How resonances synchronise with sharp thresholds

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Examples	(MeV)
$f_0(980)$ and $a_0(980)$ -> KK	991
$f_2(1565)$ -> $\omega\omega$	1566
X(3872) -> D(1865)D*(2007)	3872
Y(4660) -> ψ'(3686)$f_0(980)$	4666
$\Lambda_c(2940)$ -> D*(2007)N	2945
$P_{11}(1710)$, $P_{13}(1720)$ ->ωN	1720
$K_0(1430)$ -> Kη' ?	1453
$K_1(1420)$ -> KK*	1388

Simple explanation:

phase space

$$D(s) = M^2 - s - \Sigma_i\ \Pi_i(s)$$

$$\mathrm{Im}\,\Pi_i = g_i^2\,\rho_i(s) FF_i(s)$$

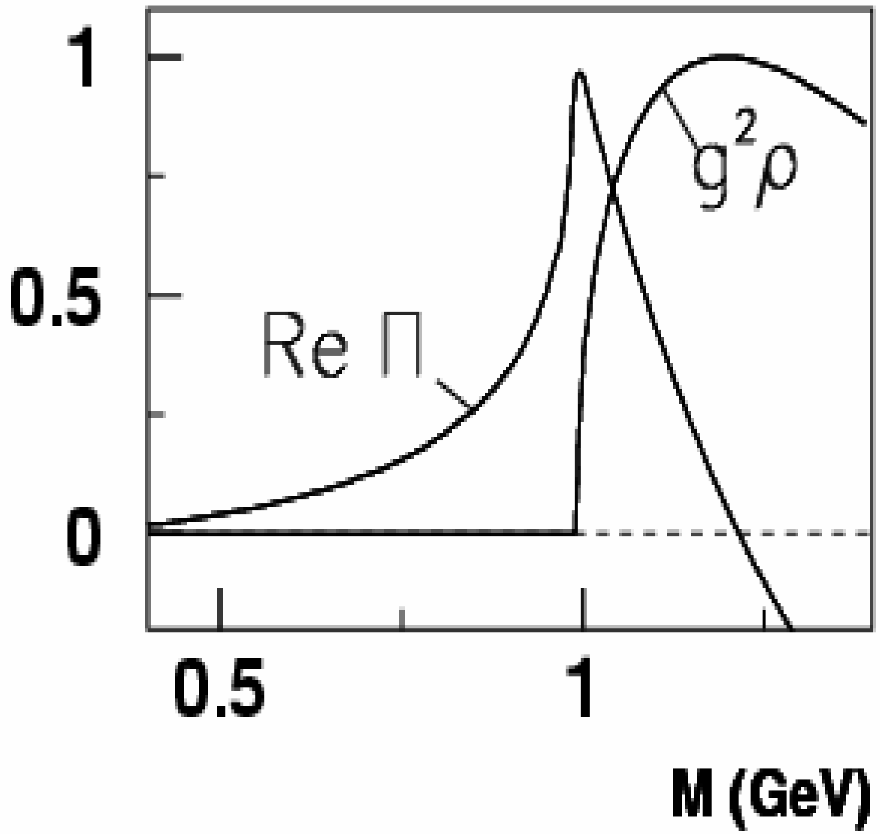
$$\mathrm{Re}\,\Pi_i = \frac{1}{\pi} P \int_{thr_i} ds' \frac{\mathrm{Im}\,\Pi_i(s')}{(s' - s)}$$

(Im Π_i arises from the pole at s = s');

At threshold, Re Π is positive definite.

