Modern View on the Resonance Parameter Extraction

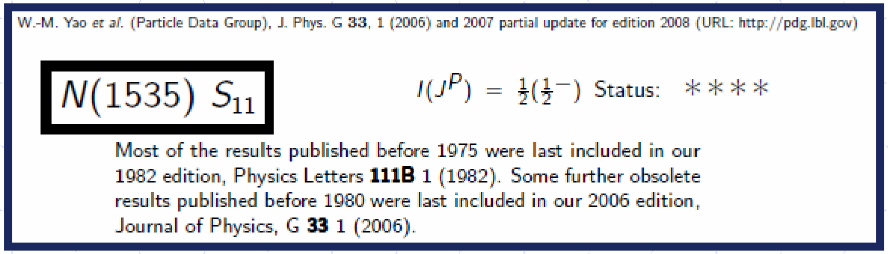
S. Ceci, H. Haberzettl, S. Krewald, K. Nakayama,
A. Svarc and B. Zauner (with S. Watson and J. Stahov)

The resolution of the (big) speed plot problem

Meson 2008, 10th International Workshop on Meson Production, Properties and Interaction

June 9th, 2008

PDG:
Poles and BW



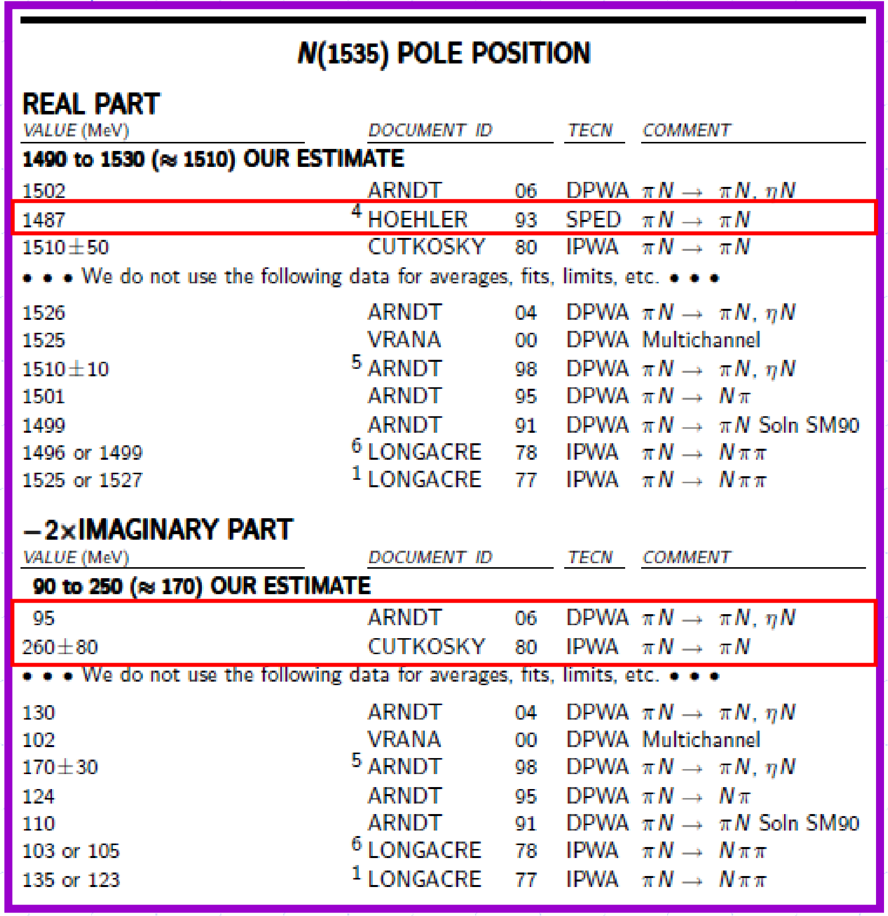
W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) and 2007 partial update for edition 2008 (URL: http://pdg.lbl.gov)

$N(1535)\ S_{11}$ $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ Status: ∗∗∗∗

Most of the results published before 1975 were last included in our 1982 edition, Physics Letters **111B** 1 (1982). Some further obsolete results published before 1980 were last included in our 2006 edition, Journal of Physics, G **33** 1 (2006).

