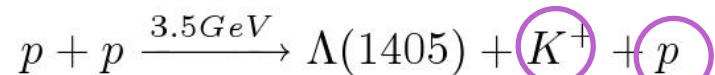
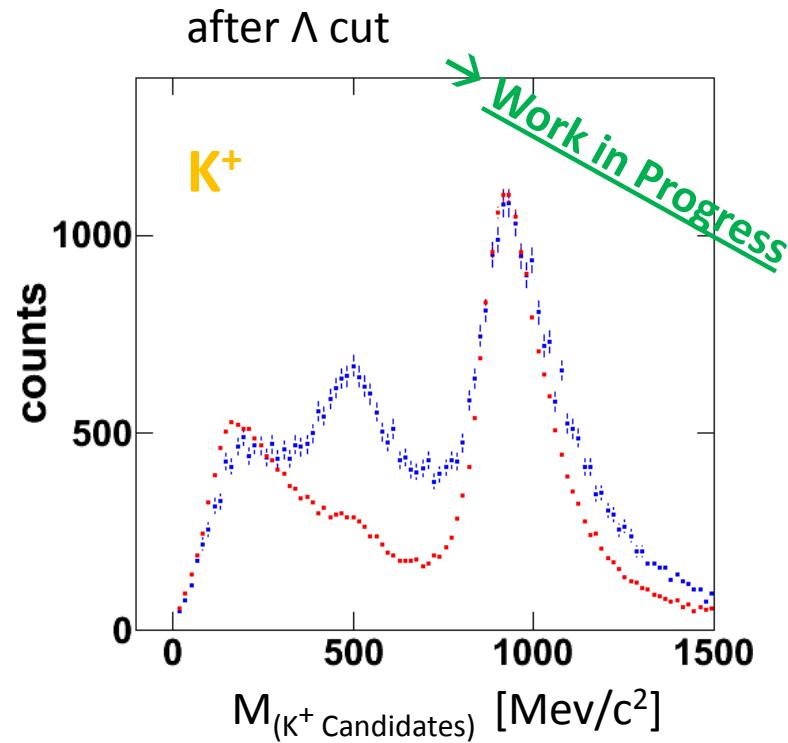
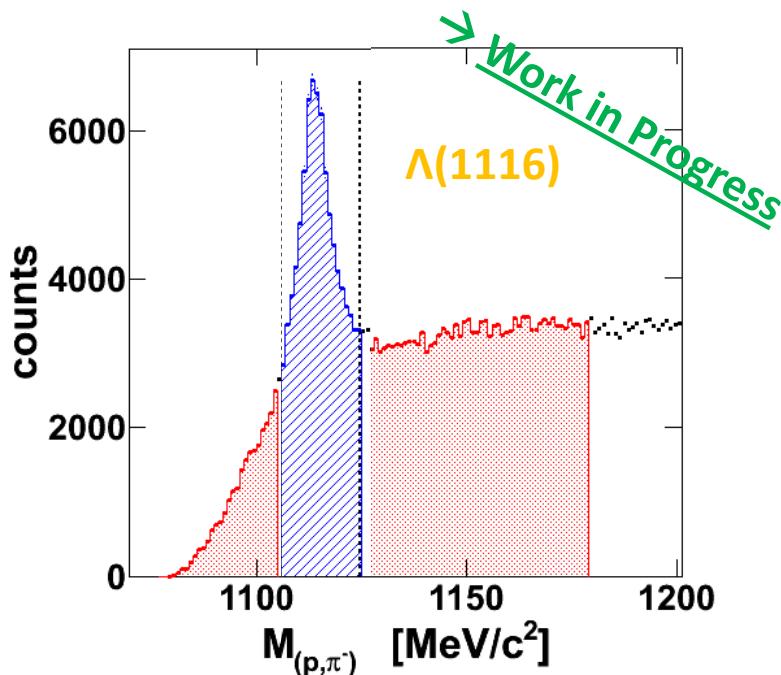
PDG Entries

- M. Baubillier et al., Z. Phys. C23 213 (1984)
- S.R. Borenstein et al., Phys. Rev. D 9 3006 (1974)

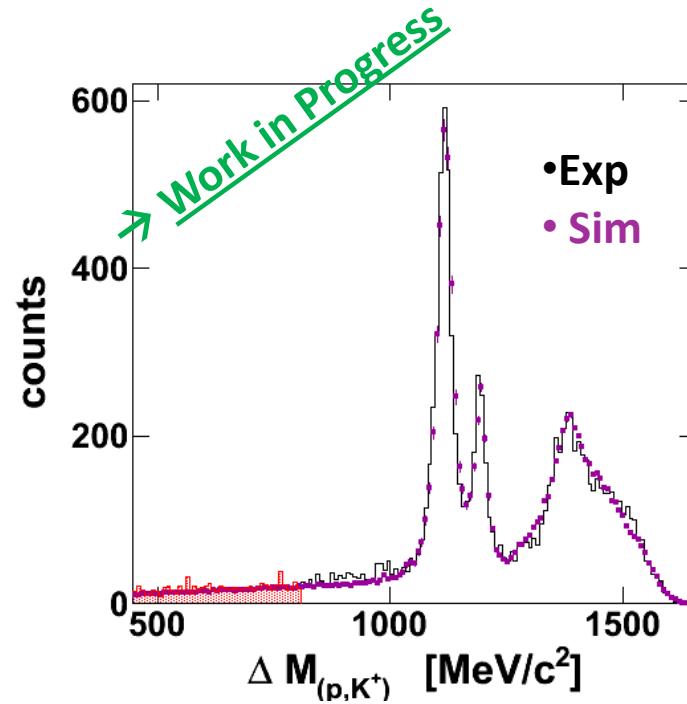
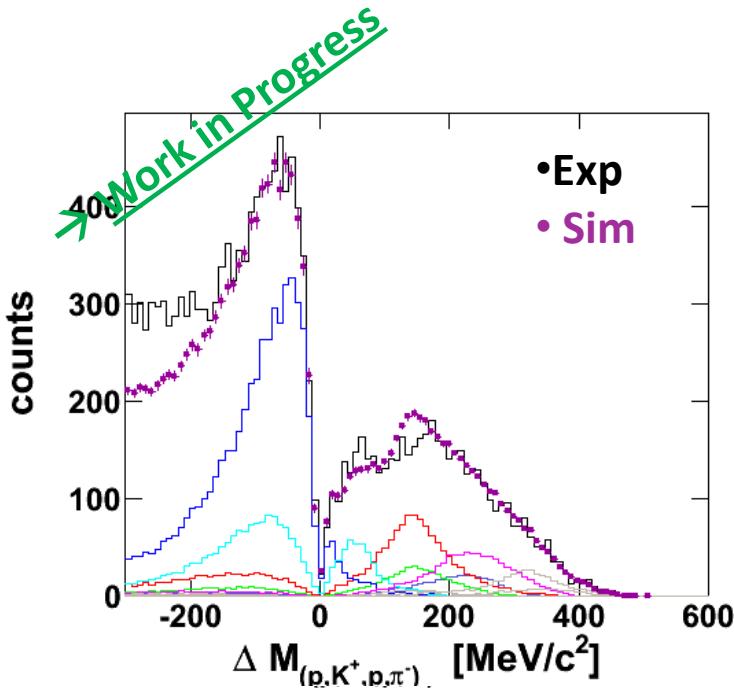
Sideband Analysis