Form factor needed to make integral converge

f0(980) -> KK as an example

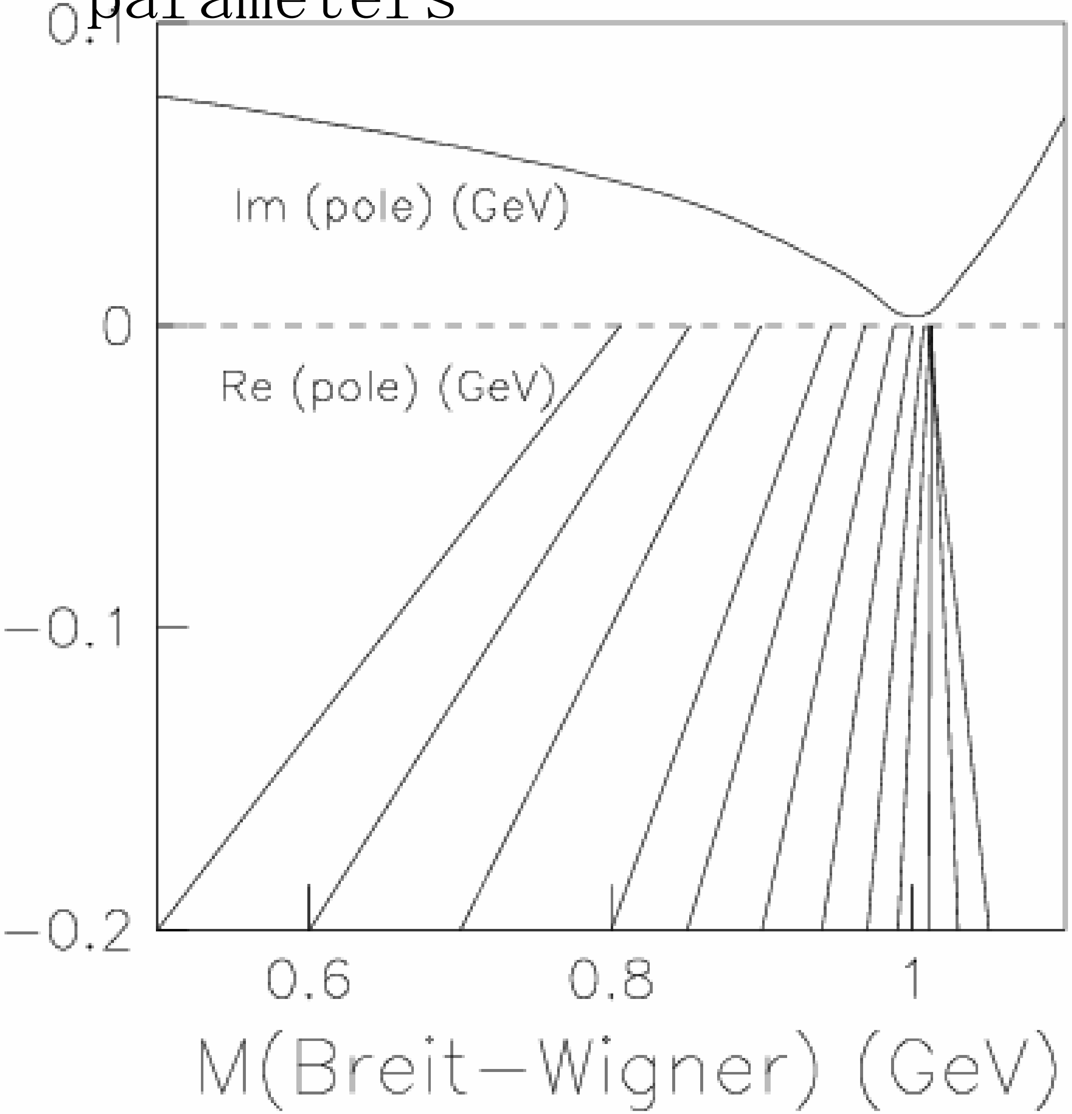


FF = exp(-3k^2)

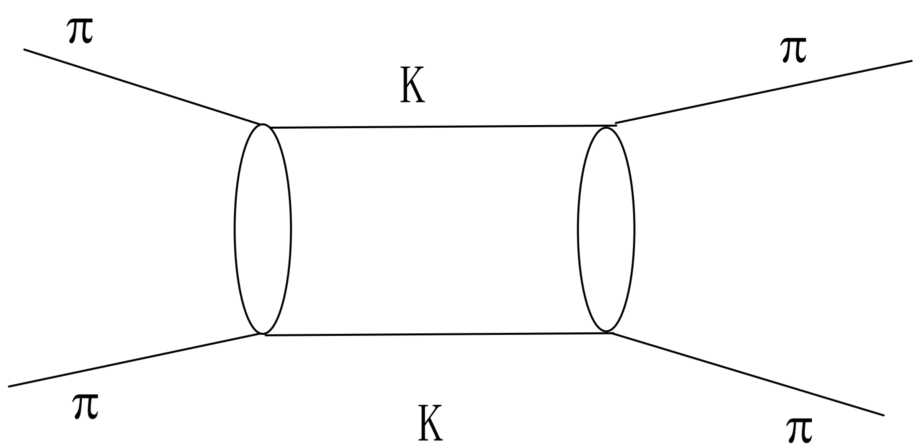
(R=0.8 fm)

