# Chiral effective potential with Delta degrees of freedom

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# Outline

- ChPT and low energy QCD
- Δ-less few nucleon forces within chiral EFT
  - Expectation from inclusion of Δ explicitely
- Isospin-conserving nuclear forces with explicit Δ
- Isospin-violating nuclear forces with explicit Δ
- Summary & Outlook

# ChPT and low energy QCD

Spontaneous + explicit (by small quark masses) breaking of chiral symmetry in QCD

Existence of light weakly interacting Goldstone bosons

Systematic description of QCD by ChPT in low energy sector

- Free parameters of QCD

$\alpha_s$, quark masses

- Free parameters of ChPT

$F_\pi, g_{\pi N}, c_i \ldots, d_i \ldots, C_S, C_T \ldots$

Computable by Lattice QCD / fit to data

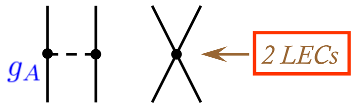
# Nucleon-nucleon force up to N$^3$LO

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; …

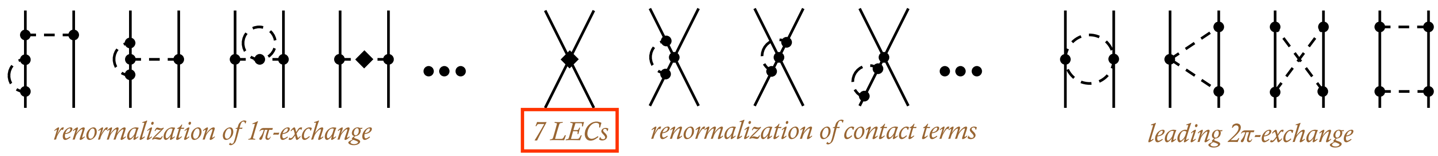
## Chiral expansion for the 2N force:

$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \ldots$$

- **LO:**

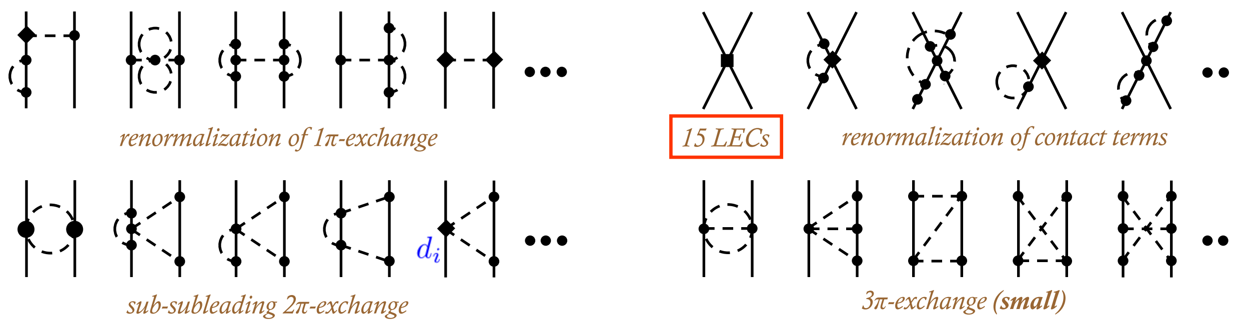



- **NLO:**



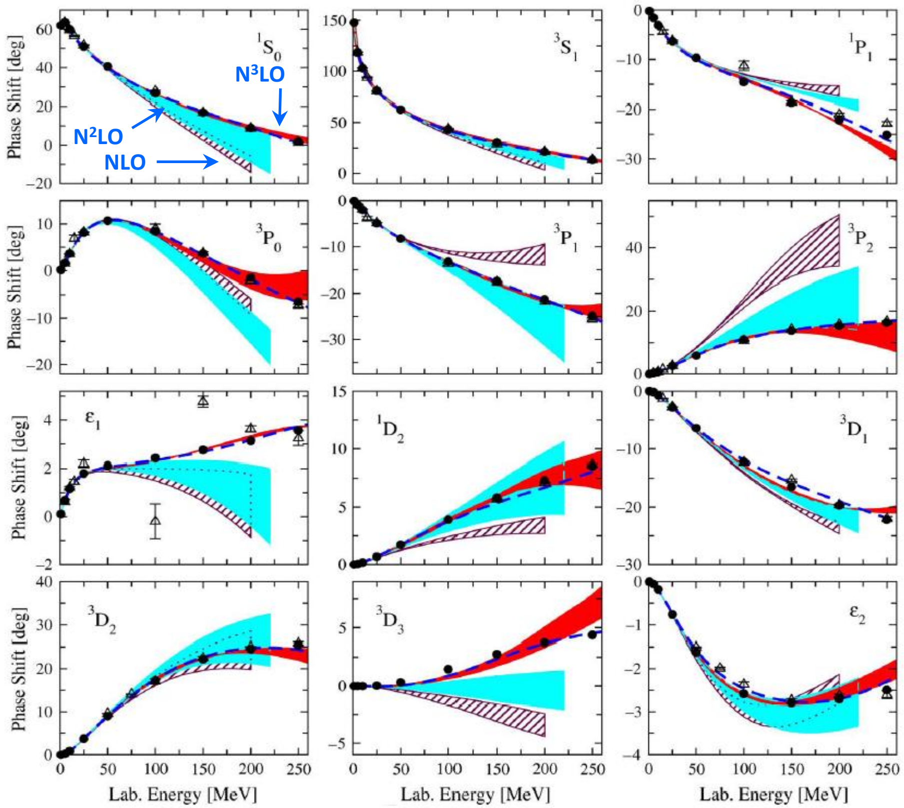

- **N$^2$LO:**


- **N$^3$LO:**

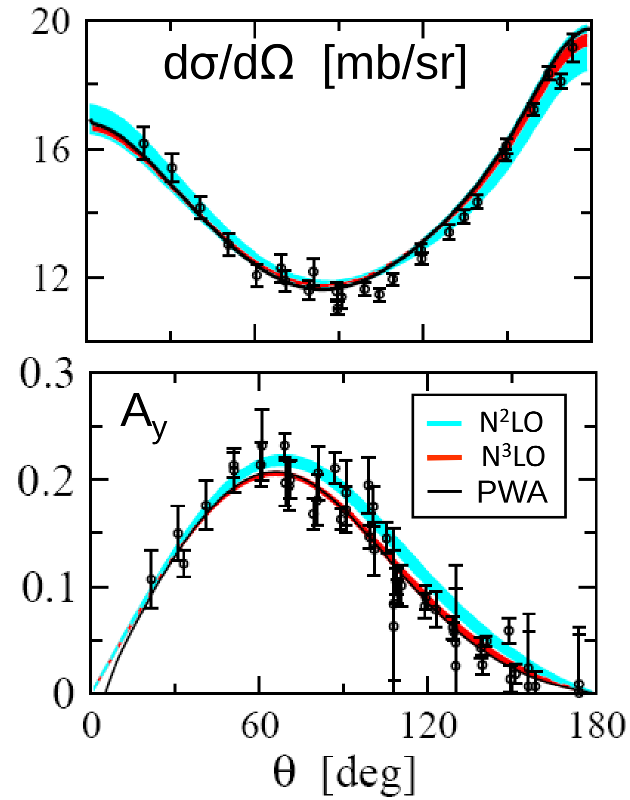



+ 1/m and isospin-breaking corrections…

## Neutron-proton phase shifts up to $N^3LO$




## np scattering at 50 MeV



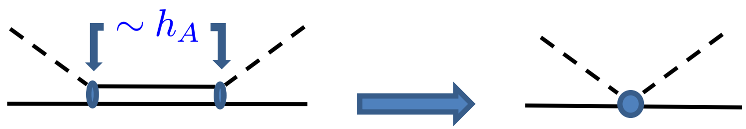

## Deuteron binding energy & asymptotic normalizations $A_s$ and $\eta_d$

| | NLO | $N^2LO$ | $N^3LO$ | Exp |
|---|---|---|---|---|
| $E_d$ [MeV] | $-2.171\ldots-2.186$ | $-2.189\ldots-2.202$ | $-2.216\ldots-2.223$ | $-2.224575(9)$ |
| $A_S$ [fm$^{-1/2}$] | $0.868\ldots0.873$ | $0.874\ldots0.879$ | $0.882\ldots0.883$ | $0.8846(9)$ |
| $\eta_d$ | $0.0256\ldots0.0257$ | $0.0255\ldots0.0256$ | $0.0254\ldots0.0255$ | $0.0256(4)$ |