The analysis of the $\Lambda(1405)$

HADES + FW DATA

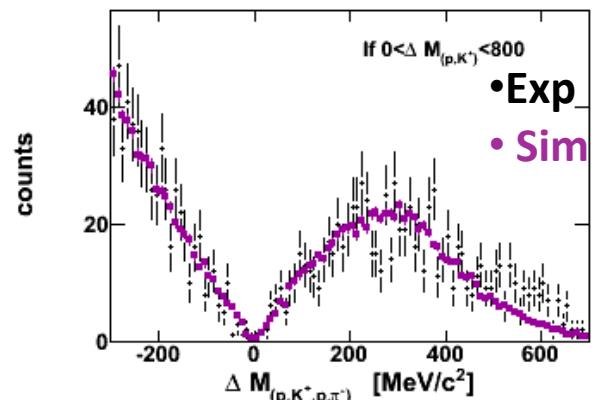


multi parameter fit

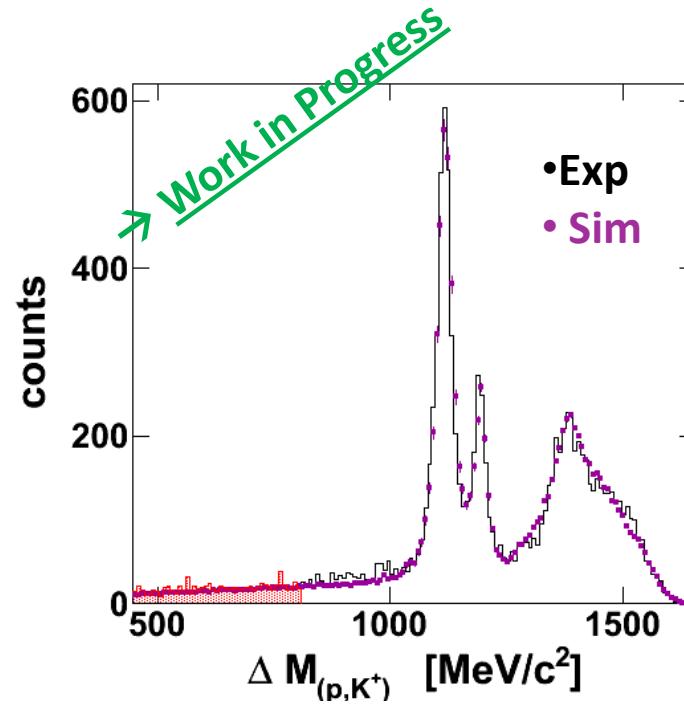
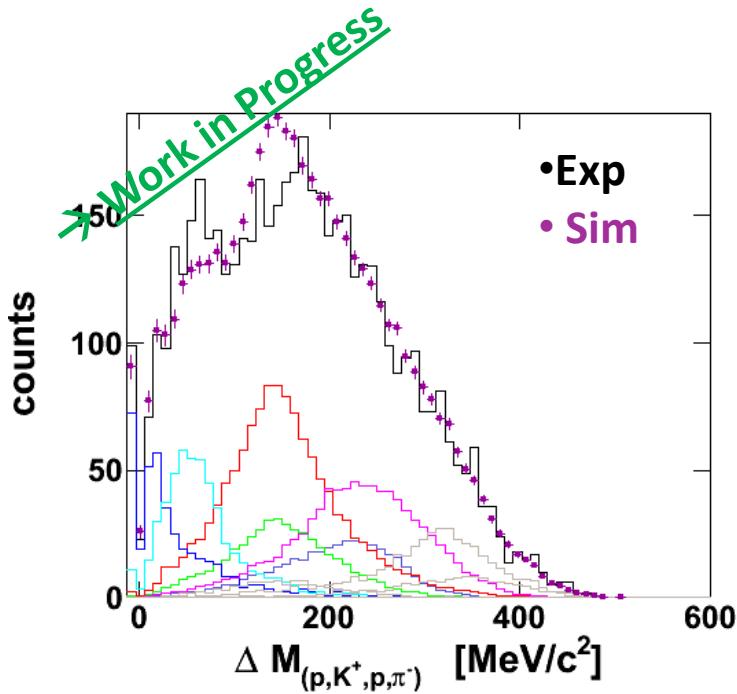


Contributing channels

- | | |
|-------------------------------|-----------------------------|
| → $\Lambda + p + K^+$ | → $\Lambda(1405) + p + K^+$ |
| → $\Lambda + p + K^+ + \pi^0$ | → $\Lambda(1520) + p + K^+$ |
| → $\Sigma + p + K^+$ | → $\Sigma(1385) + p + K^+$ |
| → $\Sigma + p + K^+ + \pi^0$ | + K^+ misidentification |

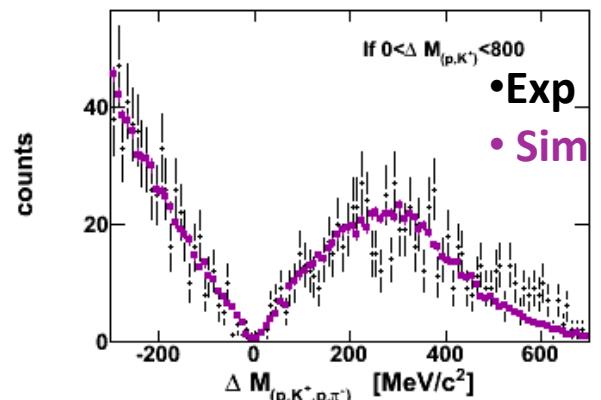


multi parameter fit

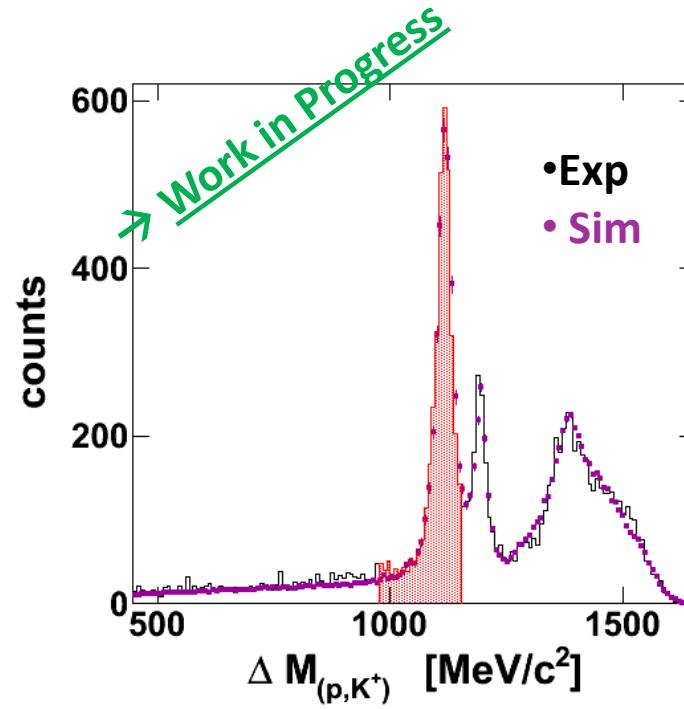
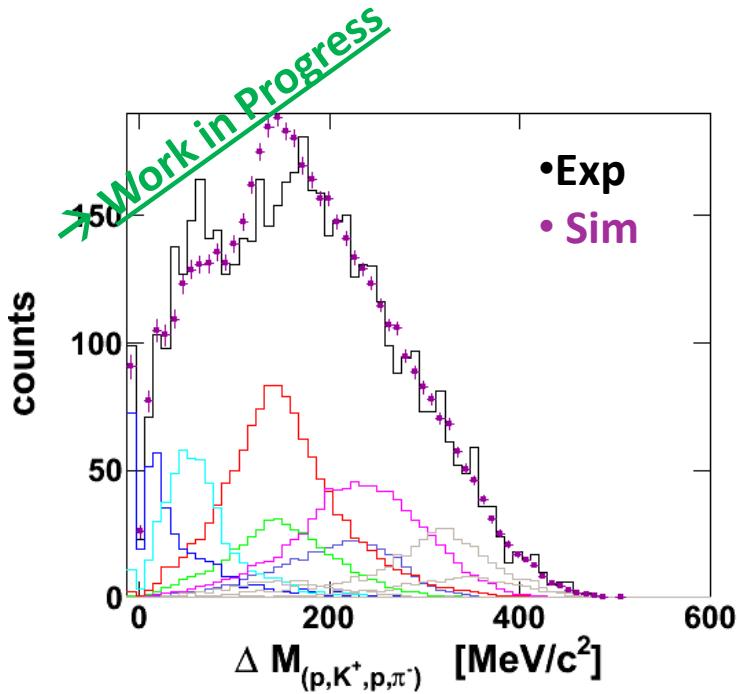


Contributing channels

- | | |
|-------------------------------|-----------------------------|
| → $\Lambda + p + K^+$ | → $\Lambda(1405) + p + K^+$ |
| → $\Lambda + p + K^+ + \pi^0$ | → $\Lambda(1520) + p + K^+$ |
| → $\Sigma + p + K^+$ | → $\Sigma(1385) + p + K^+$ |
| → $\Sigma + p + K^+ + \pi^0$ | + K^+ misidentification |