Zero-point energy also helps attract the

Illustration with $f_0(980)$ parameters



Incidentally, the dispersive term Re Π is equivalent to the loop diagram for producing the open channel:



A bit more algebra:

$$\mathrm{Re}\ D(s) = M^2 - s + g^2\ j$$

Above threshold, $\pi\ j = -2\rho^2 + \ldots,\quad \rho = 2k/s^{1/2}$

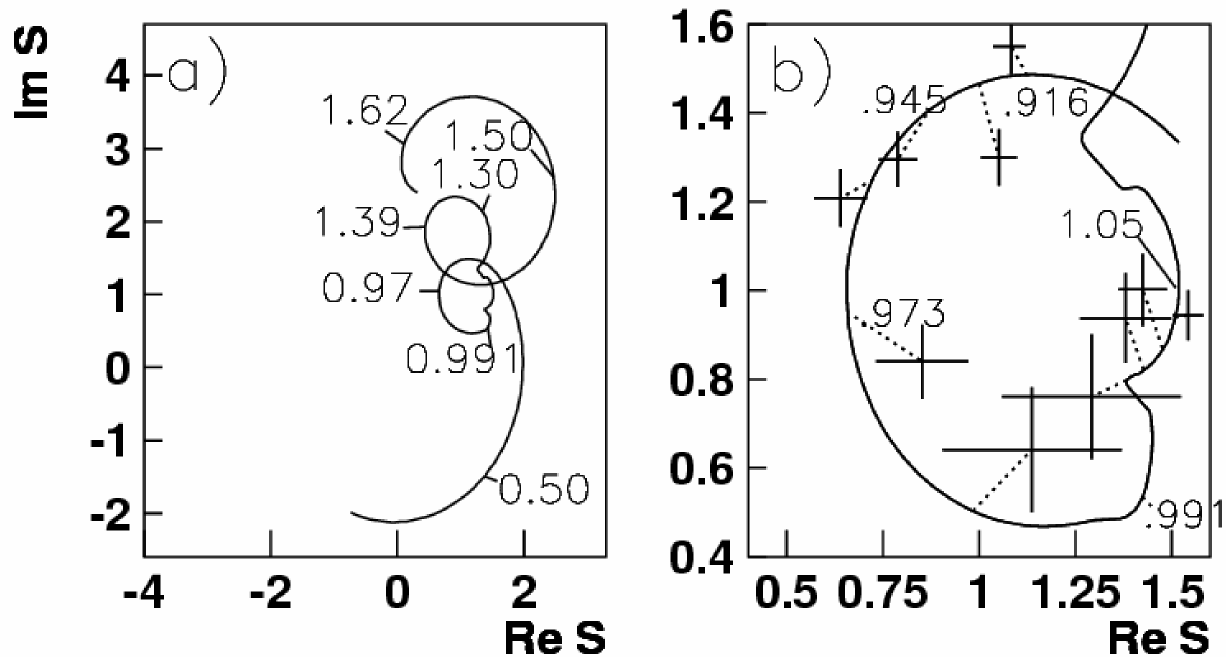
Below, $\pi\ j = \pi[(4m^2_K - s)/s]^{1/2} - 2v^2 + \ldots\ v = 2|k|/s^{1/2}$

$$= \pi[(4m^2_K - s)/s]^{1/2} - 2(4m^2_K - s)/s + \ldots$$

(Flatte term)

i.e. the cusp contributes like a resonant term with respect to the $K\bar{K}$ threshold; g^2_K and M(res) are very

Is f0(980) actually resonant? Answer: Yes, but



for the $\bar{p}p \to 3\pi^0$ at rest

Likewise $\bar{p}p \to \eta\pi^0\pi^0$ data require resonant

phase variation for $a_0(980)$

Tornqvist gives a formula for the KK components:

$$\Psi = \frac{|q\bar{q}q\bar{q}> + [(d/ds)\,\mathrm{Re}\,\Pi(s)]^{1/2}\,|KK>}{1 + (d/ds)\,\mathrm{Re}\,\Pi(s)}$$

For $f_0(980)$, KK intensity > 60%

For $a_0(980)$, > 35%.