*Entem & Machleidt '03; Epelbaum, Glöckle & Meißner '05*

# Delta-less effective potential

- Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293 \text{ MeV}$
- Delta contributions encoded in LECs
  *(Bernard, Kaiser & Meißner '97)*

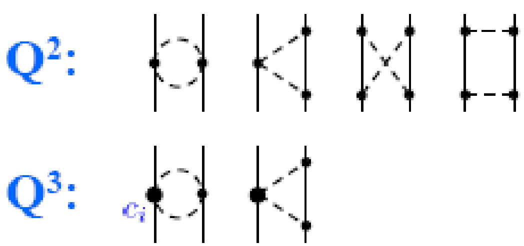


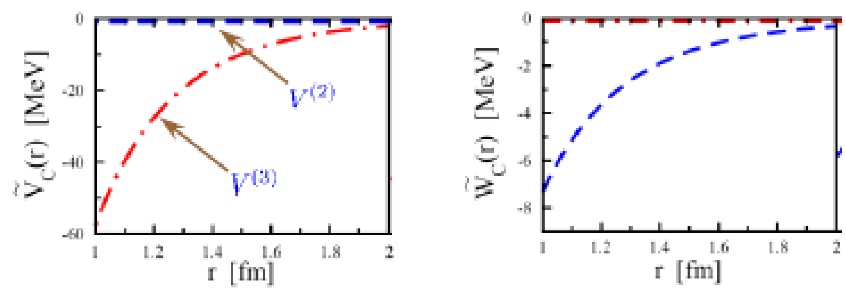

Delta-resonance saturation

$$c_3 = -2c_4 = c_3(\not{\Delta}) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

- Convergence of EFT potential



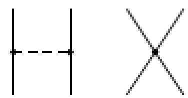
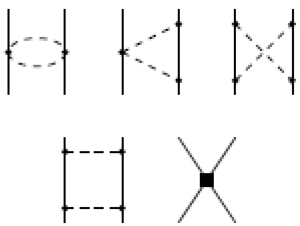
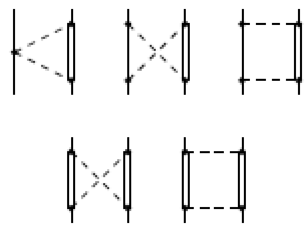
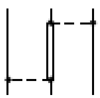
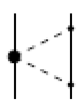
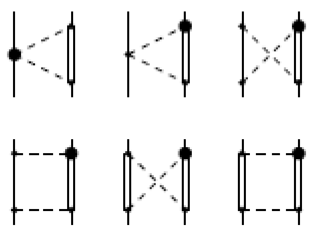
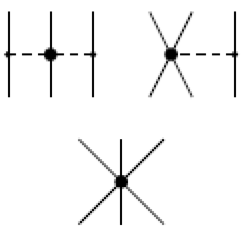

The subleading contribution is bigger than the leading one!

Expectation from inclusion of Δ explicitely

- more natural size of LECs
- better convergence
- applicability at higher energies

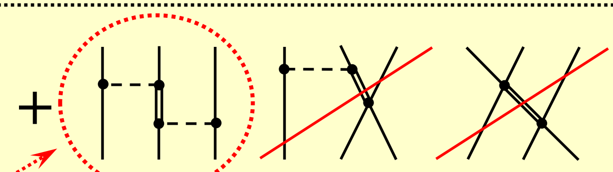
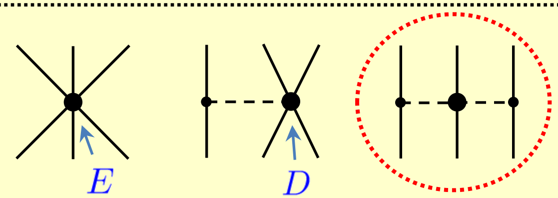
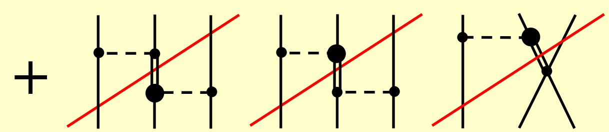
# Few-nucleon forces with the Delta