analyses w/o poles

CMB analyses



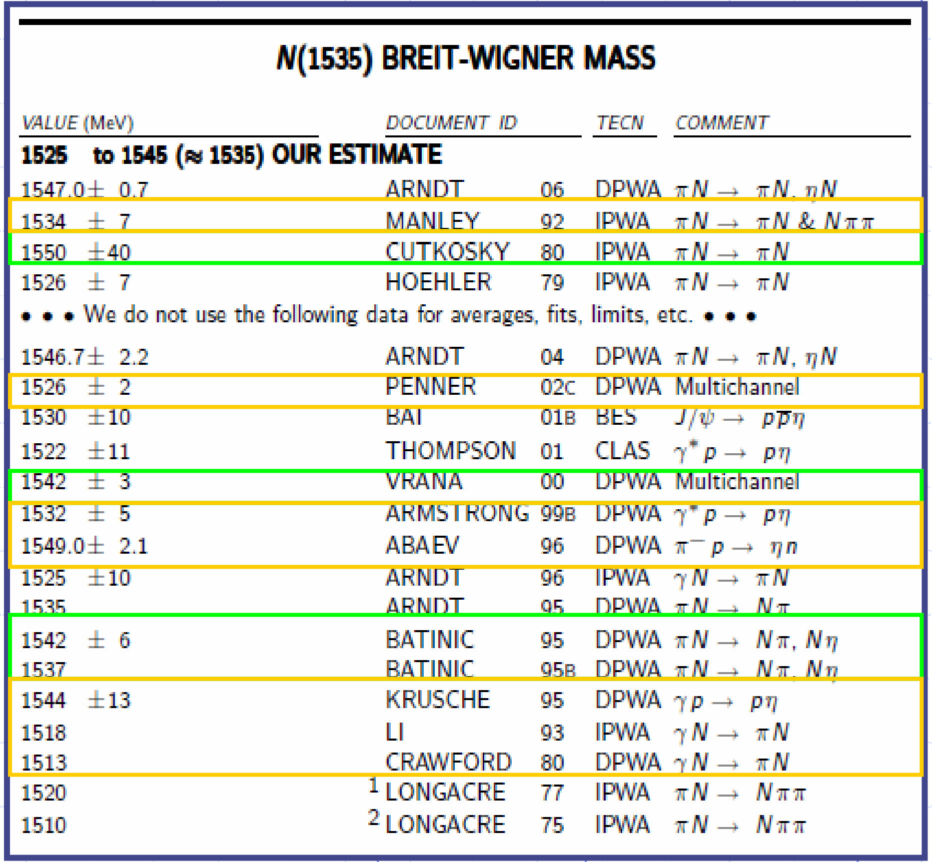
N(1535) POLE POSITION

REAL PART

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
1490 to 1530 (≈ 1510) OUR ESTIMATE				
1502	ARNDT	06	DPWA	$\pi N \to \pi N, \eta N$
1487	[4] HOEHLER	93	SPED	$\pi N \to \pi N$
1510±50	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1526	ARNDT	04	DPWA	$\pi N \to \pi N, \eta N$
1525	VRANA	00	DPWA	Multichannel
1510±10	[5] ARNDT	98	DPWA	$\pi N \to \pi N, \eta N$
1501	ARNDT	95	DPWA	$\pi N \to N\pi$
1499	ARNDT	91	DPWA	$\pi N \to \pi N$ Soln SM90
1496 or 1499	[6] LONGACRE	78	IPWA	$\pi N \to N\pi\pi$
1525 or 1527	[1] LONGACRE	77	IPWA	$\pi N \to N\pi\pi$

−2×IMAGINARY PART

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
90 to 250 (≈ 170) OUR ESTIMATE				
95	ARNDT	06	DPWA	$\pi N \to \pi N, \eta N$
260±80	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
130	ARNDT	04	DPWA	$\pi N \to \pi N, \eta N$
102	VRANA	00	DPWA	Multichannel
170±30	[5] ARNDT	98	DPWA	$\pi N \to \pi N, \eta N$
124	ARNDT	95	DPWA	$\pi N \to N\pi$
110	ARNDT	91	DPWA	$\pi N \to \pi N$ Soln SM90
103 or 105	[6] LONGACRE	78	IPWA	$\pi N \to N\pi\pi$
135 or 123	[1] LONGACRE	77	IPWA	$\pi N \to N\pi\pi$

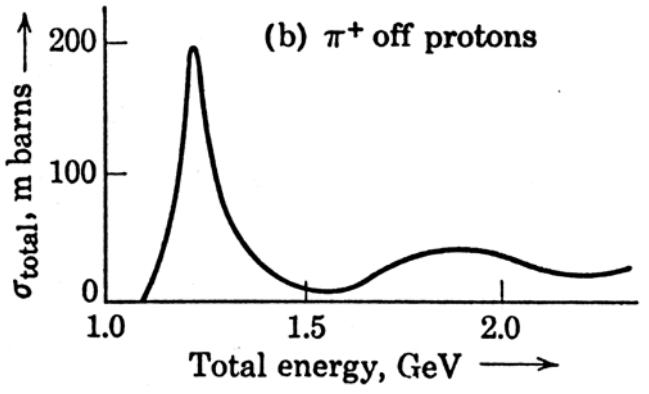


N(1535) BREIT-WIGNER MASS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
1525 to 1545 (≈ 1535) OUR ESTIMATE				
1547.0± 0.7	ARNDT	06	DPWA	$\pi N \to \pi N, \eta N$
1534 ± 7	MANLEY	92	IPWA	$\pi N \to \pi N$ & $N\pi\pi$
1550 ±40	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
1526 ± 7	HOEHLER	79	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1546.7± 2.2	ARNDT	04	DPWA	$\pi N \to \pi N, \eta N$
1526 ± 2	PENNER	02C	DPWA	Multichannel
1530 ±10	BAI	01B	BES	$J/\psi \to p\overline{p}\eta$
1522 ±11	THOMPSON	01	CLAS	$\gamma^* p \to p\eta$
1542 ± 3	VRANA	00	DPWA	Multichannel
1532 ± 5	ARMSTRONG	99B	DPWA	$\gamma^* p \to p\eta$
1549.0± 2.1	ABAEV	96	DPWA	$\pi^- p \to \eta n$
1525 ±10	ARNDT	96	IPWA	$\gamma N \to \pi N$
1535	ARNDT	95	DPWA	$\pi N \to N\pi$
1542 ± 6	BATINIC	95	DPWA	$\pi N \to N\pi, N\eta$
1537	BATINIC	95B	DPWA	$\pi N \to N\pi, N\eta$
1544 ±13	KRUSCHE	95	DPWA	$\gamma p \to p\eta$
1518	LI	93	IPWA	$\gamma N \to \pi N$
1513	CRAWFORD	80	DPWA	$\gamma N \to \pi N$
1520	[1] LONGACRE	77	IPWA	$\pi N \to N\pi\pi$
1510	[2] LONGACRE	75	IPWA	$\pi N \to N\pi\pi$