Note that $f_0(980)$ has $g^2_{\pi\pi}$ = 165 MeV, g^2_{KK}= 695 MeV and $a_0(980)$ has $g^2_{\eta\pi}$=221 MeV, g^2_{KK}=256. Form factors are required to cut off their high mass tails.

What are σ, κ, $a_0(980)$ and $f_0(980)$?

(i) Leutwyler et al fit $\pi\pi$ elastic scattering and find the

σ pole using the Roy equations; these have left-hand

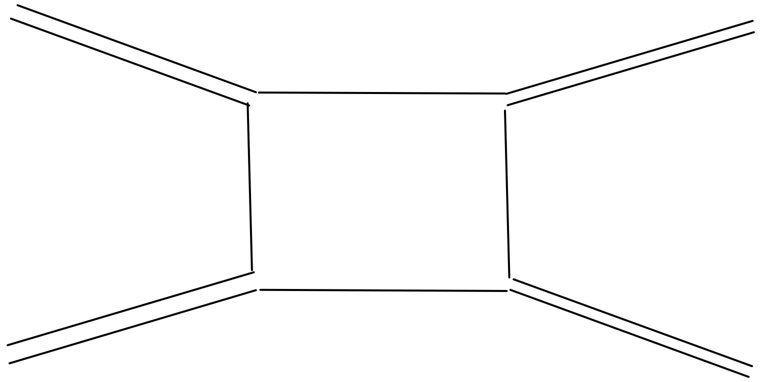
cuts which account for meson exchanges, mostly ρ.

Moussallam et al fit $K\pi$ elastic scattering likewise and

determine the κ pole. The success of the calculations

suggests σ and κ are due to meson exchanges.

