Meson spectroscopy and properties in a Dyson-Schwinger approach

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Work with:

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Work performed at/supported by/in collaboration with:

Austrian Research Foundation

Argonne National Laboratory

University of Graz

Outline

- QCD and hadrons
- DSE-BSE
- Symmetries ↔ exact results
- Mesons and their properties
- Example (sophisticated) Ansatz
- Example results
- Conclusions and outlook

Motivation

$$\frac{R_E}{D_{GK}}$$

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$$= 10.2$$

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$$= 10.2$$

$$= \frac{5\pi}{N_C}\, e^{f_\pi/m_\pi}$$

DSEs

- Dyson Schwinger Equations:
  a modern method in relativistic QFT

  P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12** (2003) 297
  R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281
  C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. 45 (2000) S1
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- Study hadrons as composites of quarks and gluons . . .
- . . . including:
  - Chiral symmetry and $D\chi SB$
  - correct perturbative limit (via $\alpha_p(Q^2)$)
  - quark and gluon confinement
  - Poincaré covariance
- Propagators and Bethe-Salpeter amplitudes $\rightarrow$ observables

DSEs

- Solutions: Schwinger functions
  (Euclidean Green functions)
  (also calculated on the lattice)

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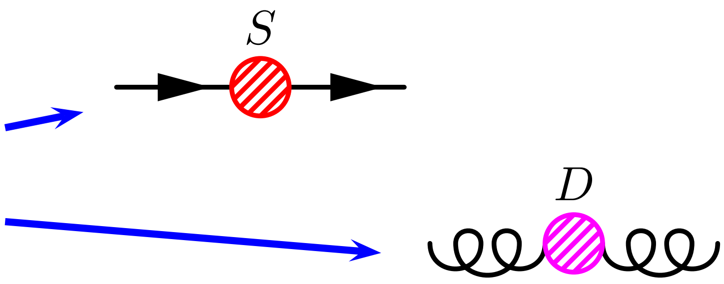
- Each function satisfies integral equation involving other functions ⇒
- Infinite set of coupled integral equations
- Truncation scheme necessary ⇒
- Generating tool for perturbation theory

DSEs

- Solutions: Schwinger functions
  (Euclidean Green functions)
  (also calculated on the lattice)
- Each function satisfies integral equation involving other functions $\Rightarrow$
- Infinite set of coupled integral equations
- Truncation scheme necessary $\Rightarrow$
- Nonperturbative truncation scheme
- Respect symmetries
- Prove exact (model independent) results
- Devise (sophisticated) models to illustrate them

Building Blocks

- 2-point functions:
    - quark propagator
    - gluon propagator



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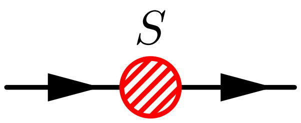
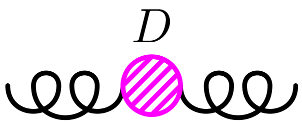
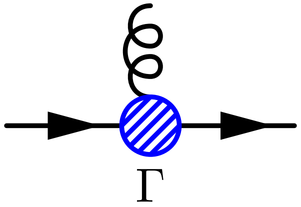
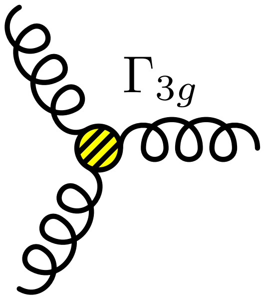




- 3-point functions:
  - quark-gluon vertex
  - three-gluon vertex . . .





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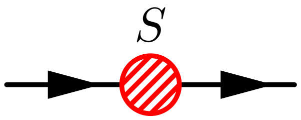