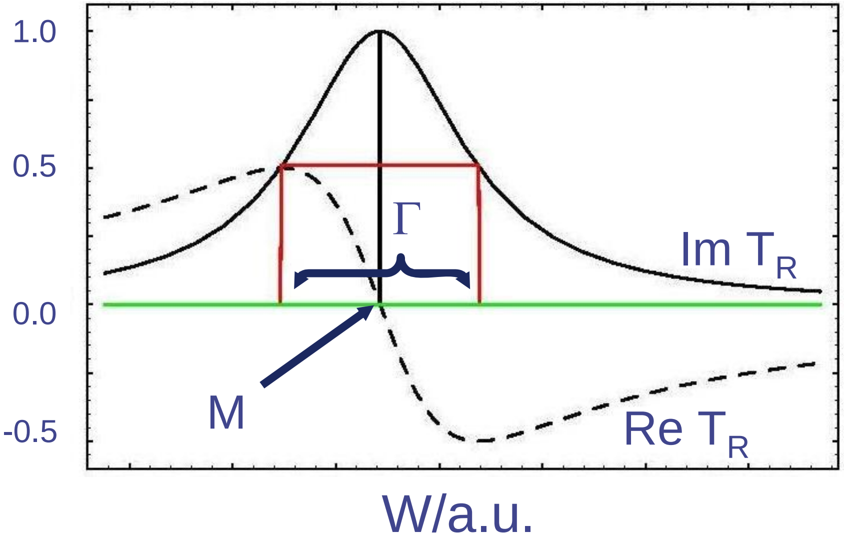
small SP problem

Resonance parameters



PWA

Ideal BW case: all approaches give same RP



Resonance Parameters (RP)

- BW mass, width, BR
- T-matrix pole and residue
- K-matrix pole and residue
- speed plot (T pole?)
- other parameterizations (time delay,...)

Extraction method: The Speed Plot (SP)

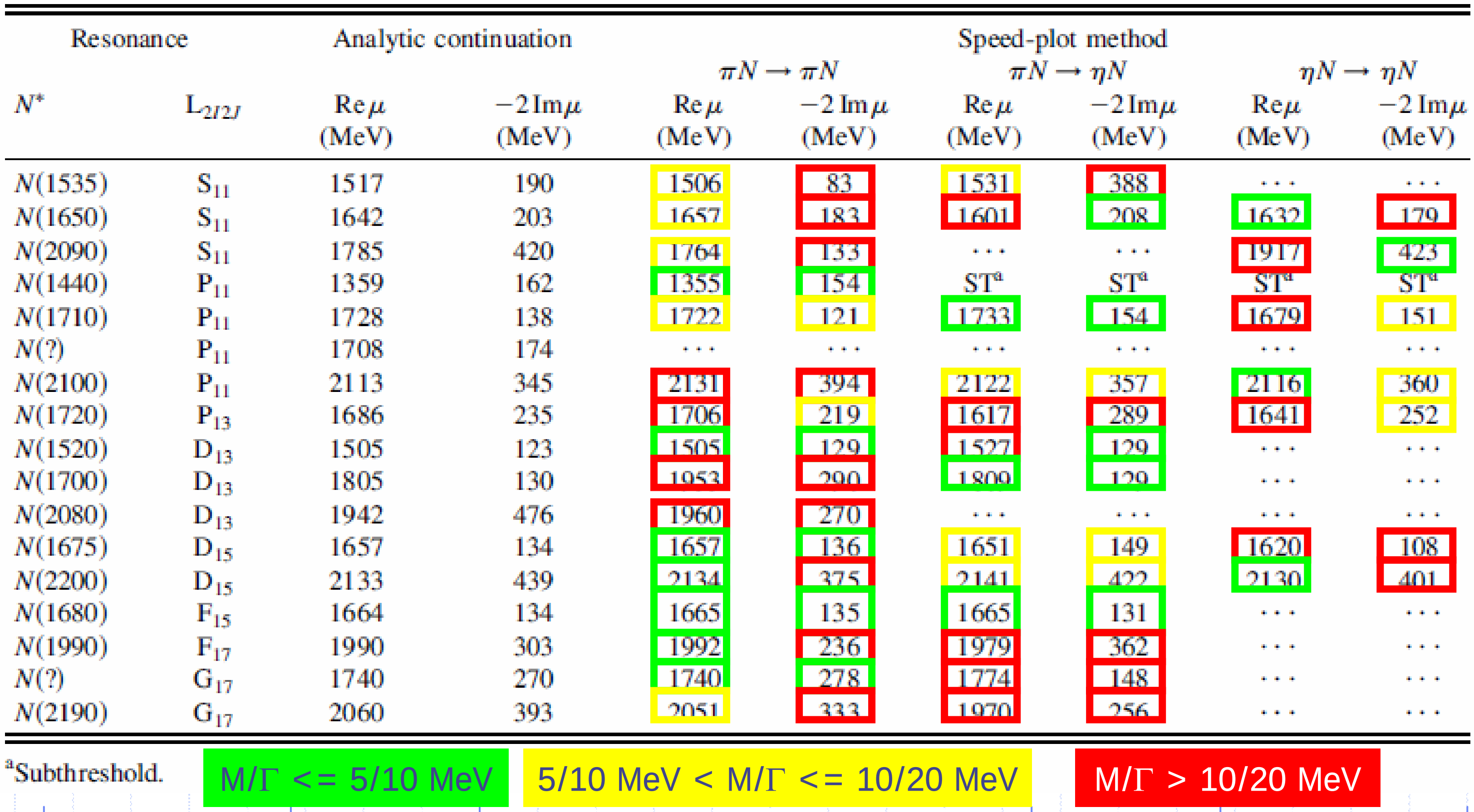
- T matrix (BW)
  - $T_R = R / (M - W - i\,\Gamma/2)$
  - $|T_R|^2 = |R|^2 / [(M - W)^2 + (\Gamma/2)^2]$
  - $T = T_R + T_B$

