Phenomenological $\bar{K}N$ interaction with isospin-breaking effects and $\bar{K}NN$ system

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K^-pp bound state

Prediction of the existence of deep and narrow K^- pp bound state
with E_B = - 48 MeV, Γ = 61 MeV (optical $\bar{K}N$ potential):
T. Yamazaki and Y. Akaishi, Phys. Lett. B535 (2002) 70

FINUDA collaboration: evidence for a deeply bound state
(correlated Λ and *p*) with E_B = -115 MeV, Γ = 67 MeV:
M. Agnello et. al., Phys. Rev. Lett. 94 (2005) 212303

another interpretation of the experiment:
V.K. Magas, E. Oset, A. Ramos, H. Toki, Phys. Rev. C 74 (2006) 025206

Non-relativistic coupled-channel 3-body Faddeev equations in AGS form
for the $\bar{K}NN - \pi\Sigma N$ system: E_B = - (50-70) MeV, Γ ~ 100 MeV :
- *N.V. Shevchenko, A. Gal, J. Mareš; Phys. Rev. Lett. 98 (2007) 082301*
- *N.V. Shevchenko, A. Gal, J. Mareš, J. Révai; Phys. Rev. C 76 (2007) 044004*

Other theoretical calculations of K^-pp system

Properies of $\bar{K}N$ interaction :

- Strongly coupled with $\pi\Sigma$ channel through $\Lambda(1405)$ resonance

  PDG : $E_\Lambda = 1406.5 - i\,25.0$ MeV, $I = 0$

  Usual assuption :

  a resonance in I = 0 $\pi\Sigma$ and a quasi - bound state in I = 0 $\bar{K}N$ channel

  Alternative version : $\Lambda(1405)$ is an effect of two close poles

  J. A. Oller, U. G. Meissner, Phys. Lett. B 500 *(*2001*)* 263,

  D. Jido et. al, Nucl. Phys. A 725 *(*2003*)* 181

- Measured scattering data :

  - Cross - sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions,

  - Threshold branching ratio $\gamma = \dfrac{\sigma(K^- p \to \pi^+\Sigma^-)}{\sigma(K^- p \to \pi^-\Sigma^+)} = 2.36$

  from *D.N. Tovee et al., Nucl. Phys. B*33 *(*1971*)* 493,

  *R.J. Nowak et al., Nucl. Phys. B*139 *(*1978*)* 61

- $K^- p$ scattering length :

$$a_{K^- p}^{\mathrm{KEK}} = -(0.78 \pm 0.15 \pm 0.03) + i\,(0.49 \pm 0.25 \pm 0.12)\ \mathrm{fm}$$

from *M. Iwasaki et al.*, Phys. Rev. Lett. 78 (1997) 3067,

T.M. Ito e t al., Phys. Rev. C 58 (1998) 2366

Obtained from Deser-Trueman formula:

S. Deser et al., Phys. Rev. 96 (1954) 774, T.L. Trueman, Nucl. Phys. 26 (1961) 57

$$\Delta E_{1\mathrm{s}} + i\,\frac{\Gamma_{1\mathrm{s}}}{2} = 2\,\alpha^3 \mu^2 a_{K^- p}, \quad \Delta E_{1\mathrm{s}} = E_{2\mathrm{p}\to 1\mathrm{s}}^{\exp} - E_{2\mathrm{p}\to 1\mathrm{s}}^{\mathrm{Coulomb}}$$

model-independent, but not accurate. There are many theoretical articles calculating corrections to the formula (quantum-mechanical, field theory)

Experimentally measured are: strong interaction shift and width of the kaonic hydrogen atom *1s* level state

$$\Delta E_{1s}^{KEK} = -323 \pm 63 \pm 11\ \mathrm{eV}, \ \Gamma_{1s}^{KEK} = 407 \pm 208 \pm 100\ \mathrm{eV}$$

Our aim

To construct $\bar{K}N-\pi\Sigma(-other\ channels)$ phenomenological potential, reproducing:

- $\Lambda(1405)$ resonance : one - resonance and two - resonance structure (dynamically generated)
- Measured $1s$ K^-p level shift and width,
- Cross - sections of $K^- p \rightarrow K^- p$ and $K^- p \rightarrow MB$ reactions,
- Threshold branching ratio γ

Isospin-breaking effects

1. Kaonic hydrogen: inclusion **Coulomb interaction**
2. Inclusion of the **mass difference**:

$$m_{K^-}, m_{\bar{K}^0}, m_p, m_n \quad \text{instead of} \quad m_{\bar{K}}, m_N$$

Coupled-channel equations for $\bar{K}N - \pi\Sigma$ system plus Coulomb interaction in $K^- p$ subsystem (no mass difference)