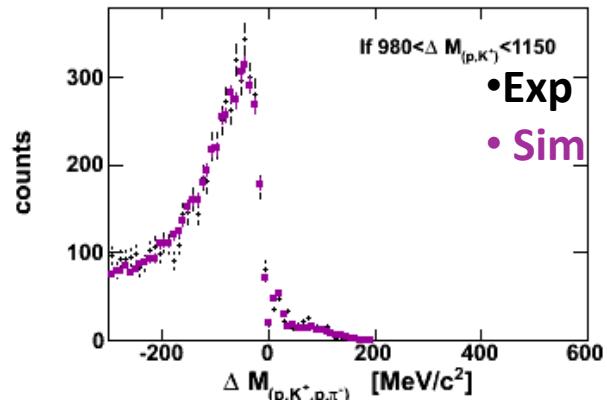


multi parameter fit

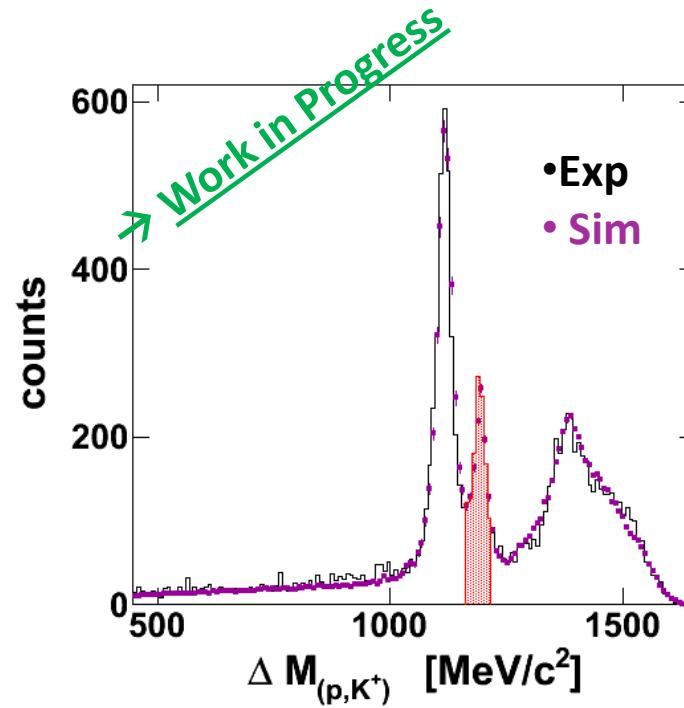
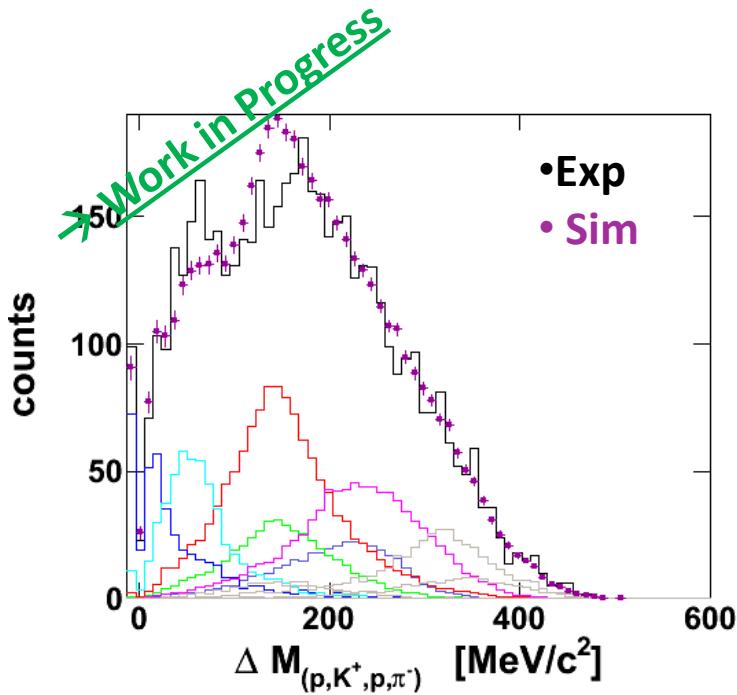


Contributing channels

- | | |
|-------------------------------|-----------------------------|
| → $\Lambda + p + K^+$ | → $\Lambda(1405) + p + K^+$ |
| → $\Lambda + p + K^+ + \pi^0$ | → $\Lambda(1520) + p + K^+$ |
| → $\Sigma + p + K^+$ | → $\Sigma(1385) + p + K^+$ |
| → $\Sigma + p + K^+ + \pi^0$ | + K^+ misidentification |

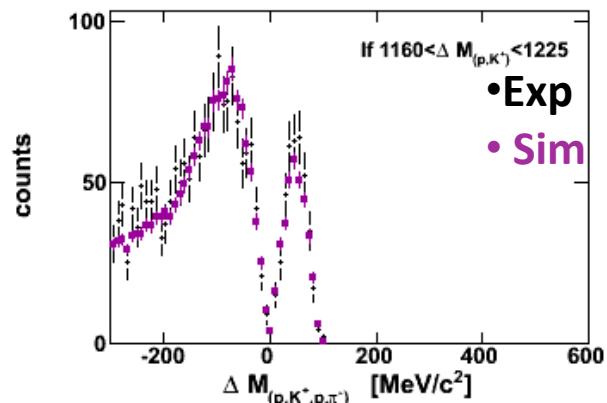


multi parameter fit

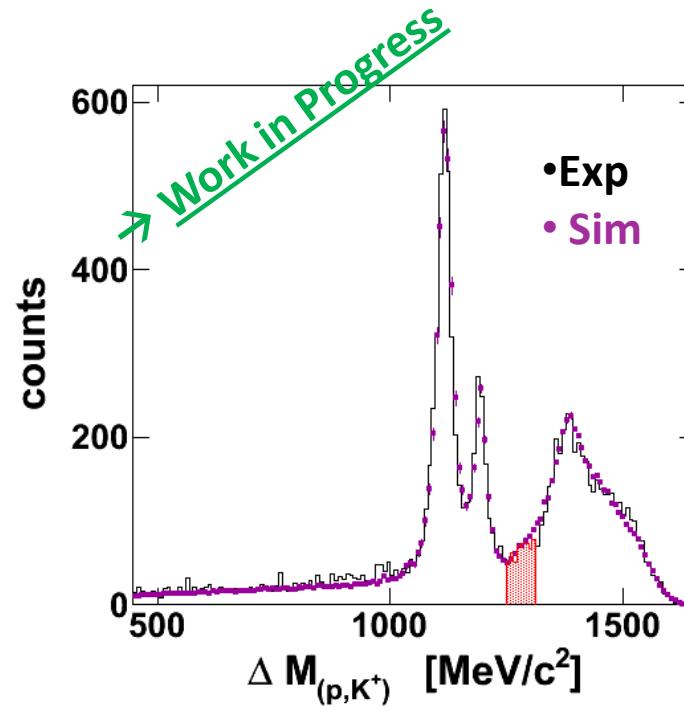
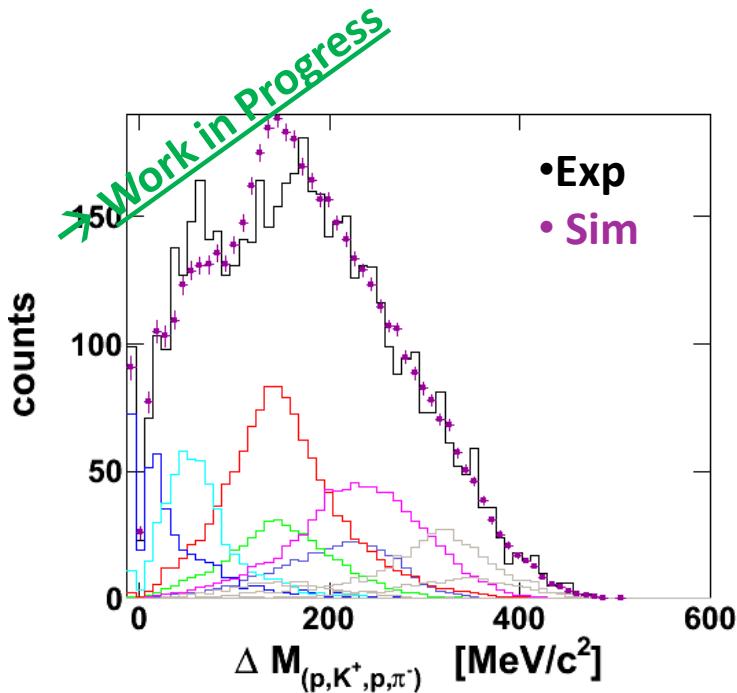


Contributing channels

- | | |
|-------------------------------|-----------------------------|
| → $\Lambda + p + K^+$ | → $\Lambda(1405) + p + K^+$ |
| → $\Lambda + p + K^+ + \pi^0$ | → $\Lambda(1520) + p + K^+$ |
| → $\Sigma + p + K^+$ | → $\Sigma(1385) + p + K^+$ |
| → $\Sigma + p + K^+ + \pi^0$ | + K^+ misidentification |