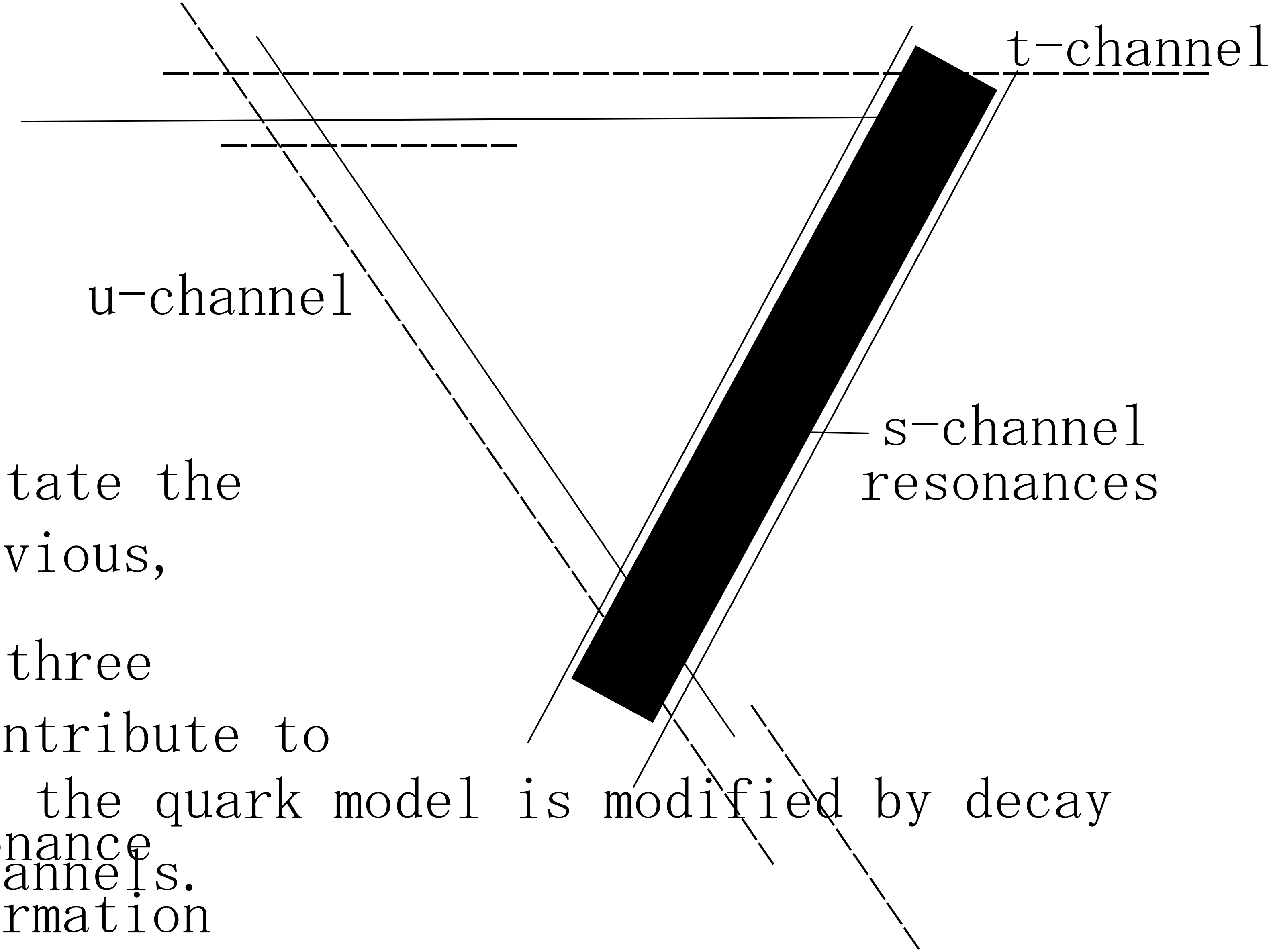
Rupp, van Beveren and I have modelled all 4 states with a short-range confining potential coupled at r~0.65 fm to outgoing waves. Adler zeros are included in all cases. This successfully fits data for all four states with a universal coupling constant, except for SU3 coefficients, confirming they make a nonet. [Phys.Lett. 92 (2006) 265]



An interesting point emerges from this model. The $a_0(980)$ is <u>not</u> attracted to the $\eta\pi$ threshold because of the nearby Adler zero at $s = m^2_\eta - m^2_\pi/2$; the Adler zero in KK is at $s = m^2_K/2$, far removed from the KK threshold, so there is no inhibition of the attraction to the KK threshold.

BUT of course meson exchanges do not account for all states, e.g. the $\rho(770)$.

In the Mandelstam diagram, there are:



To state the obvious,

all three contribute to resonance formation

i.e. the quark model is modified by decay channels.

Oset, Oller et al find they can generate many states from meson exchanges (including Adler zeros). Hamilton and Donnachie found in 1965 that meson exchanges have the right signs to generate P_{33}, D_{13}, D_{15} and F_{15} baryons. Suppose contributions to the Hamiltionian are H_{11} and H_{22}; the eigenvalue equation is

$$\begin{pmatrix} H_{11} & V \\ V & H_{22} \end{pmatrix} \Psi = E\,\Psi$$

The Variational Principle ensures the minimum E is the Eigenstate. Most non $q\bar{q}$ states are pushed up and become too broad to observe. There is an analogy to the covalent bond in chemistry