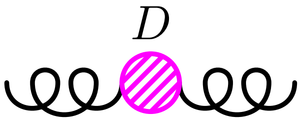

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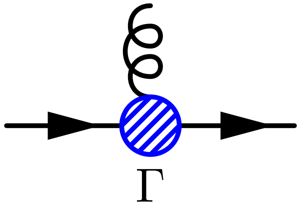
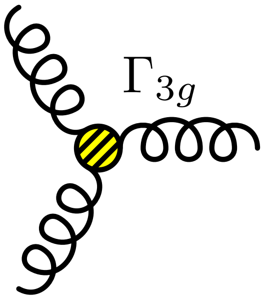
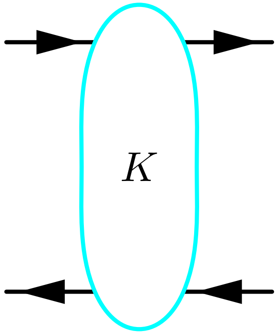
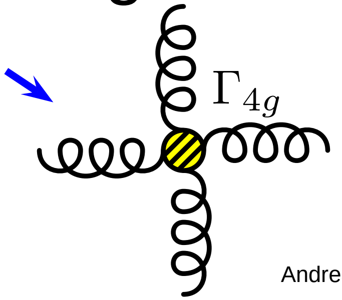




- 4-point functions:
  - quark-antiquark scattering kernel
  - four-gluon vertex . . .





Motivation (ctd.)

- Pion is very light →
  made out of (heavy) quark constituents?
- Current quarks in perturbative region
- Constituent quarks at low energies

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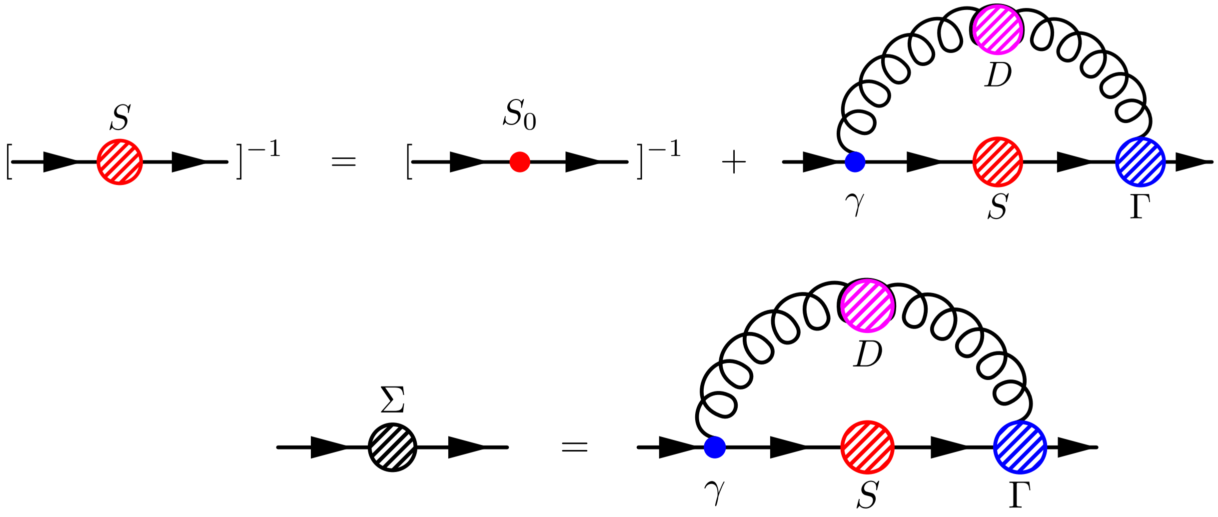
  made out of (heavy) quark constituents?
- Current quarks in perturbative region
- Constituent quarks at low energies
- Need to . . .
  - have a well-defined chiral limit
  - understand dynamical chiral symmetry breaking
  - understand the relation between
    current and constituent quarks
  - have a covariant framework