Isospin-symmetric contributions

| | Two-nucleon force | | Three-nucleon force | |
|---|---|---|---|---|
| | $\Delta$–less EFT | $\Delta$–contributions | $\Delta$–less EFT | $\Delta$–contributions |
| ***LO*** |  | — | — | — |
| ***NLO*** |  |  *Ordonez et al.'96, Kaiser et al. '98* | — |  |
| ***NNLO*** |  |  *H.K., Epelbaum & Meißner '07* |  | — |

# Delta excitations and the three-nucleon force

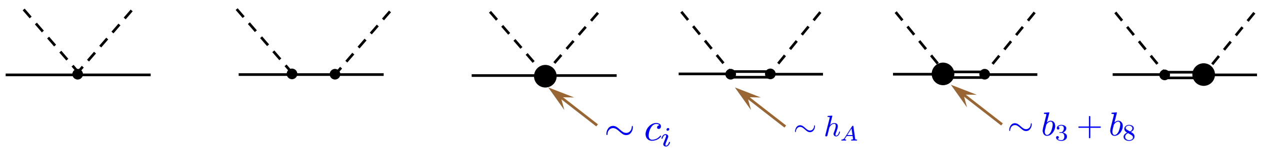
*Epelbaum, H.K., Meißner, Nucl. Phys. A806 (2008) 65*

| | standard chiral EFT | Including Δ as an explicit DOF |
|---|---|---|
| LO | — | |
| NLO | — |   |
| N²LO |   |   |

- The LO NNNΔ contact interaction $\bar{T}_i^\mu N \bar{N} S_\mu \tau^i N + \text{ h.c.}$ vanishes due to the Pauli principle
  ⟹ the LECs $D$ and $E$ are not saturated by the delta.
- No contributions from subleading 2π –exchange due to $\partial^0$ at the $b_3 + b_8$ vertex.
- The entire effect of the Δ is given by a partial shift of the N²LO TPE 3NF to NLO…

# Fit of LECs to $\pi$N scattering data

*Epelbaum, H.K., Meißner, Eur. Phys. J. A32 (2007) 127*




+ crossed graphs

## Two possible values for $h_A$

- fit 1: $h_A = \dfrac{3 g_A}{2\sqrt{2}} \sim 1.34$ *(SU(4), large N$_c$)*
- fit 2: $h_A = 1.05$ *(Fettes & Meißner '01)*

| LECs | $Q^2$, no $\Delta$ | $Q^2$, fit 1 | $Q^2$, fit 2 |
|---|---|---|---|
| $c_1$ | $-0.57$ | $-0.57$ | $-0.57$ |
| $c_2$ | $2.84$ | $-0.25$ | $0.83$ |
| $c_3$ | $-3.87$ | $-0.79$ | $-1.87$ |
| $c_4$ | $2.89$ | $1.33$ | $1.87$ |
| $h_A$ | – | $1.34^\star$ | $1.05^\star$ |
| $b_3 + b_8$ | – | $1.40$ | $2.95$ |

## Results of the fit

- Improved description of P-wave parameters when $\Delta$ is included
- Strongly reduced values for $c_i$
- Resulting $c_i$ depend strongly on $h_A$ while the thresh. param. do not