- **Speed Plot**
  - SP = |dT/dW|
  - (ignoring background contribution)
  - $|dT_R/dW| = |R| / [(M - W)^2 + (\Gamma/2)^2]$

- SP is assumed to be more than yet another extraction method:
  - SP corresponds to the time delay, the first derivative of phase shift over energy (in single channel case)
  - if TD is physical, SP must be physical too
  - no SP peak means there is no pole, meaning no resonance (and vice versa); the resonance existence criteria

Speed plot shows a problem

TABLE II. The N^* resonance pole parameters obtained by the analytic continuation method and speed plot in various channels. The $N(?)$ stands for resonances unnamed in the RPP.



Resonance		Analytic continuation		Speed-plot method					
				$\pi N \to \pi N$		$\pi N \to \eta N$		$\eta N \to \eta N$	
N^*	$L_{2I\,2J}$	Re μ (MeV)	-2 Im μ (MeV)	Re μ (MeV)	-2 Im μ (MeV)	Re μ (MeV)	-2 Im μ (MeV)	Re μ (MeV)	-2 Im μ (MeV)
$N(1535)$	S_{11}	1517	190	🟨 1506	🟥 83	🟨 1531	🟥 388	...	...
$N(1650)$	S_{11}	1642	203	🟨 1657	🟥 183	🟥 1601	🟩 208	🟩 1632	🟥 179
$N(2090)$	S_{11}	1785	420	🟨 1764	🟥 133	...	...	🟥 1917	🟩 423
$N(1440)$	P_{11}	1359	162	🟩 1355	🟩 154	ST^a	ST^a	ST^a	ST^a
$N(1710)$	P_{11}	1728	138	🟨 1722	🟨 121	🟩 1733	🟩 154	🟥 1679	🟨 151
$N(?)$	P_{11}	1708	174	...	...	...	...	...	...
$N(2100)$	P_{11}	2113	345	🟥 2131	🟥 394	🟨 2122	🟨 357	🟩 2116	🟨 360
$N(1720)$	P_{13}	1686	235	🟥 1706	🟨 219	🟥 1617	🟥 289	🟥 1641	🟨 252
$N(1520)$	D_{13}	1505	123	🟩 1505	🟩 129	🟥 1527	🟩 129	...	...
$N(1700)$	D_{13}	1805	130	🟥 1953	🟥 290	🟩 1809	🟩 129	...	...
$N(2080)$	D_{13}	1942	476	🟥 1960	🟥 270	...	...	...	...
$N(1675)$	D_{15}	1657	134	🟩 1657	🟩 136	🟨 1651	🟨 149	🟥 1620	🟥 108
$N(2200)$	D_{15}	2133	439	🟩 2134	🟥 375	🟨 2141	🟨 422	🟩 2130	🟥 401
$N(1680)$	F_{15}	1664	134	🟩 1665	🟩 135	🟩 1665	🟩 131	...	...
$N(1990)$	F_{17}	1990	303	🟩 1992	🟥 236	🟥 1979	🟥 362	...	...
$N(?)$	G_{17}	1740	270	🟩 1740	🟩 278	🟥 1774	🟥 148	...	...
$N(2190)$	G_{17}	2060	393	🟨 2051	🟥 333	🟥 1970	🟥 256	...	...

^{a}Subthreshold.