Gap Equation

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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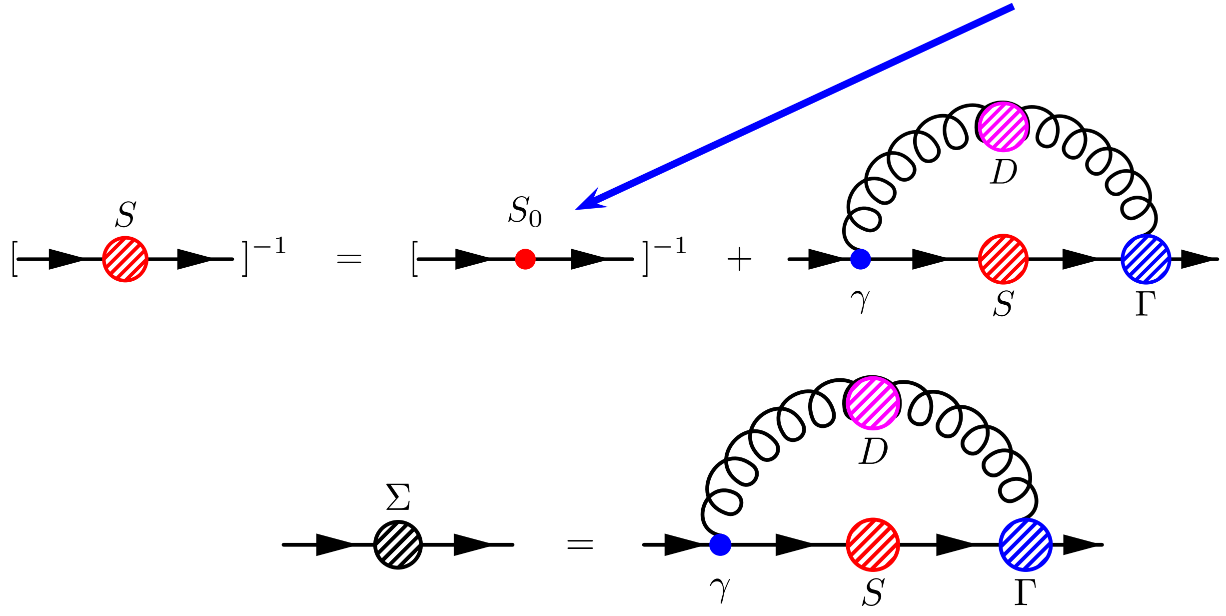
current quark mass m_ζ



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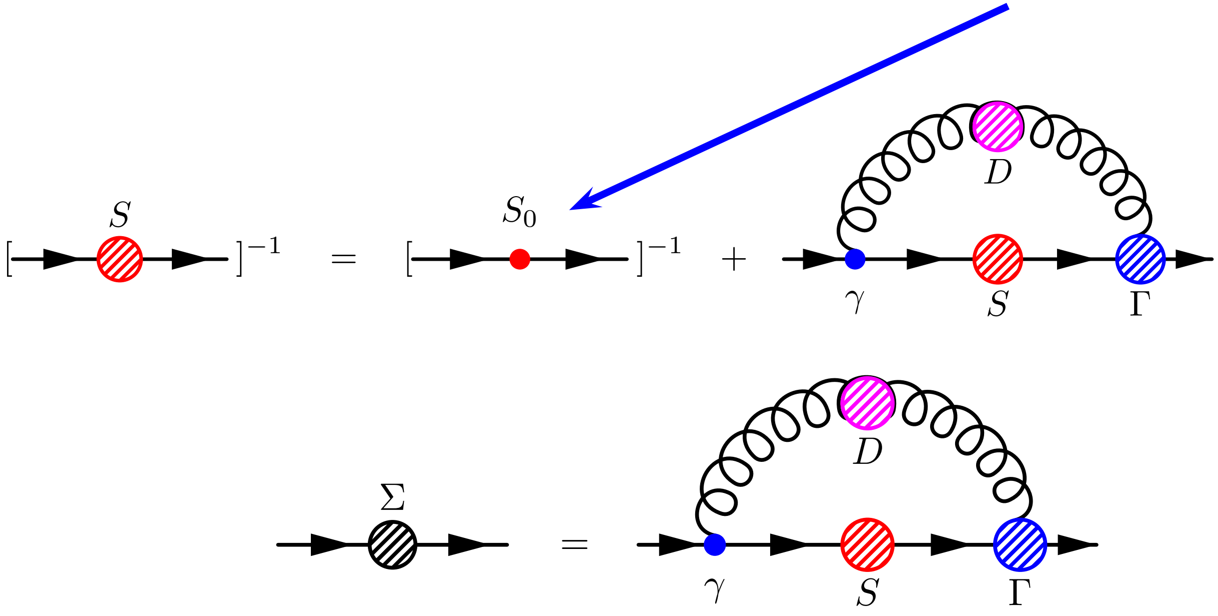