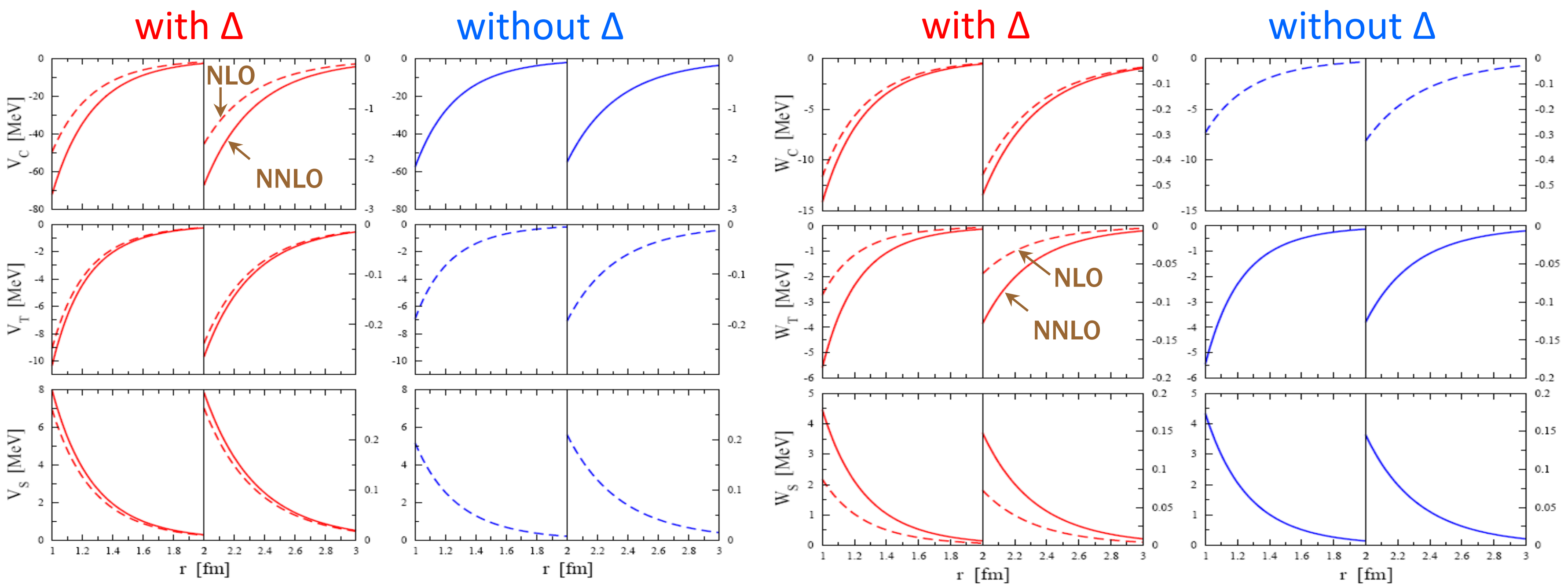
## S- and P-wave threshold parameters

| | $Q^2$, no $\Delta$ | $Q^2$ fits 1, 2 | EM98 |
|---|---|---|---|
| $a^+_{0+}$ | $0.41$ | $0.41$ | $0.41 \pm 0.09$ |
| $b^+_{0+}$ | $-4.46$ | $-4.46$ | $-4.46$ |
| $a^-_{0+}$ | $7.74$ | $7.74$ | $7.73 \pm 0.06$ |
| $b^-_{0+}$ | $3.34$ | $3.34$ | $1.56$ |
| $a^-_{1-}$ | $-0.05$ | $-1.32$ | $-1.19 \pm 0.08$ |
| $a^+_{1-}$ | $-2.81$ | $-5.30$ | $-5.46 \pm 0.10$ |
| $a^-_{1+}$ | $-6.22$ | $-8.45$ | $-8.22 \pm 0.07$ |
| $a^+_{1+}$ | $9.68$ | $12.92$ | $13.13 \pm 0.13$ |

# NN potential with explicit Δ

$$V_{\text{eff}} = V_C + W_C \vec{\tau}_1\cdot\vec{\tau}_2 + [V_S + W_S \vec{\tau}_1\cdot\vec{\tau}_2]\vec{\sigma}_1\cdot\vec{\sigma}_2 + [V_T + W_T \vec{\tau}_1\cdot\vec{\tau}_2](3\,\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2)$$