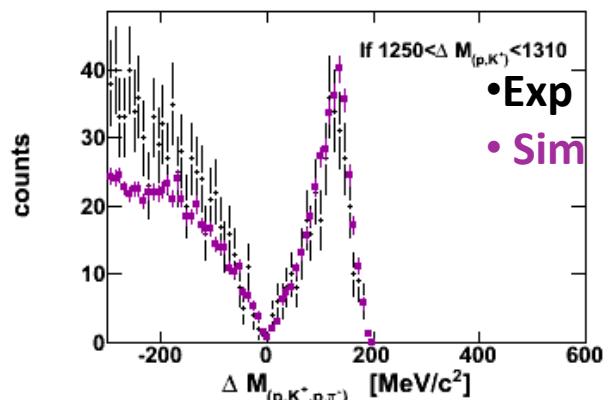


multi parameter fit

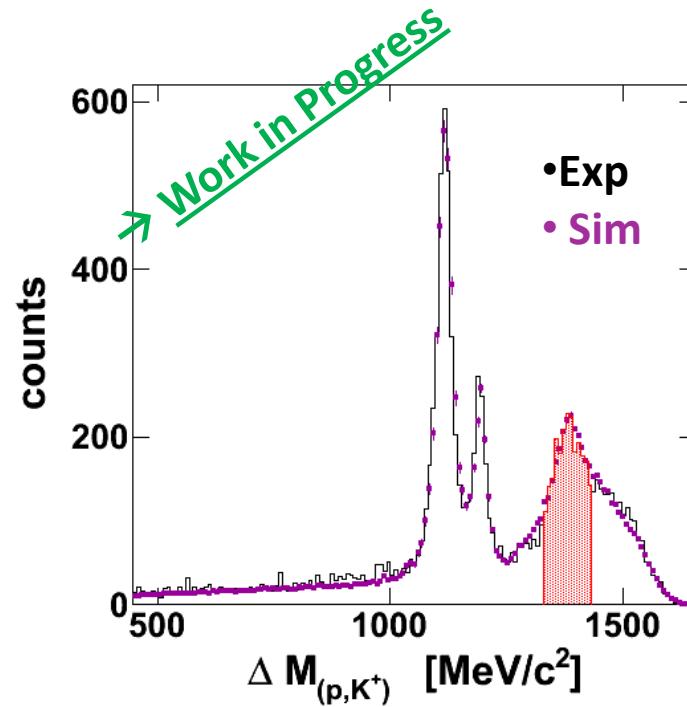
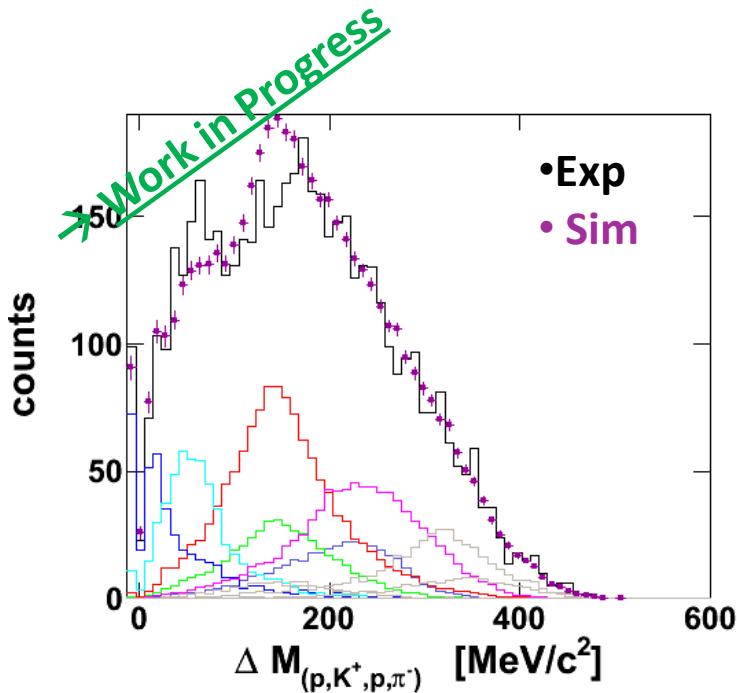


Contributing channels

- | | |
|---|---------------------------------------|
| $\rightarrow \Lambda + p + K^+$ | $\rightarrow \Lambda(1405) + p + K^+$ |
| $\rightarrow \Lambda + p + K^+ + \pi^0$ | $\rightarrow \Lambda(1520) + p + K^+$ |
| $\rightarrow \Sigma + p + K^+$ | $\rightarrow \Sigma(1385) + p + K^+$ |
| $\rightarrow \Sigma + p + K^+ + \pi^0$ | + K ⁺ misidentification |

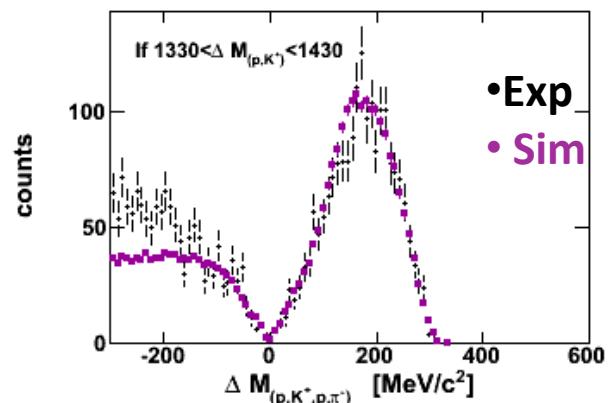


multi parameter fit

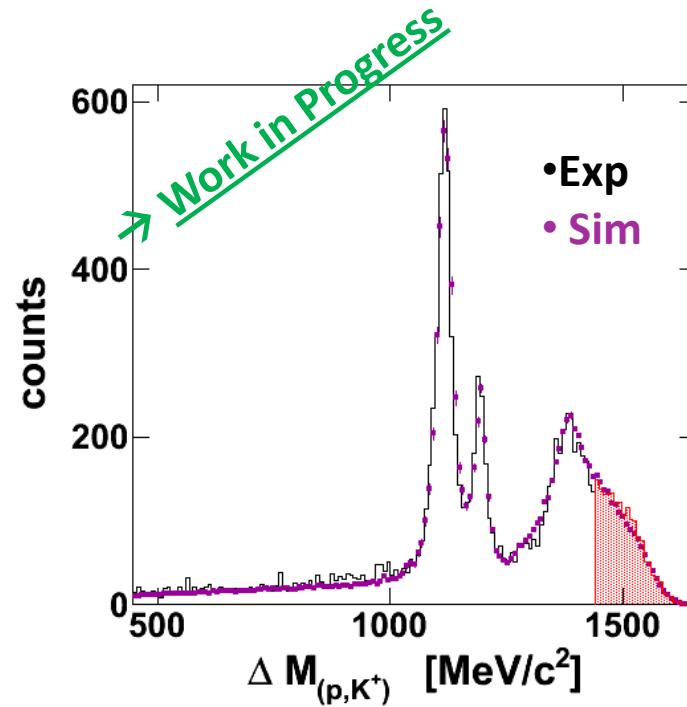
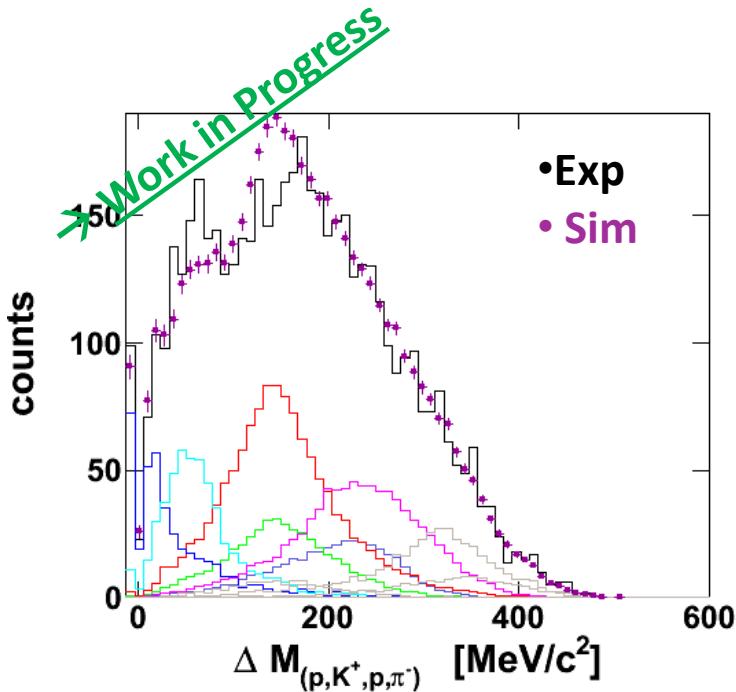


Contributing channels

$\rightarrow \Lambda + p + K^+$	$\rightarrow \Lambda(1405) + p + K^+$
$\rightarrow \Lambda + p + K^+ + \pi^0$	$\rightarrow \Lambda(1520) + p + K^+$
$\rightarrow \Sigma + p + K^+$	$\rightarrow \Sigma(1385) + p + K^+$
$\rightarrow \Sigma + p + K^+ + \pi^0$	+ K^+ misidentification