The X(3872) cannot be fitted as a pure cusp - it is too

broad. It can be fitted either as a resonance (Braaten

et al) or as a virtual state (Hanhart et al). With my

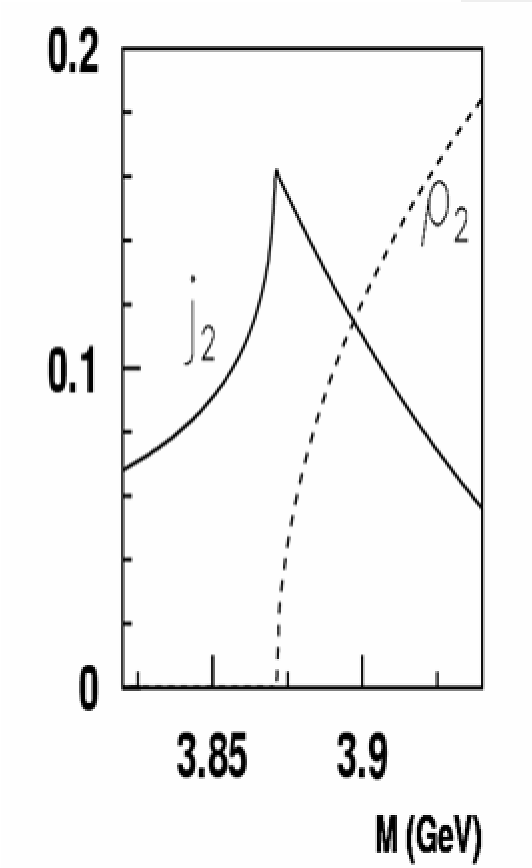
formula including the threshold cusp, I can fit it either

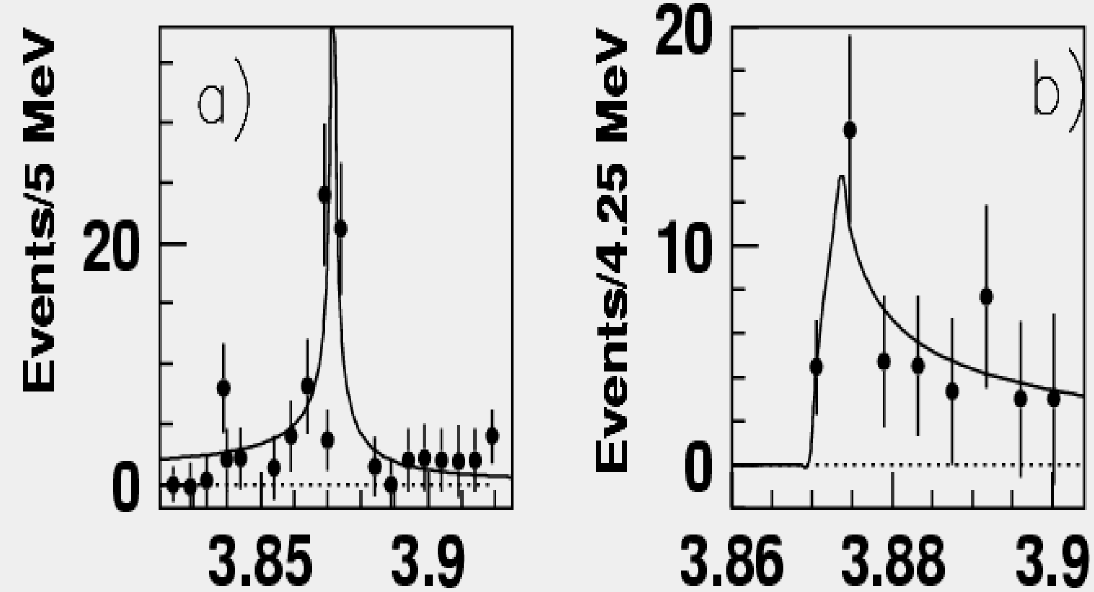
way. It could be a regular cc state captured by the D D*

threshold. [It is important to find the quantum numbers

of the X(3942) reported by Belle in D D*. If it

X(3872)