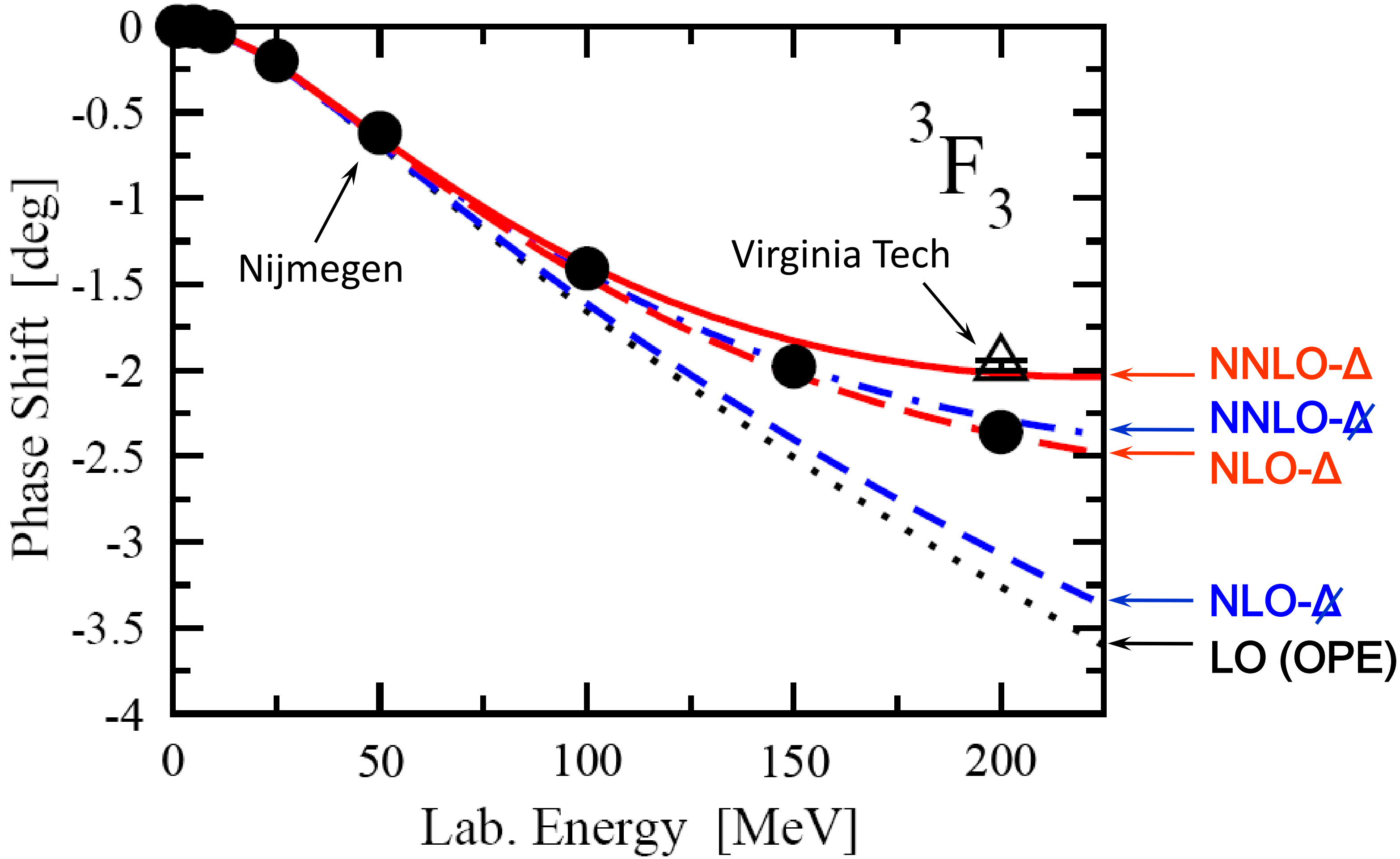
Chiral 2π- exchange potential up to NNLO




Advantages when Δ is included explicitly

- Dominant contributions already at NLO
- Much better convergence in all potentials