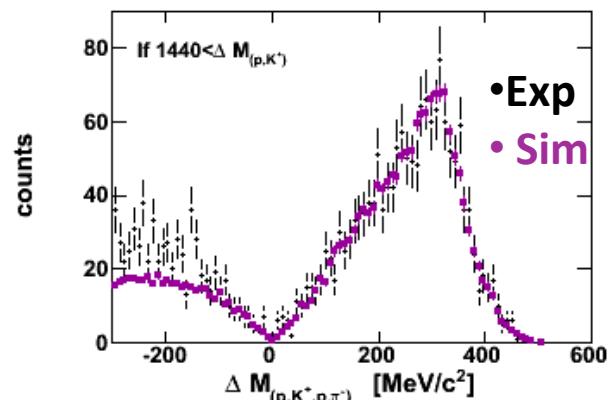


multi parameter fit



Contributing channels

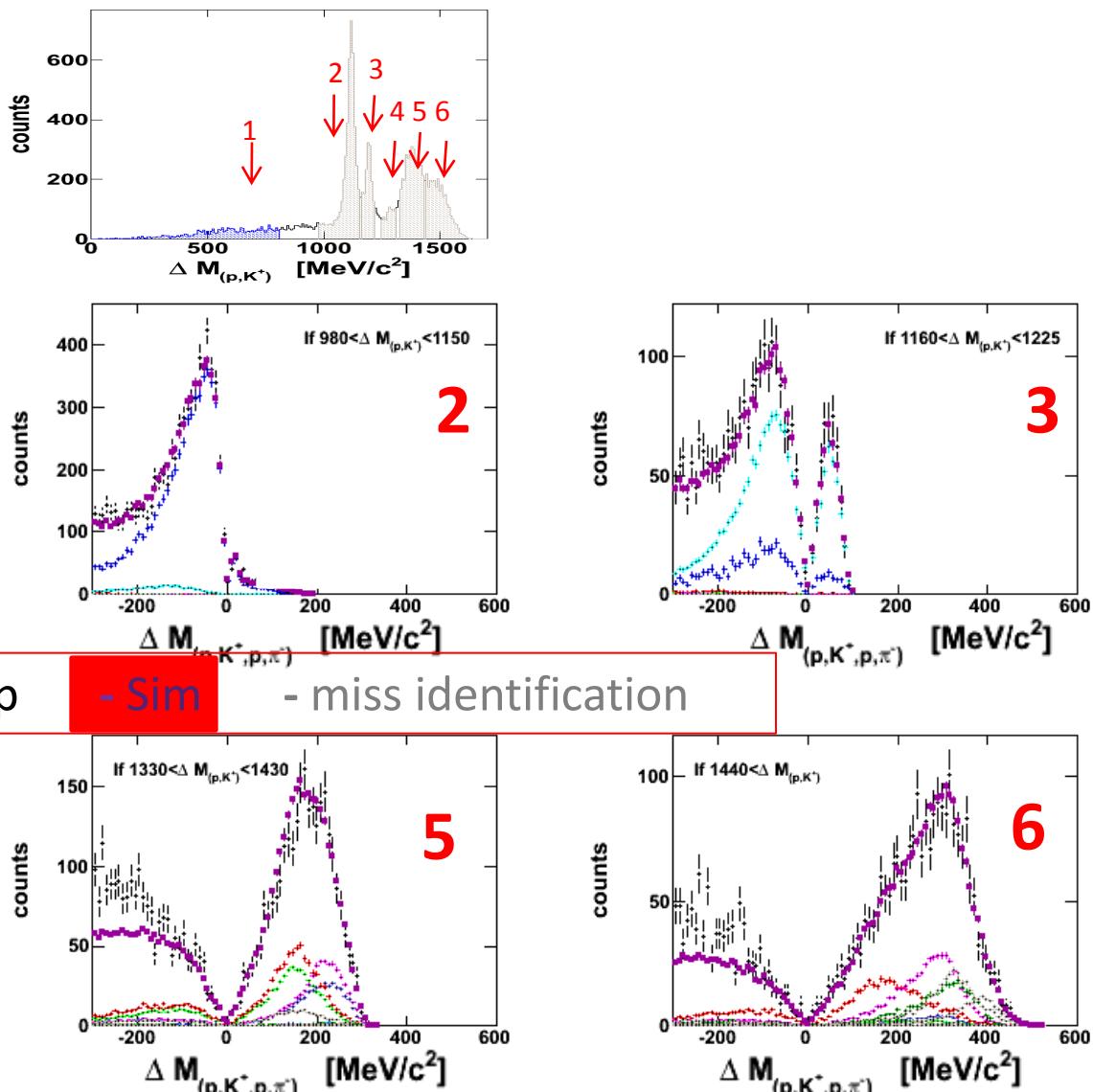
- | | |
|---|---------------------------------------|
| $\rightarrow \Lambda + p + K^+$ | $\rightarrow \Lambda(1405) + p + K^+$ |
| $\rightarrow \Lambda + p + K^+ + \pi^0$ | $\rightarrow \Lambda(1520) + p + K^+$ |
| $\rightarrow \Sigma + p + K^+$ | $\rightarrow \Sigma(1385) + p + K^+$ |
| $\rightarrow \Sigma + p + K^+ + \pi^0$ | + K^+ misidentification |



multi parameter fit

$\rightarrow \Lambda + p + K^+$
 $\rightarrow \Lambda + p + K^+ + \pi^0$
 $\rightarrow \Sigma + p + K^+$
 $\rightarrow \Sigma + p + K^+ + \pi^0$

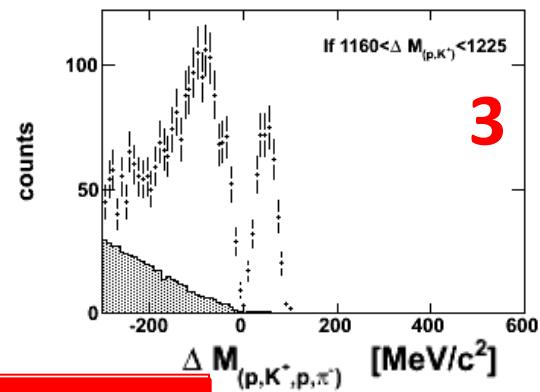
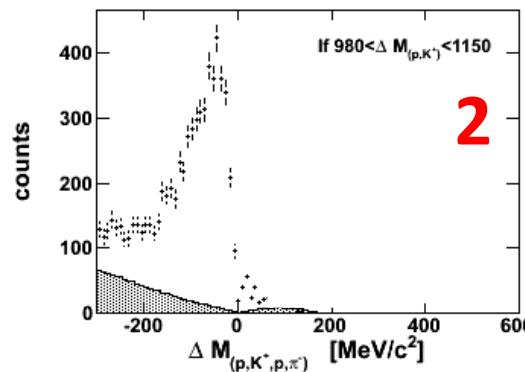
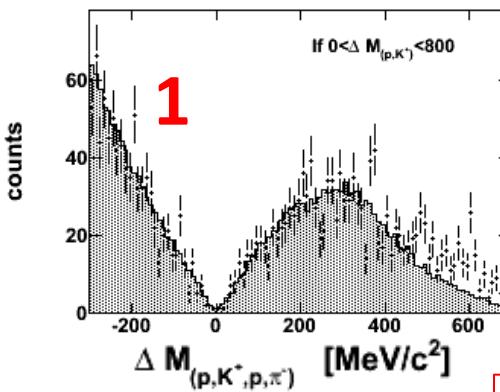
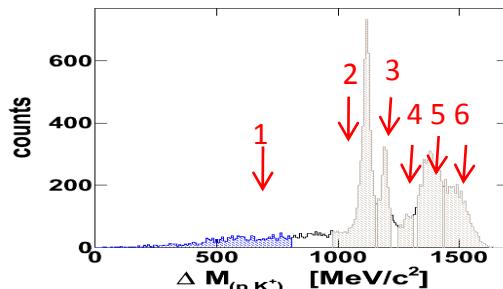
$\Lambda(1405) + p + K^+$
 $\Lambda(1520) + p + K^+$
 $\Sigma(1385) + p + K^+$