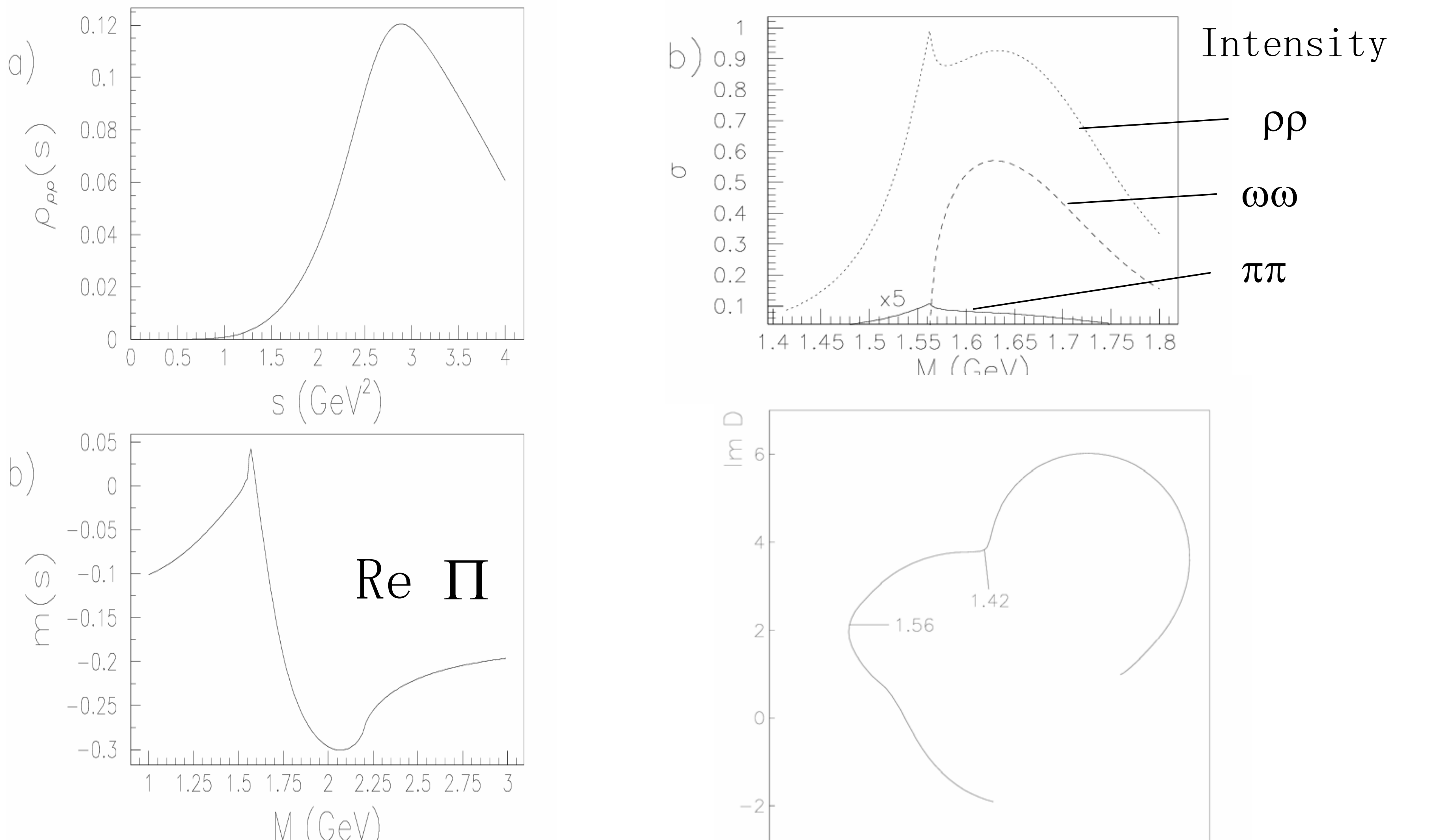


J/ψ
ρ

D D*

Pure cusp too wide

$f_2(1565)$ in $\pi\pi, \omega\omega$ and $\rho\rho$



Cusp due to $\pi\pi \rightarrow \omega\omega$ via ρ exchange and $\rho\rho \rightarrow \omega\omega$ via π exchange. Data on $\pi\pi \rightarrow 4\pi$ badly needed!

Other examples

1) $\eta(1405)$ and $\eta(1475)$ are probably the same object.

The latter is almost purely KK*(890), with L=1 decays,

hence phase space rising as k^3 from threshold, 1392

MeV. This phase space forces the KK* peak up in mass

by ~50 MeV. BES I fitted both with a single $\eta(1425)$.