# $^3F_3$ partial waves up to NNLO with and without Δ

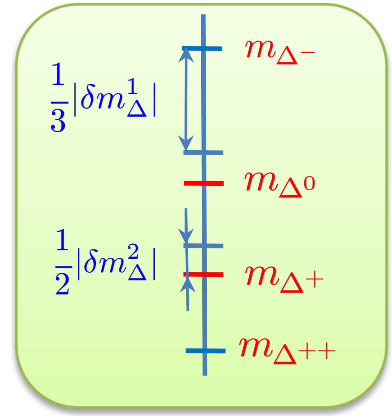



(calculated in the first Born approximation)

# Δ-mass splitting in chiral EFT

*Epelbaum, H.K., Meißner, Nucl. Phys. A806 (2008) 65*

$$\mathcal{L}^{\rm LO}_{\Delta,\rm mass} = -\bar{T}^\mu_i \left[ -\delta m^1_\Delta \frac{1}{2}\tau^3 \delta_{ij} - \delta m^2_\Delta \frac{3}{4}\delta_{i3}\delta_{j3} \right] g_{\mu\nu} T^\nu_j \,.$$




Equidistant splitting: strong & em

$$\delta m^1_\Delta = -4M_\pi^2 \epsilon c_5^\Delta - F_\pi^2 e^2 f_2^\Delta$$

Non-equidistant splitting: em

$$\delta m^2_\Delta = -\frac{4}{3} F_\pi^2 e^2 f_2^\Delta$$

- Most recent data from the PDG: $m_{\Delta^{++}} = 1230.80 \pm 0.30$ MeV, $m_{\Delta^0} = 1233.45 \pm 0.35$ MeV

  In addition, PDG's recommended value for the average mass:

$$m_\Delta = \frac{1}{4}\left(m_{\Delta^{++}} + m_{\Delta^+} + m_{\Delta^0} + m_{\Delta^-}\right) = \tilde{m}_\Delta + \frac{1}{4}\delta m^2_\Delta = 1231 \ldots 1233\,\text{MeV}$$

  On the other hand: $m_\Delta = 1233.4 \pm 0.4$ MeV *(Arndt et al. '06)* ⟹ use: $m_\Delta = 1233$ MeV

  ⟹ $\boxed{\tilde{m}_\Delta = 1233.4 \pm 0.7 \text{ MeV}\,, \quad \delta m^1_\Delta = -5.3 \pm 2.0 \text{ MeV}\,, \quad \delta m^2_\Delta = -1.7 \pm 2.7 \text{ MeV}}$

- Alternatively, use $m_{\Delta^{++}}/m_{\Delta^0}$ & the QM relation: $m_{\Delta^+} - m_{\Delta^0} = m_p - m_n$ *(Rubinstein et al. '67)*

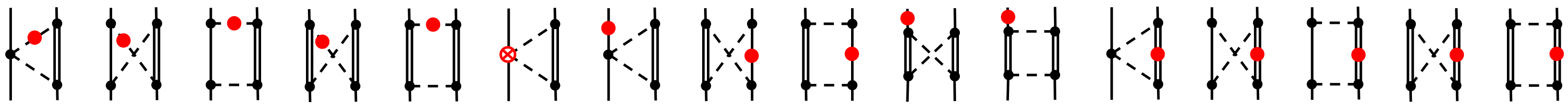
  ⟹ $\tilde{m}_\Delta = 1232.7 \pm 0.3$ MeV, $\delta m^1_\Delta = -3.9$ MeV, $\delta m^2_\Delta = 0.3 \pm 0.3$ MeV