🟩 M/Γ <= 5/10 MeV

🟨 5/10 MeV < M/Γ <= 10/20 MeV

🟥 M/Γ > 10/20 MeV

Regularization Method (RM): T matrix has pole

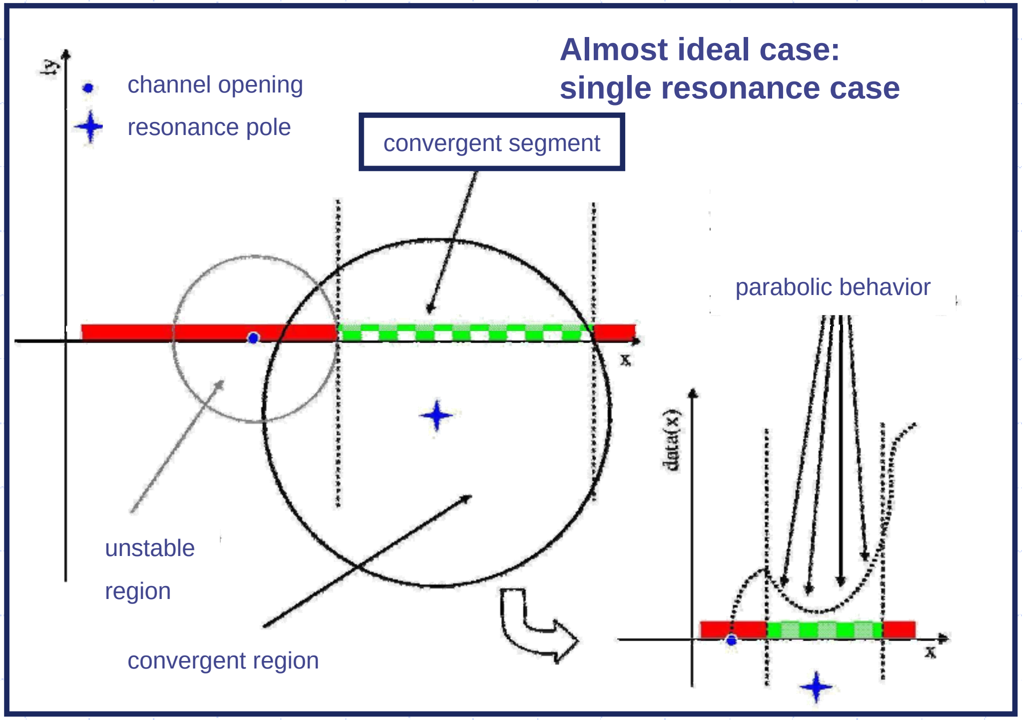
$$f(z) = (\mu - z)\, t(z)$$

$$f(\mu) = \sum_{n=0}^{N} \frac{f^{(n)}(x)}{n!}(\mu - x)^n + R_N(x,\mu).$$

$$f^{(n)}(x) = (\mu - x)\, t^{(n)}(x) - n\, t^{(n-1)}(x),$$

$$|f(\mu)| = \frac{\left|t^{(N)}(x)\right|}{N!}\, |a + i\, b - x|^{(N+1)}.$$

$$\boxed{\frac{(a-x)^2 + b^2}{\sqrt[N+1]{|f(\mu)|^2}} = \sqrt[N+1]{\frac{(N!)^2}{\left|t^{(N)}(x)\right|^2}}.}$$



Almost ideal case:
single resonance case

● channel opening

✦ resonance pole

convergent segment

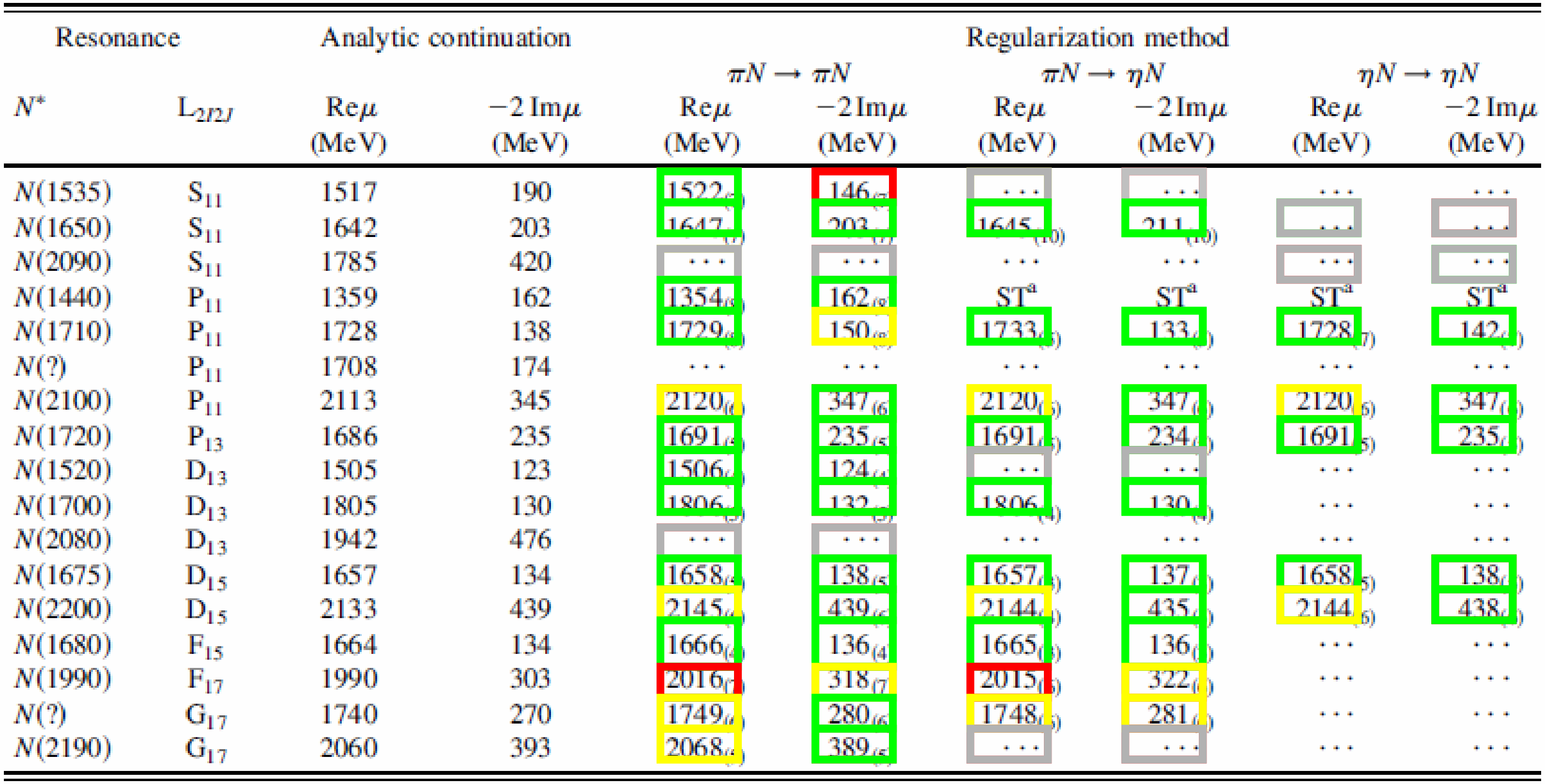
parabolic behavior

unstable region

convergent region

Regularization method:
The Results

TABLE III. The comparison of N^* resonance pole parameters obtained by the analytic continuation method, and the regularization method for πN, $\eta N \to \eta N$, and $\pi N \to \eta N$ processes. Numbers in subscript are the expansion order required to obtain convergent result.