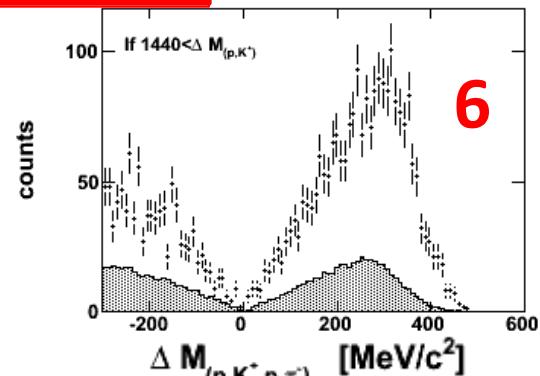
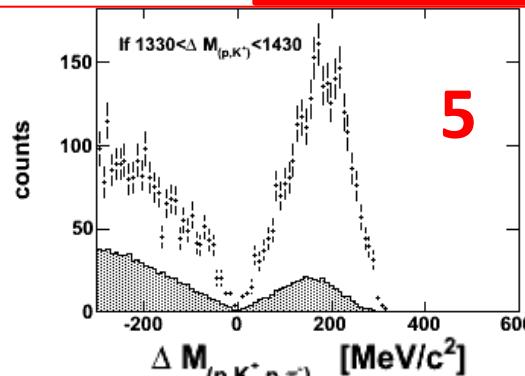
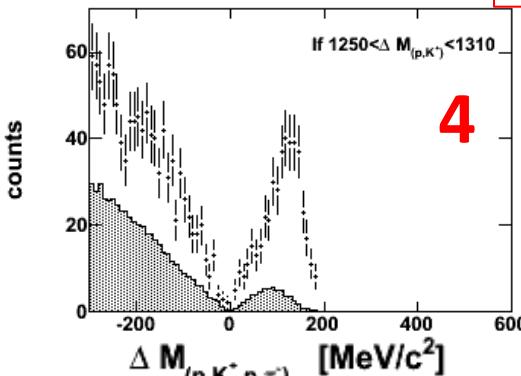
multi parameter fit

$\rightarrow \Lambda + p + K^+$
 $\rightarrow \Lambda + p + K^+ + \pi^0$
 $\rightarrow \Sigma + p + K^+$
 $\rightarrow \Sigma + p + K^+ + \pi^0$

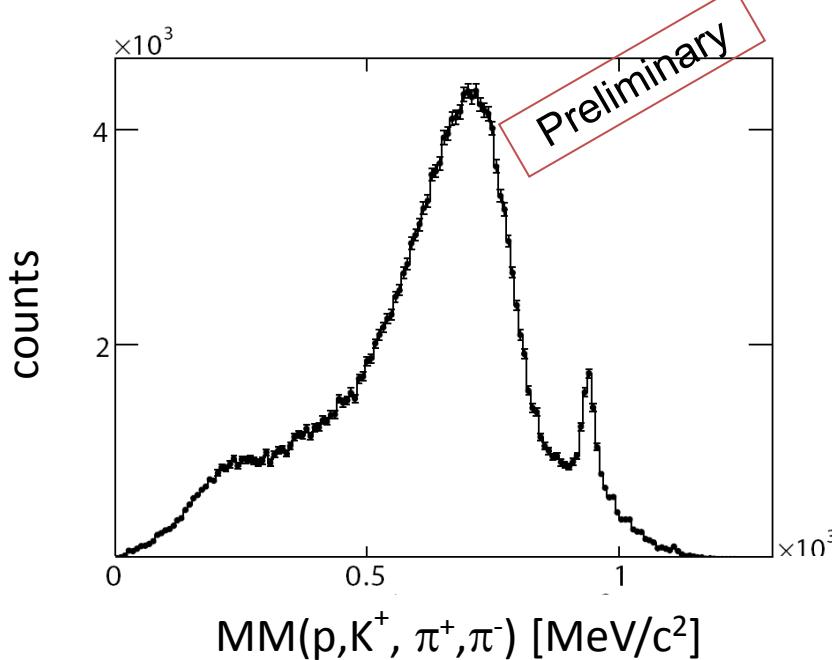
$\Lambda(1405) + p + K^+$
 $\Lambda(1520) + p + K^+$
 $\Sigma(1385) + p + K^+$