- Weak coupling expansion reproduces every diagram in perturbation theory, but:
- Perturbation theory: $m_\zeta = 0 \;\Rightarrow\; M(p^2) \equiv 0$

Quark Mass Function

Solution of gap equation: $S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)

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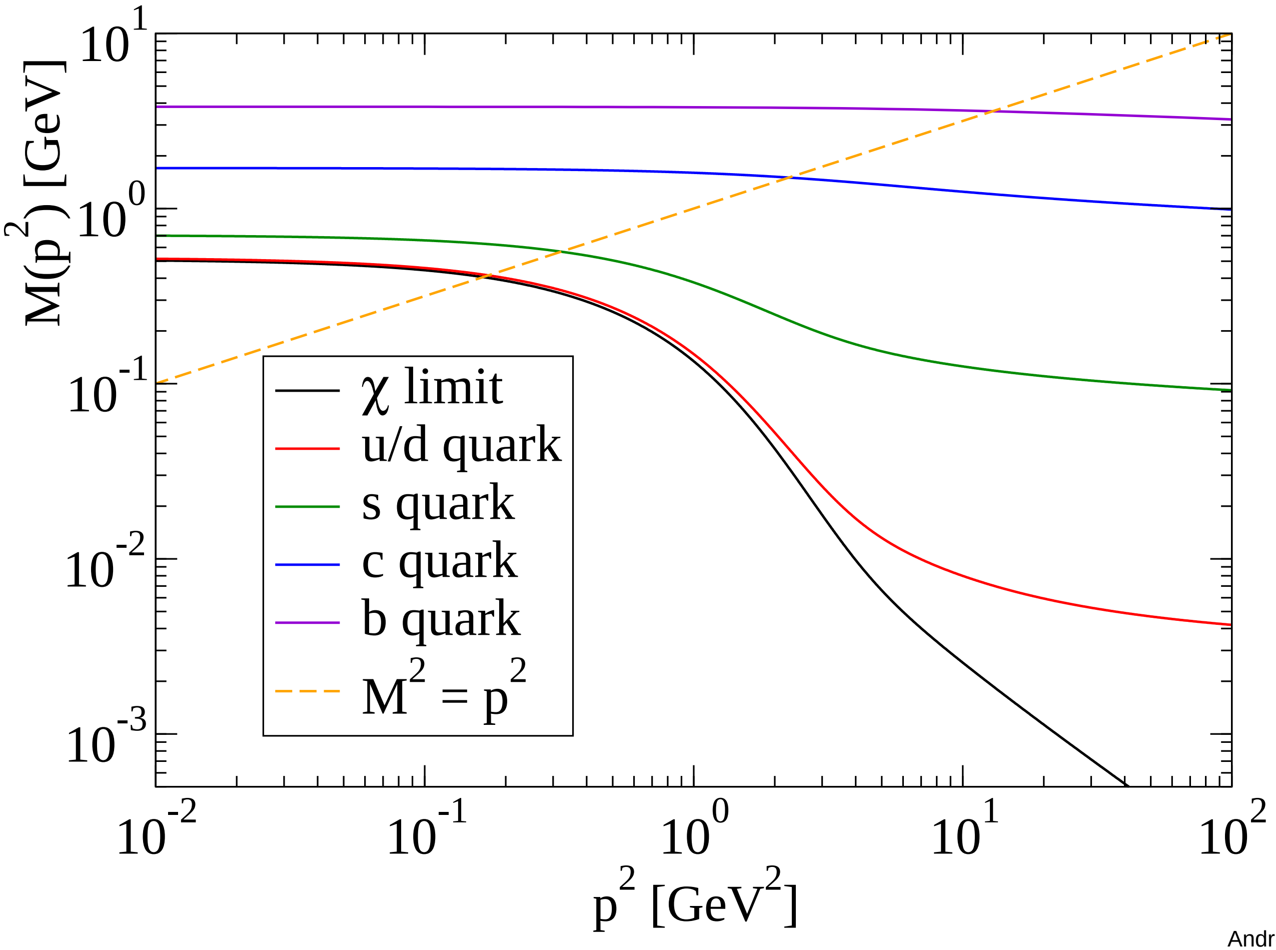

Quark Mass Function

Solution of gap equation: $\color{red}{S(p)} = \dfrac{Z(p^2)}{i\gamma\cdot p + \color{green}{M(p^2)}}$