Resonance		Analytic continuation		Regularization method					
				$\pi N \to \pi N$		$\pi N \to \eta N$		$\eta N \to \eta N$	
N^*	L_{2I2J}	Reμ (MeV)	$-2\,$Imμ (MeV)	Reμ (MeV)	$-2\,$Imμ (MeV)	Reμ (MeV)	$-2\,$Imμ (MeV)	Reμ (MeV)	$-2\,$Imμ (MeV)
$N(1535)$	S_{11}	1517	190	1522	146	⋯	⋯	⋯	⋯
$N(1650)$	S_{11}	1642	203	1647	203	$1645_{(10)}$	$211_{(10)}$	⋯	⋯
$N(2090)$	S_{11}	1785	420	⋯	⋯	⋯	⋯	⋯	⋯
$N(1440)$	P_{11}	1359	162	1354	162	ST[a]	ST[a]	ST[a]	ST[a]
$N(1710)$	P_{11}	1728	138	1729	150	1733	133	1728	142
$N(?)$	P_{11}	1708	174	⋯	⋯	⋯	⋯	⋯	⋯
$N(2100)$	P_{11}	2113	345	2120	347	2120	347	2120	347
$N(1720)$	P_{13}	1686	235	1691	235	1691	234	1691	235
$N(1520)$	D_{13}	1505	123	1506	124	⋯	⋯	⋯	⋯
$N(1700)$	D_{13}	1805	130	1806	132	1806	130	⋯	⋯
$N(2080)$	D_{13}	1942	476	⋯	⋯	⋯	⋯	⋯	⋯
$N(1675)$	D_{15}	1657	134	1658	138	1657	137	1658	138
$N(2200)$	D_{15}	2133	439	2145	439	2144	435	2144	438
$N(1680)$	F_{15}	1664	134	1666	136	1665	136	⋯	⋯
$N(1990)$	F_{17}	1990	303	2016	318	2015	322	⋯	⋯
$N(?)$	G_{17}	1740	270	1749	280	1748	281	⋯	⋯
$N(2190)$	G_{17}	2060	393	2068	389	⋯	⋯	⋯	⋯

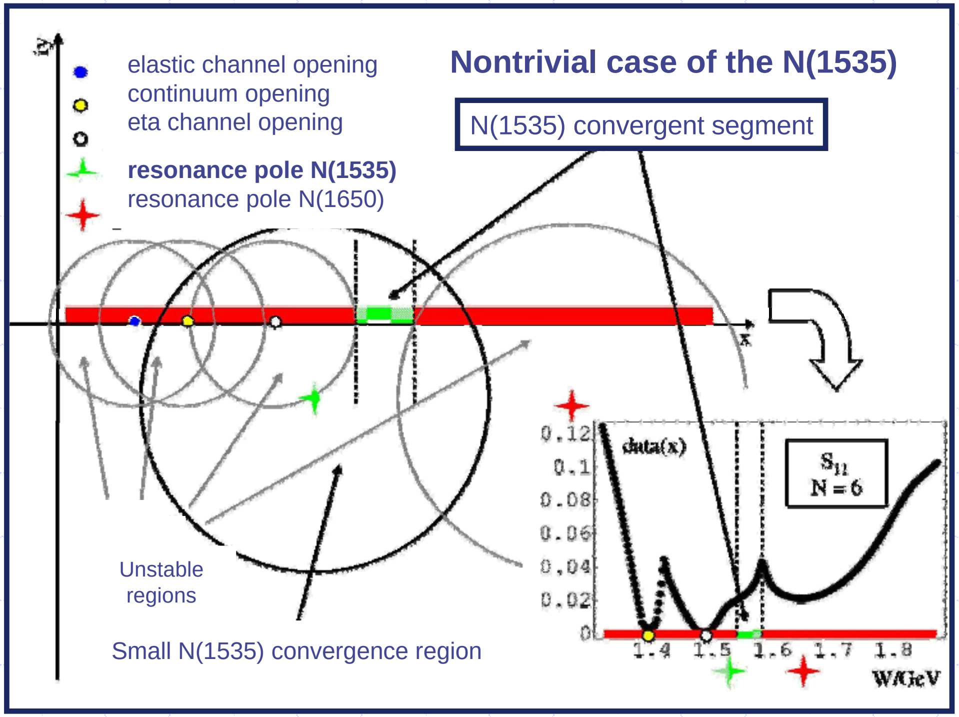
[a] Subthreshold.

🟩 M/Γ <= 5/10 MeV

🟨 5/10 MeV < M/Γ <= 10/20 MeV

🟥 M/Γ > 10/20 MeV

What about the small SP problem?