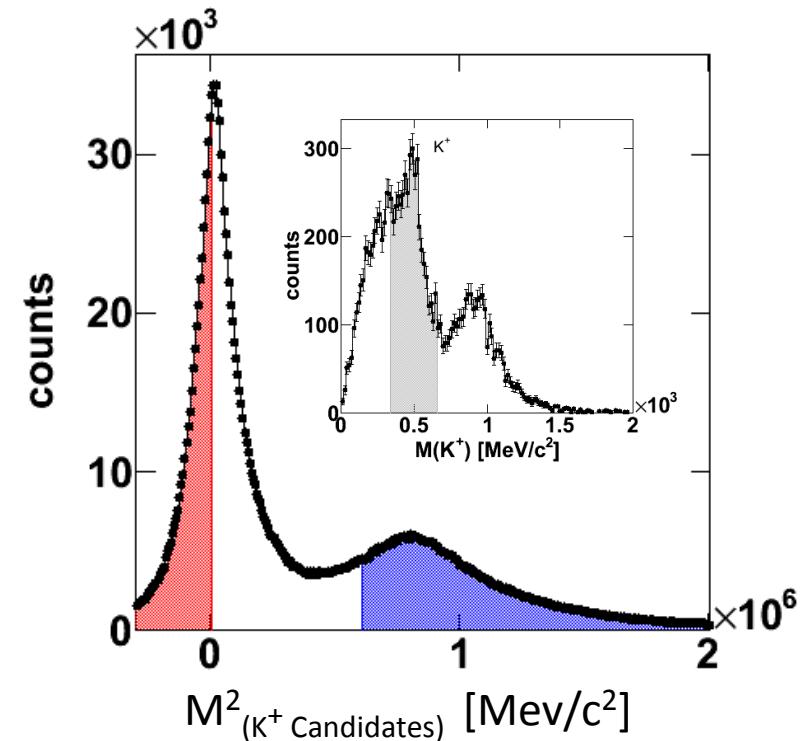
- Exp - Sim - miss identification



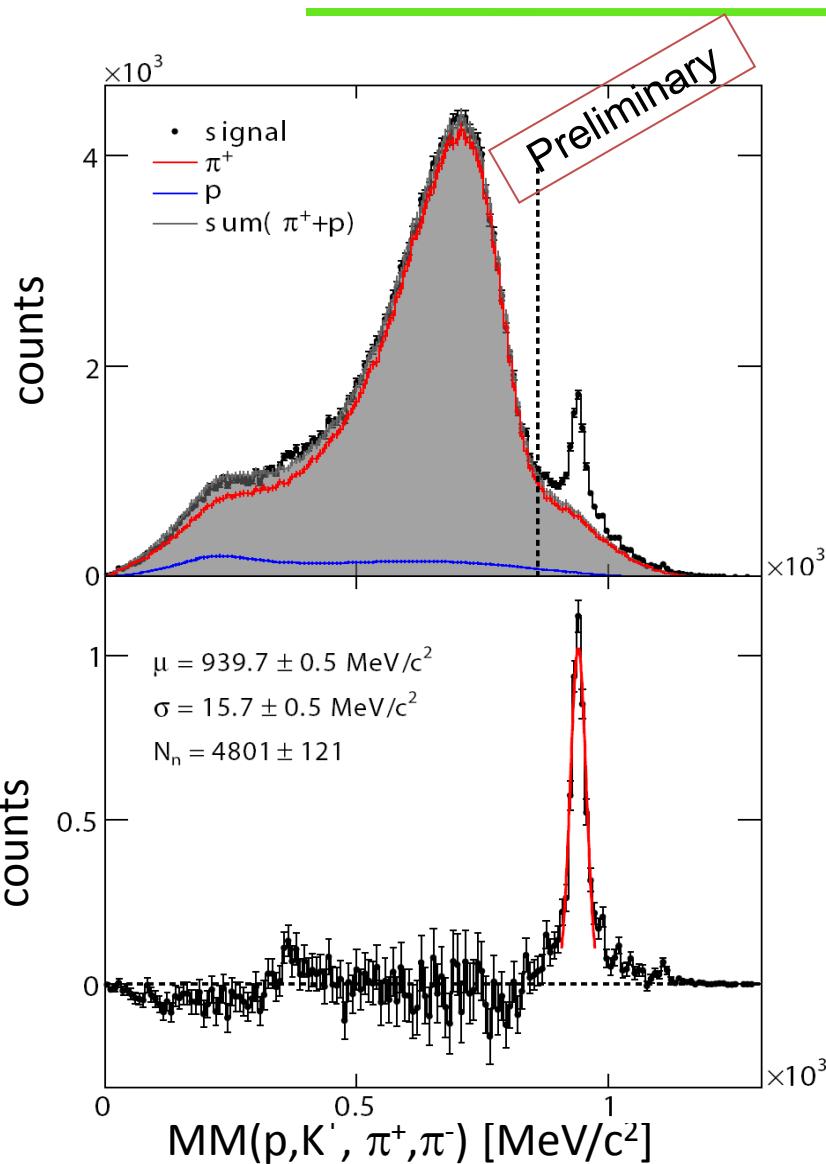
Missing Neutron



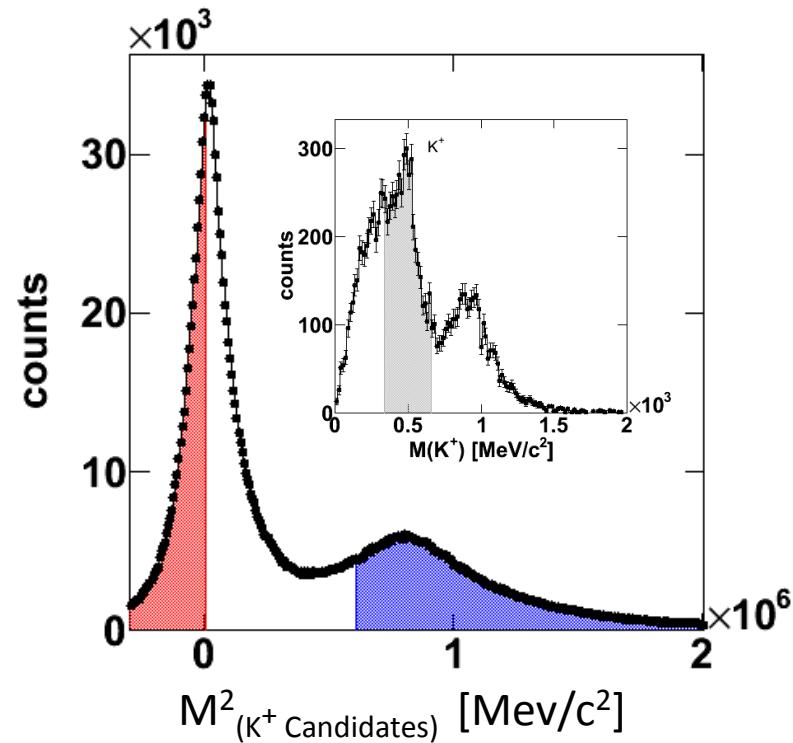
The misidentification background can be modeled with a sophisticated sideband analysis on the K^+ - mass



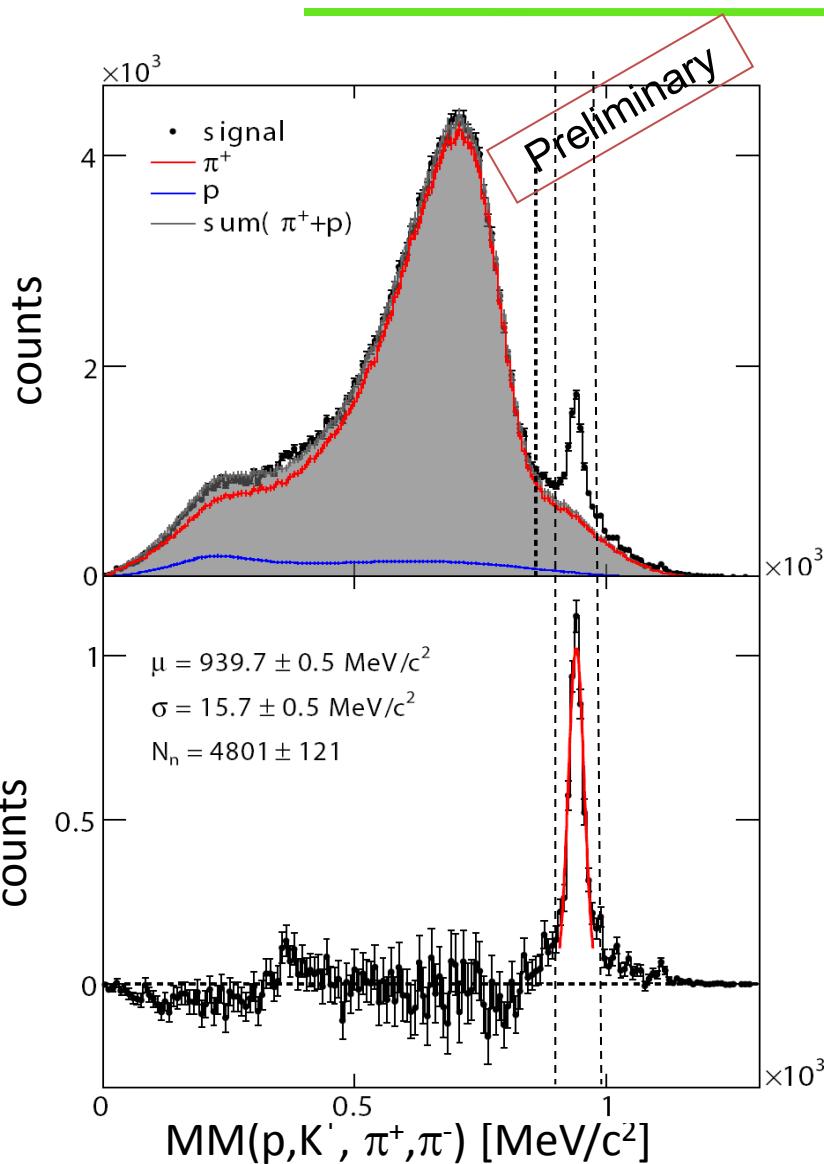
Missing Neutron



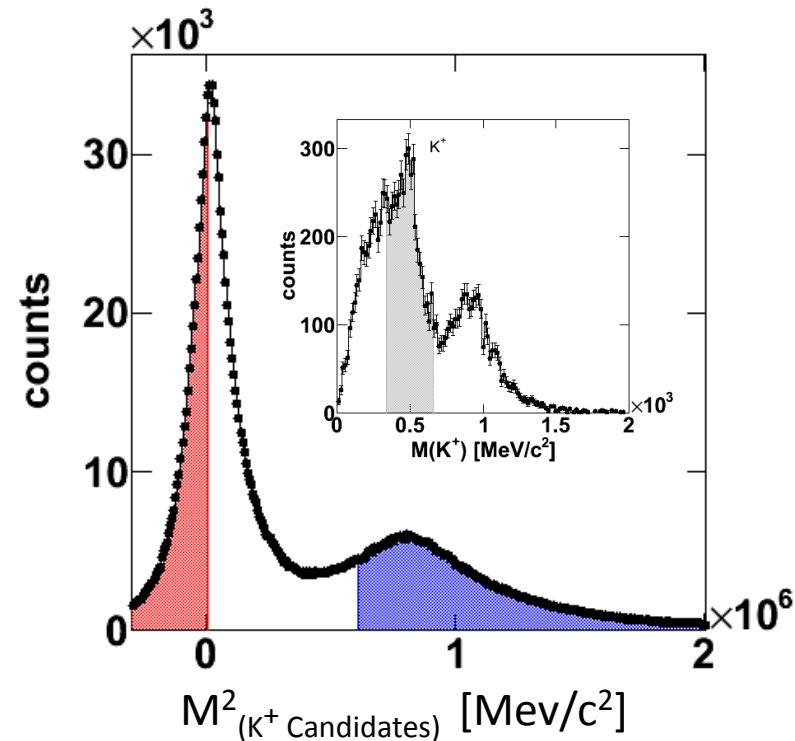
The misidentification background can be modeled with a sophisticated sideband analysis on the K^+ - mass