# Isospin-breaking NN potential

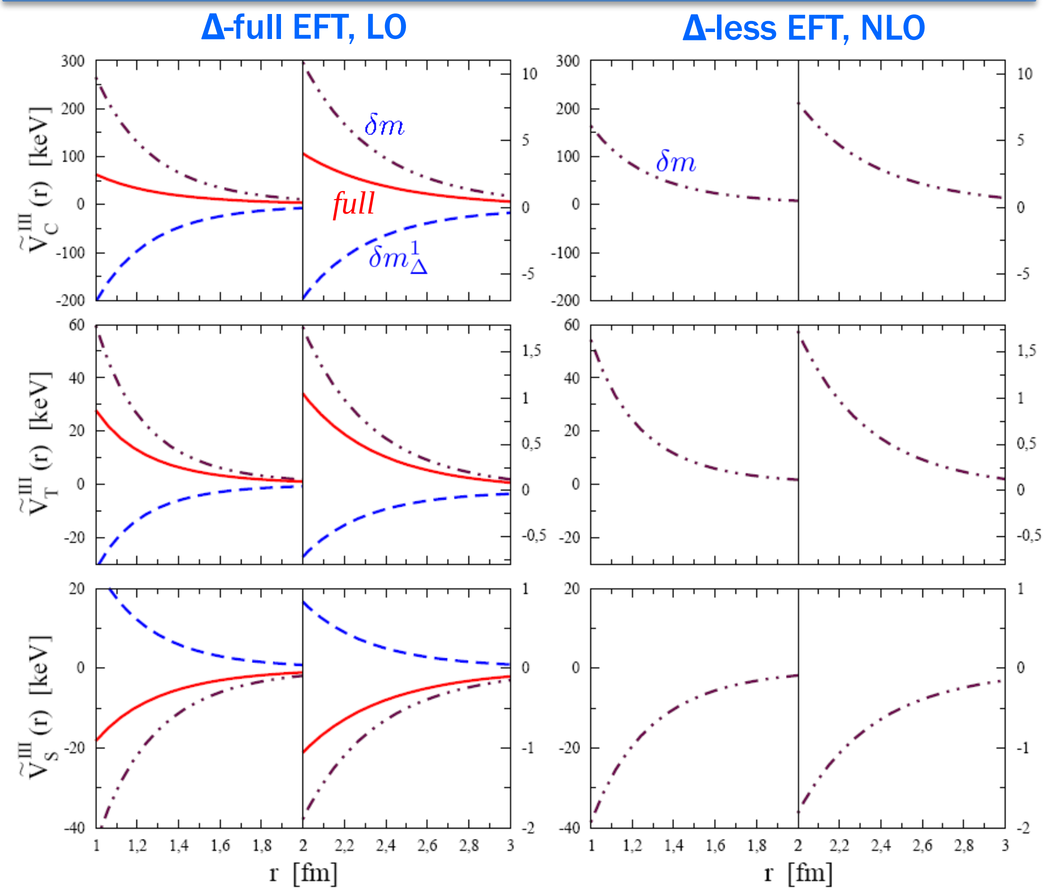
Epelbaum, H.K., Meißner, Phys. Rev. C77 (2008) 034006

**2π – exchange contributions with expicit Δ**

$$V = (\tau_1^3 + \tau_2^3)\left[V_C^{\rm III} + V_S^{\rm III}\vec\sigma_1\cdot\vec\sigma_2 + V_T^{\rm III}\vec\sigma_1\cdot\vec q\,\vec\sigma_2\cdot\vec q\right] + \ldots$$



**Charge-symmetry-breaking 2π–exchange potential**




- Similar $\sim \delta m$ contr. to $\tilde V_{\rm S,T}^{\rm III}$ in the Δ-less and Δ-full EFT
- Sizeable deviation in $\sim \delta m$ contr. for $\tilde V_{\rm C}^{\rm III}$
- Strong cancelations between $\sim \delta m$ and $\sim \delta m_\Delta^1$ terms

⇓

Big contributions beyond the subleading corrections in the Δ-less EFT

# Summary

- Nuclear forces with explicit Δ analyzed up to NNLO in chiral EFT
- The unknown LECs fixed from πN S- & P-waves; $c_i$ strongly reduced
- A more natural convergence pattern for nuclear forces.
  - The most important effects of the Δ are well approximated by the saturation of LECs $c_i$ in the Δ-less theory
- Leading isospin-violating Δ-contr. to nuclear forces are worked out
  - Delta mass splitting analyzed at LO
  - Found effects beyond resonance saturation of $c_i$

# Outlook

- Few nucleon observables within the Δ-full chiral EFT at NNLO
- Extension to N$^3$LO: important effects expected for 3NF and 4NF