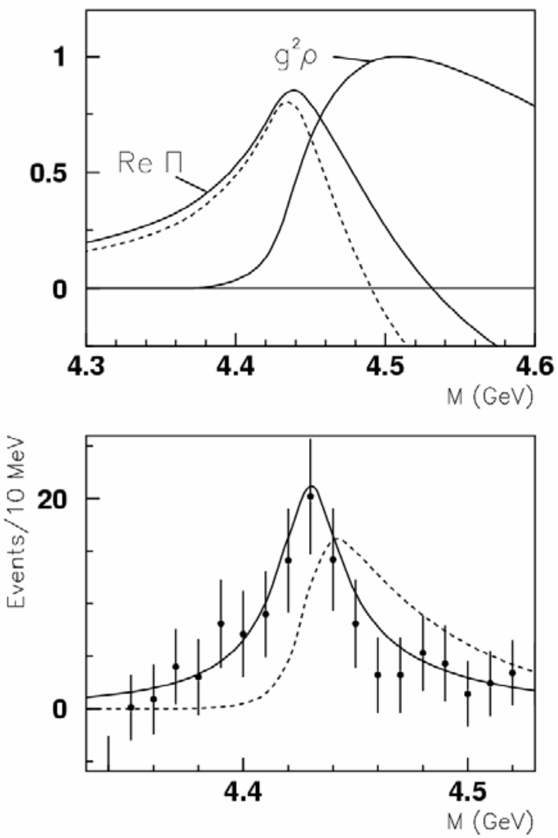
BES II data needed.

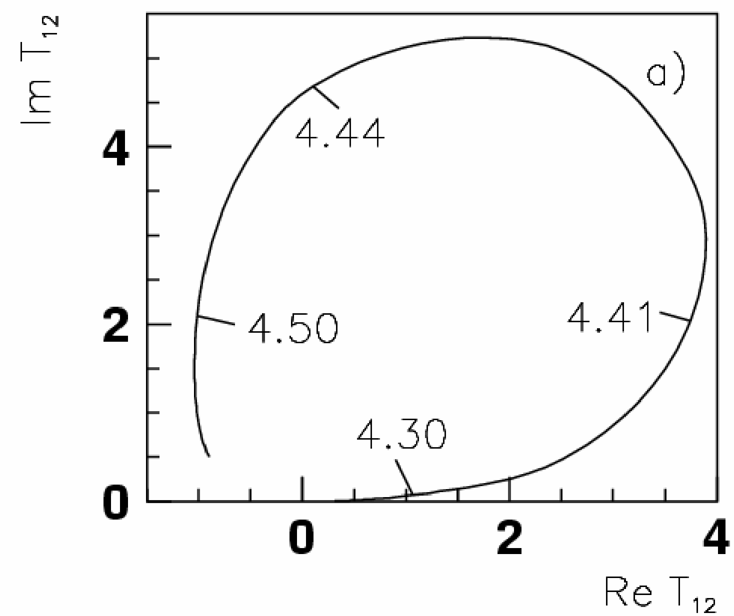
Z(4430) -> ψ' π+ at D*(2010)D1(2420) threshold

Many de-excitations modes, e.g. DD* or D*D*

Is it a resonance or just a threshold cusp?

Fit with $D(s) = M^2 - \mathrm{Re}\,\Pi(s) - ig^2\,\rho(D^*D_1)$





The π_1 (1600) looks a reasonable candidate for an exotic $J^{PC} = 1^{-+}$ hybrid.

There is also a claim for a π_1(1405). It may be just a threshold cusp due to the thresholds b_1(1235)π and f_1(1235)π. Needs fitting to this possibility.

The di-baryons of the 1980s are threshold cusps, e.g. in $N\Delta$ and $K^+\Delta$.

News on $a_0(1450)$
(preliminary)

I have just finished refitting $\bar{p}p \to \eta\pi^0\pi^0$ at rest using the

full dispersive forms for its decays to $\rho\omega$ and $a_0(980)\sigma$.

The fit to $a_0(1450)$ improves substantially, the fitted

signal increases (due to better line-shape) and becomes

much more stable. Central mass -> 1446 +- 8(stat) +-

16(syst) MeV for both $\eta\pi\pi$ and $\rho\omega\pi^0$; $KK\pi$ remains to be

Conclusions

1) Dispersive effects due to rapidly opening thresholds

are important in the mass range 1–1.7 GeV.

2) At sharp thresholds, the cusp in the real part of the

amplitude can attract resonances over a mass range

of at least 100 MeV. Zero point energy also helps to

stabilise resonances at thresholds.

3) More work is needed allowing for these dispersive