Nontrivial case of the N(1535)

- ● elastic channel opening
- ○ continuum opening
- ○ eta channel opening
- ✦ **resonance pole N(1535)**
- ✦ resonance pole N(1650)

N(1535) convergent segment

Unstable regions

Small N(1535) convergence region

Connection between RM and SP

$$\frac{(a-x)^2+b^2}{\sqrt[N+1]{|f(\mu)|^2}} = \sqrt[N+1]{\frac{(N!)^2}{|t^{(N)}(x)|^2}}.$$

N = 1

$|dt\backslash dx| = |r| / [(a-x)^2+b^2]$

Speed plot

$|dT_R/dW| = |R| / [(M - W)^2 + (\Gamma/2)^2]$

$$\frac{(a-x)^2+b^2}{\sqrt[N+1]{|f(\mu)|^2}} = \sqrt[N+1]{\frac{(N!)^2}{|t^{(N)}(x)|^2}}.$$

N = 0

$|t|^2 = |r|^2 / [(a-x)^2+b^2]$

Breit-Wigner

$|T_R|^2 = |R|^2 / [(M - W)^2 + (\Gamma/2)^2]$

Summary

- RM produces consistent pole positions (unlike SP)
- it can easily be applied to other PWA results (yellow marked)
- RM clearly shows resonance signal (parabolic shapes, cusps between resonances, ...)
- having exact and simple form enables us to build extensions and enhancements to it (the same goes for SP and BW)
- close future:
  - solution of the small SP (and RM) problem
  - finding poles from other PW analyses
  - analyzing KH80 SP poles

Q&A: 1 Invariant amplitude from the PW T matrices

The $\pi N \to \eta N$ process is given by the invariant amplitude

$A(W, \cos\theta^*) + \not{q}_\eta B(W, \cos\theta^*)$

with the standard on-shell partial waves decomposition of A and B:

$$
\begin{aligned}
A(W, \cos\theta^*) = \frac{4\pi}{\sqrt{q_\pi^{*3} q_\eta^{*3}}} \Bigg\{ & \sum_{l=0}^{\infty} T_{l+} \Big[\sqrt{(E_i^* + m)(E_f^* + m)}(W - m) P_l'(\cos\theta^*) \\
& + \sqrt{(E_i^* - m)(E_f^* - m)}(W + m) P_{l+1}'(\cos\theta^*) \Big] \\
& - \sum_{l=1}^{\infty} T_{l-} \Big[\sqrt{(E_i^* + m)(E_f^* + m)}(W - m) P_{l+1}'(\cos\theta^*) \\
& + \sqrt{(E_i^* - m)(E_f^* - m)}(W + m) P_l'(\cos\theta^*) \Big] \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
B(W, \cos\theta^*) = \frac{4\pi}{\sqrt{q_\pi^{*3} q_\eta^{*3}}} \Bigg\{ & - \sum_{l=0}^{\infty} T_{l+} \Big[\sqrt{(E_i^* + m)(E_f^* + m)} P_l'(\cos\theta^*) \\
& - \sqrt{(E_i^* - m)(E_f^* - m)} P_{l+1}'(\cos\theta^*) \Big] \\
& + \sum_{l=1}^{\infty} T_{l-} \Big[\sqrt{(E_i^* + m)(E_f^* + m)} P_{l+1}'(\cos\theta^*) \\
& - \sqrt{(E_i^* - m)(E_f^* - m)} P_l'(\cos\theta^*) \Big] \Bigg\}.
\end{aligned}
$$

W is the total c.m. energy,

θ^* is the c.m. scattering angle,

q_π^* and q_η^* are the initial pion and final η c.m. momenta,

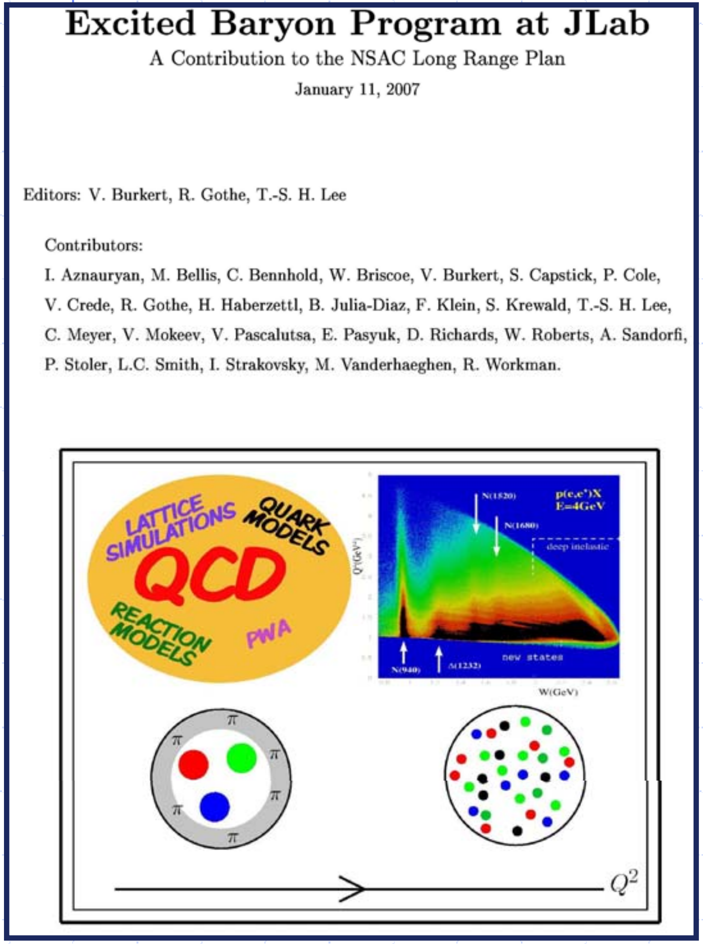
E_i^* and E_f^* are the initial and final nucleon c.m. energies,

$P_l'(z)$ are derivatives of Legendre polynomials,

$T_{l\pm}$ are the $\pi N \to \eta N$ T-matrices,

and m is the nucleon mass.

Q&A: 2
Who (else) needs the speed plot?