Strong interaction

$$V = V_s$$

$$T_s = \langle \Phi | V_s | \Psi^+ \rangle$$

$$|\Psi^+\rangle = |\Phi\rangle + G_0 V_s |\Psi^+\rangle$$

$$G_0 = (z - H_0)^{-1}$$

Isospin conserving:

$$\langle I\ I_z | V_s | I'\ I'_z \rangle = \delta_{I\,I'}\ \delta_{I_z I'_z} V_s$$

$$\langle I\ I_z | G_0 | I'\ I'_z \rangle = \delta_{I\,I'}\ \delta_{I_z I'_z} G_0$$

Strong interaction plus Coulomb

$$V = V_c + V_s$$

$$T_{s+c} = T_c + \langle \Phi_c^- | V_s | \Psi^+ \rangle$$

$$|\Psi^+\rangle = |\Phi_c^+\rangle + G_c V_s |\Psi^+\rangle$$

$$G_c = (z - H_0 - V_c)^{-1}$$

Isospin mixing:

$$\langle I\ I_z | V_c | I'\ I'_z \rangle = V_c\ C^{I\,I_z}_{i_1\,i_{z1}\,i_2\,i_{z2}}\ C^{I'\,I'_z}_{i_1\,i_{z1}\,i_2\,i_{z2}}$$

$$\boxed{\begin{aligned}\langle I\ I_z | G_c | I'\ I'_z \rangle &= \delta_{I\,I'}\ \delta_{I_z I'_z} G_0 + \\ &(G_c - G_0)\ C^{I\,I_z}_{i_1\,i_{z1}\,i_2\,i_{z2}}\ C^{I'\,I'_z}_{i_1\,i_{z1}\,i_2\,i_{z2}}\end{aligned}}$$

For separable strong part of the total potential:

$$V_{s,I}^{\alpha\beta} = \left| g_I^{\alpha} \right\rangle \lambda_I^{\alpha\beta} \left\langle g_I^{\beta} \right|; \qquad \alpha, \beta = K \ \text{ or } \ \pi; \qquad I = 0 \ \text{ or } \ 1$$

bound state condition is $\qquad \mathrm{Det}\left(\lambda^{-1} - \left\langle g \right| G_c \left| g \right\rangle\right) = 0$

and *T*-matrix: $\qquad T_{c+s} = T_c + \left\langle \Phi_c^- \right| g \rangle \left(\lambda^{-1} - \left\langle g \right| G_c \left| g \right\rangle\right)^{-1} \langle \mathrm{g} \left| \Phi_c^+ \right\rangle$

where states are vectors *4x1*, operators – matrices *4x4*

$$\left|\Psi_I^{\alpha}\right\rangle = \begin{pmatrix} \left|\Psi_0^K\right\rangle \\ \left|\Psi_0^{\pi}\right\rangle \\ \left|\Psi_1^K\right\rangle \\ \left|\Psi_1^{\pi}\right\rangle \end{pmatrix}, \qquad \left|\Phi_{c,I}^{\alpha}\right\rangle = \begin{pmatrix} \left|\Phi_c^K\right\rangle \\ \left|\Phi^{\pi}\right\rangle \\ \left|\Phi_c^K\right\rangle \\ \left|\Phi^{\pi}\right\rangle \end{pmatrix}, \qquad G_{c,II'}^{\alpha\beta} = \begin{pmatrix} G_{c,00}^K & 0 & G_{c,01}^K & 0 \\ 0 & G_{0,00}^{\pi} & 0 & 0 \\ G_{c,10}^K & 0 & G_{c,11}^K & 0 \\ 0 & 0 & 0 & G_{0,11}^{\pi} \end{pmatrix}$$

Nuclear $\Lambda(1405)$ *and* *atomic* $1s\ K^- p$ *states are described by the same equations*

Results

The parameters of the $\bar{K}N - \pi\Sigma(-\textit{other channels})$ potential were found with one - and two - resonance dynamically generated $\Lambda(1405)$

$\underline{K^- p \text{ scattering length}}$:

strong interaction: $K^- p$ scat. length, obtained from Deser formula

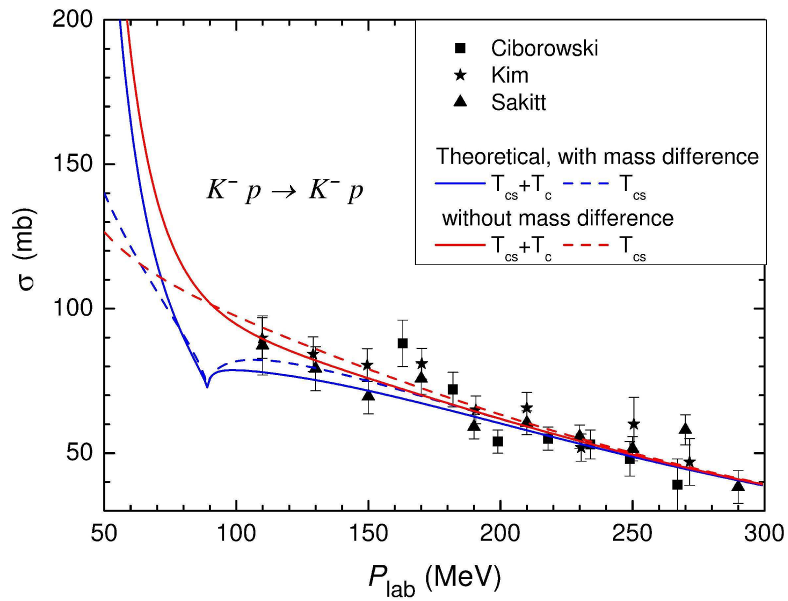
$$a_{K^- p}^{\text{KEK}}(Deser) = -0.78 + i\,0.49 \text{ fm}$$

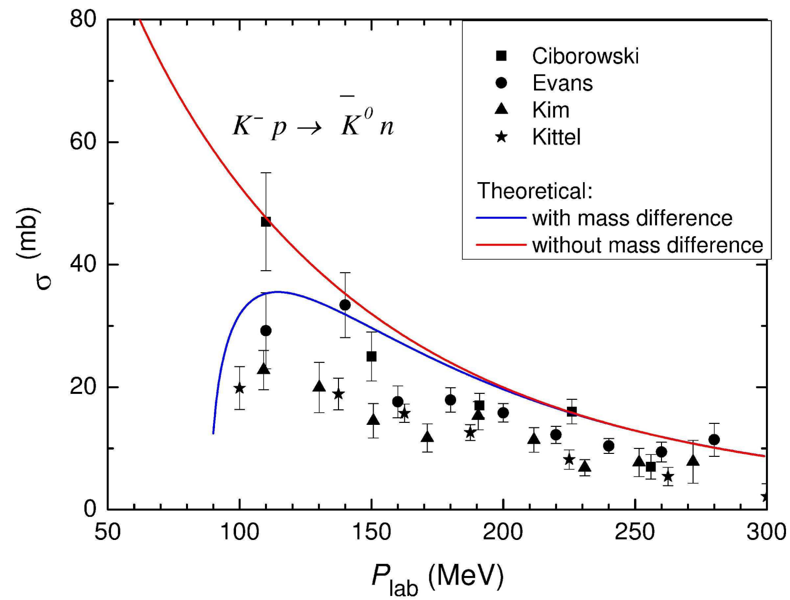
strong plus Coulomb interaction (*mass difference*):

reproducing directly $1s$ level shift of kaonic hydrogen atom

$$\boxed{a_{K^- p}^{\text{KEK}}(accurate) = -0.84 + i\,0.62 \text{ fm}}$$

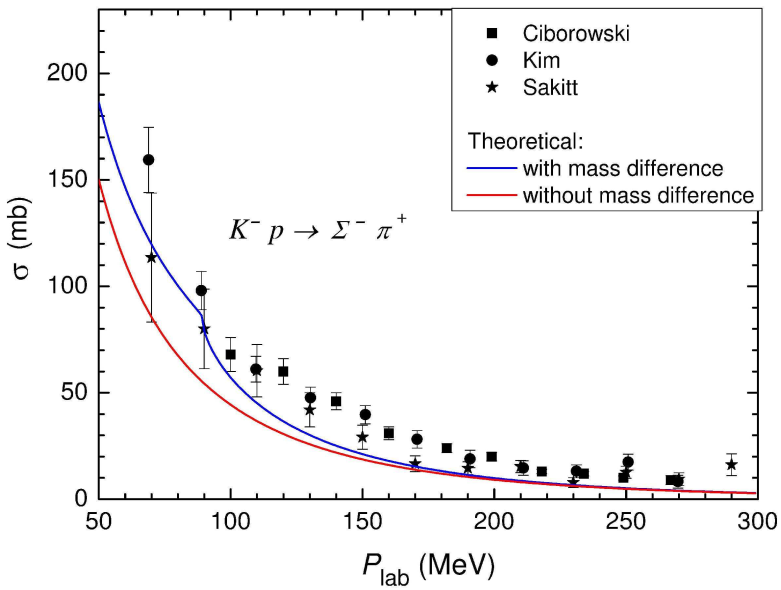
Switching off mass difference: $a_{K^- p}^{\text{KEK}}(avegared\ masses) = -0.54 + i\,0.63 \text{ fm}$