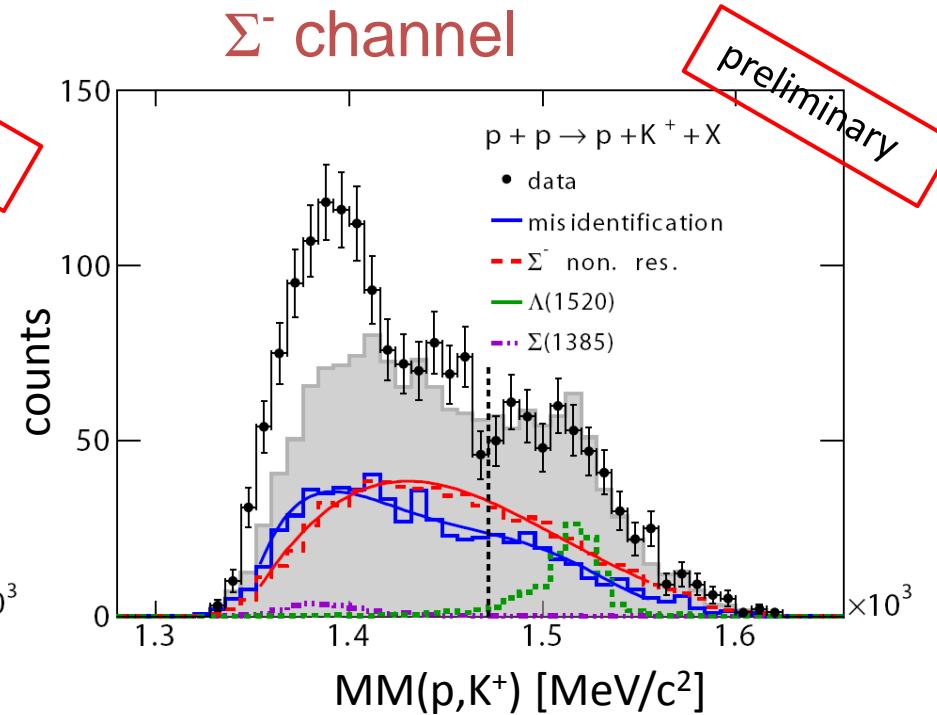
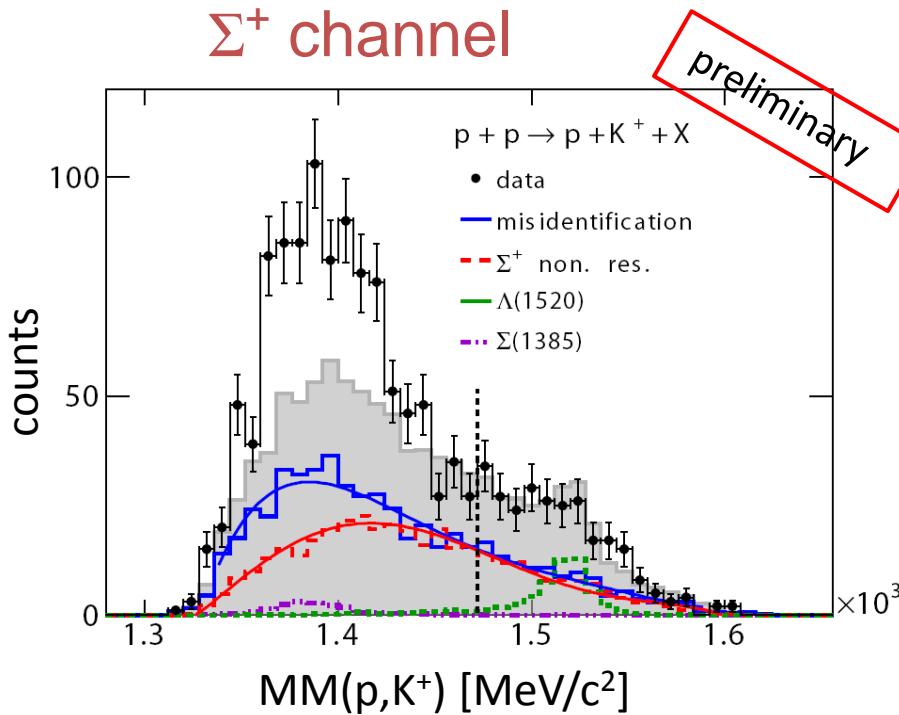
Missing Neutron



The misidentification background can be modeled with a sophisticated sideband analysis on the K^+ - mass



$\Lambda(1405)$ in Σ^+ and Σ^- channel

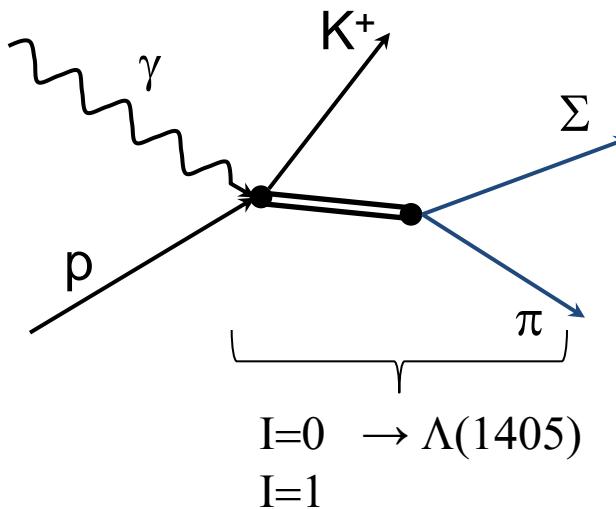


Besides the fit to the Σ^+ and Σ^- resonances, also the mass range above the vertical dashed line is simultaneously fitted with the different contributions.

Introduction

Different decay channels of the $\Lambda(1405)$

$\gamma + p \rightarrow \Lambda(1405) + K^+$:



Final states:

$$|\pi^+\Sigma^-\rangle = -\frac{1}{\sqrt{2}}|1,0\rangle - \frac{1}{\sqrt{3}}|0,0\rangle$$

$$|\pi^-\Sigma^+\rangle = +\frac{1}{\sqrt{2}}|1,0\rangle - \frac{1}{\sqrt{3}}|0,0\rangle$$

$$|\pi^0\Sigma^0\rangle = -\frac{1}{\sqrt{3}}|0,0\rangle$$

Different cross sections:

$$\frac{\partial \sigma(\pi^+\Sigma^-)}{\partial M_I} \approx \frac{1}{2}|T^{(1)}|^2 + \frac{1}{3}|T^{(0)}|^2 + \frac{2}{\sqrt{6}}\text{Re}(T^{(0)}T^{(1)*})$$

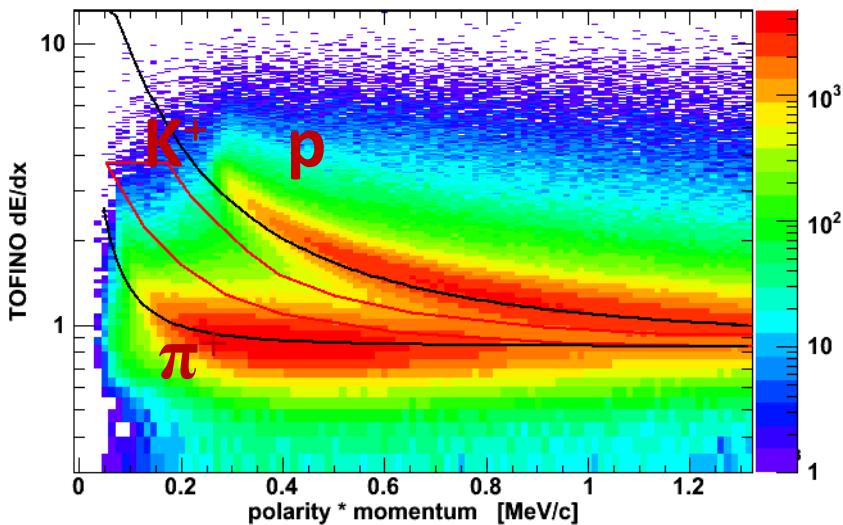
$$\frac{\partial \sigma(\pi^-\Sigma^+)}{\partial M_I} \approx \frac{1}{2}|T^{(1)}|^2 + \frac{1}{3}|T^{(0)}|^2 - \frac{2}{\sqrt{6}}\text{Re}(T^{(0)}T^{(1)*})$$

$$\frac{\partial \sigma(\pi^0\Sigma^0)}{\partial M_I} \approx \frac{1}{3}|T^{(0)}|^2$$

Hadron identification with HADES

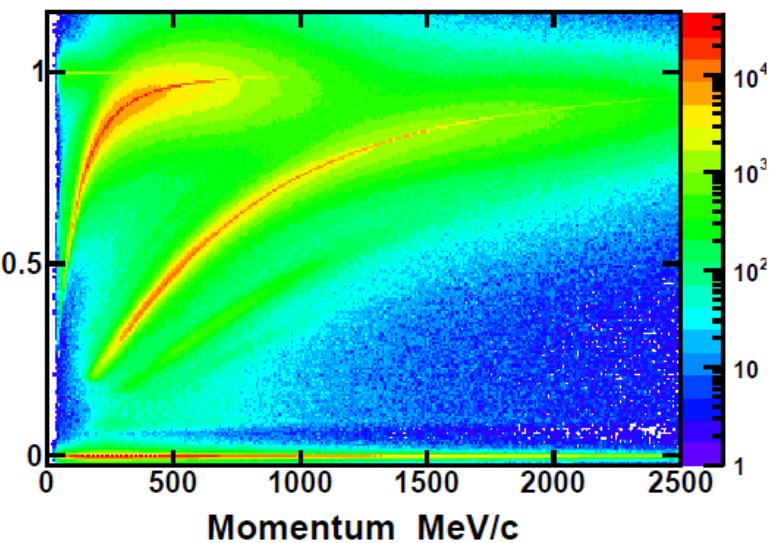
Energy loss

dE/dx and p are measured



Time-of-flight

ΔT and p are measured



NO start detector in pp

Time -of – flight reconstruction

Identify p and π via energy loss in MDC
 \rightarrow assign mass
 recalculate average start time of event

$$-\frac{dE}{dx} = \frac{4\pi}{m_e c^2} \cdot \frac{n z^2}{\beta^2} \cdot \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \cdot \left[\ln \left(\frac{2m_e c^2 \beta^2}{I \cdot (1 - \beta^2)} \right) - \beta^2 \right]$$

$$\beta = v / c$$

I mean excitation potential of the target

x distance travelled by the particle

n electron density of the target