Excited Baryon Program at JLab

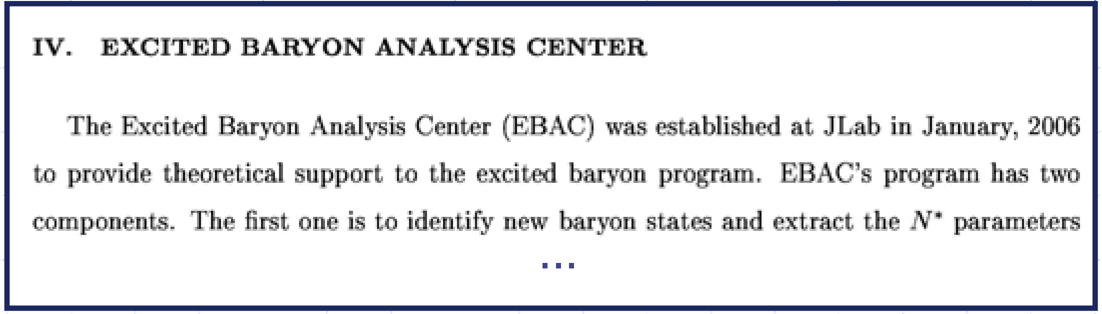
A Contribution to the NSAC Long Range Plan

January 11, 2007

Editors: V. Burkert, R. Gothe, T.-S. H. Lee

Contributors:

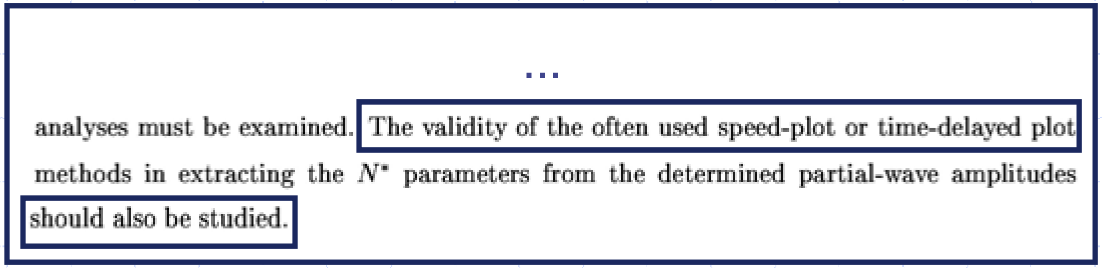
I. Aznauryan, M. Bellis, C. Bennhold, W. Briscoe, V. Burkert, S. Capstick, P. Cole, V. Crede, R. Gothe, H. Haberzettl, B. Julia-Diaz, F. Klein, S. Krewald, T.-S. H. Lee, C. Meyer, V. Mokeev, V. Pascalutsa, E. Pasyuk, D. Richards, W. Roberts, A. Sandorfi, P. Stoler, L.C. Smith, I. Strakovsky, M. Vanderhaeghen, R. Workman.



IV. EXCITED BARYON ANALYSIS CENTER

The Excited Baryon Analysis Center (EBAC) was established at JLab in January, 2006 to provide theoretical support to the excited baryon program. EBAC's program has two components. The first one is to identify new baryon states and extract the N^* parameters

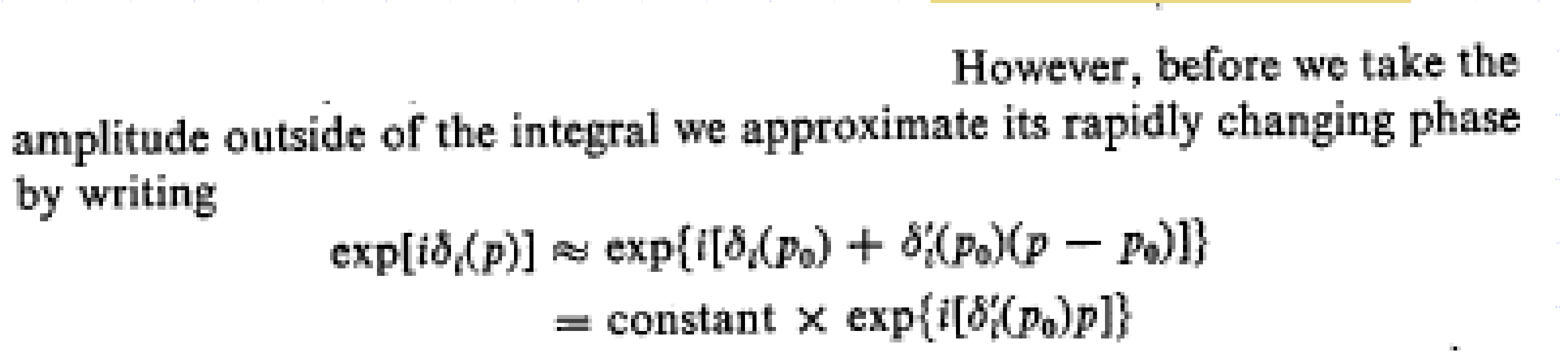
…



…

analyses must be examined. The validity of the often used speed-plot or time-delayed plot methods in extracting the N^* parameters from the determined partial-wave amplitudes should also be studied.

Q&A: 3
Time Delay and physicality of SP?



However, before we take the amplitude outside of the integral we approximate its rapidly changing phase by writing

$$\exp[i\delta_\ell(p)] \approx \exp\{i[\delta_\ell(p_0) + \delta'_\ell(p_0)(p - p_0)]\}$$
$$= \text{constant} \times \exp\{i[\delta'_\ell(p_0)p]\}$$

- J.R. Taylor: “Scattering theory”

- Just <u>the first term</u> in Taylor expansion (again)
- So much for the physicality …