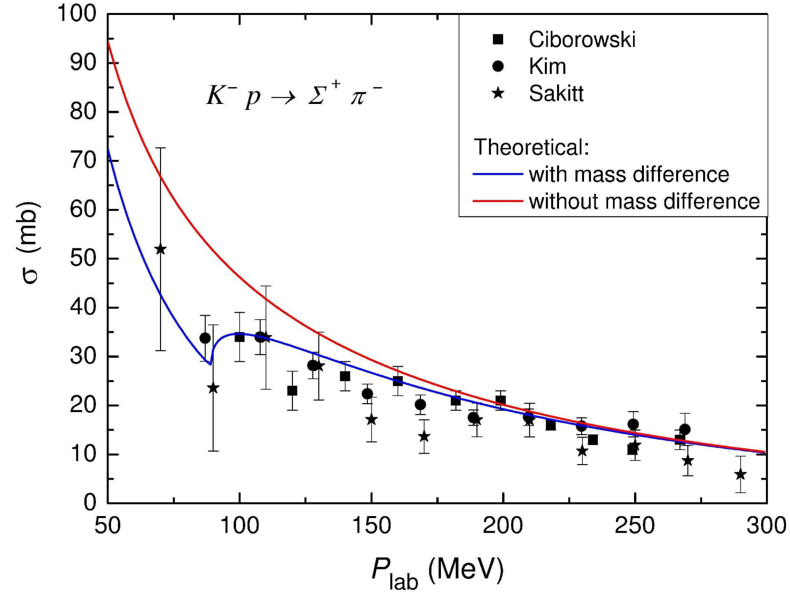


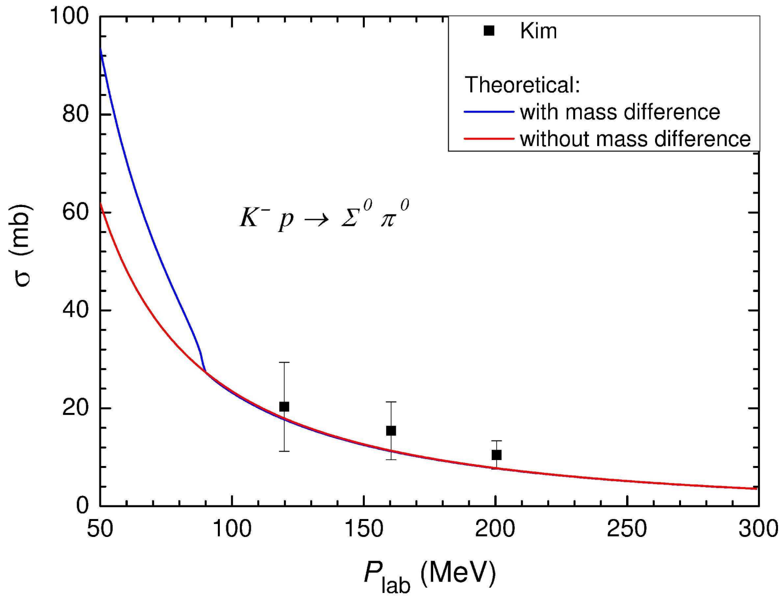


Comparison with experimental cross-sections
(theoretical model includes Coulomb interaction in $K^- p$ subsystem):

- blue lines: mass difference effect is included,
- red lines: averaged masses are used







Comparison with experimental cross-sections (continuation)

Three-body calculation (averaged masses)

Coulomb interaction is included in two-body $\bar{K}N-\pi\Sigma$,
but not in three-body $\bar{K}NN-\pi\Sigma N$ calculation.

For the three-body system it is assumed, that Coulomb interaction plays a minor role and can be omitted. Only the strong part of isospin-mixing interaction is used.

Calculated three-body pole energy of the I=1/2, $J^{\pi}=0^-$ quasi-bound state of the $\bar{K}NN-\pi\Sigma N$ system with respect to the K^-pp threshold:

$$E=-55.3-i\,51.1\ \text{MeV}$$

(fit to Deser K^-p scattering length)

$$E=-62.3-i\,49.6\ \text{MeV}$$

(fit to 1s strong level shift of kaonic hydrogen atom)

Conclusions

• Coupled-channel equations for strong isospin-dependent $\bar{K}N - \pi\Sigma(-other)$ interaction with direct inclusion of Coulomb interaction in $K^- p$ subsystem were derived and solved using physical masses. Parameters of the potential were found for one-resonance and two-resonance $\Lambda(1405)$.

• KEK $K^- p$ scattering length, obtained from measured *1s* level shift of kaonic hydrogen atom is

$$a_{K^- p}^{\mathrm{KEK}}\textit{(accurate)} = -0.84 + i\,0.62\ \mathrm{fm}$$

• Strong part of isospin mixing $\bar{K}N - \pi\Sigma$ *T*-matrix provides more deep and narrow I=1/2, $J^{\pi}=0^-$ quasi-bound state in $\bar{K}NN - \pi\Sigma N$ system, than pure strong $\bar{K}N - \pi\Sigma$, fitted to Deser's $K^- p$ scattering length