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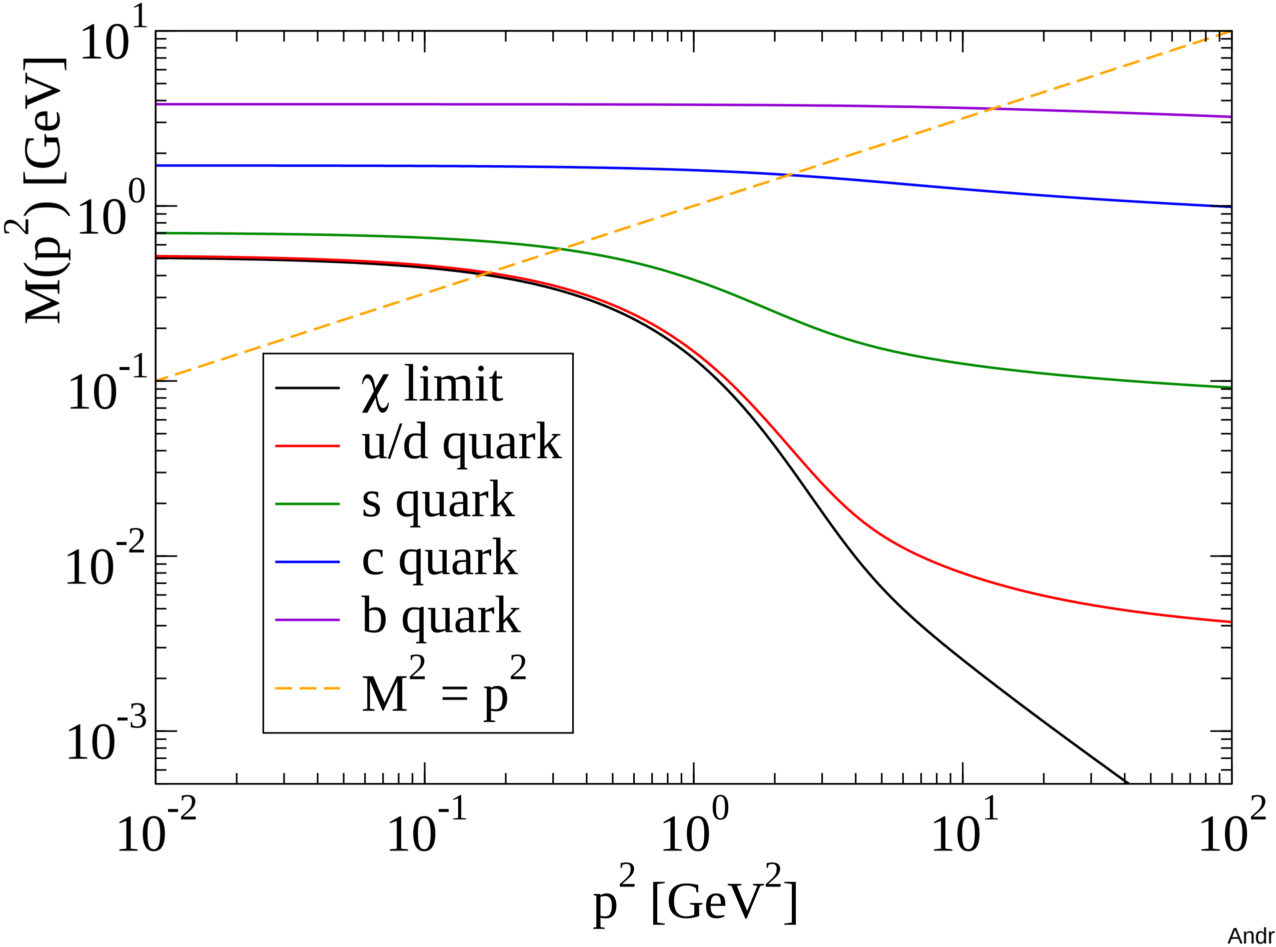
$M^2(p^2) = p^2 \;\Rightarrow$ Euclidean constituent quark mass M_E



q	M_E/m_ζ
χ	∞
u/d	100
s	7
c	1.7
b	1.2

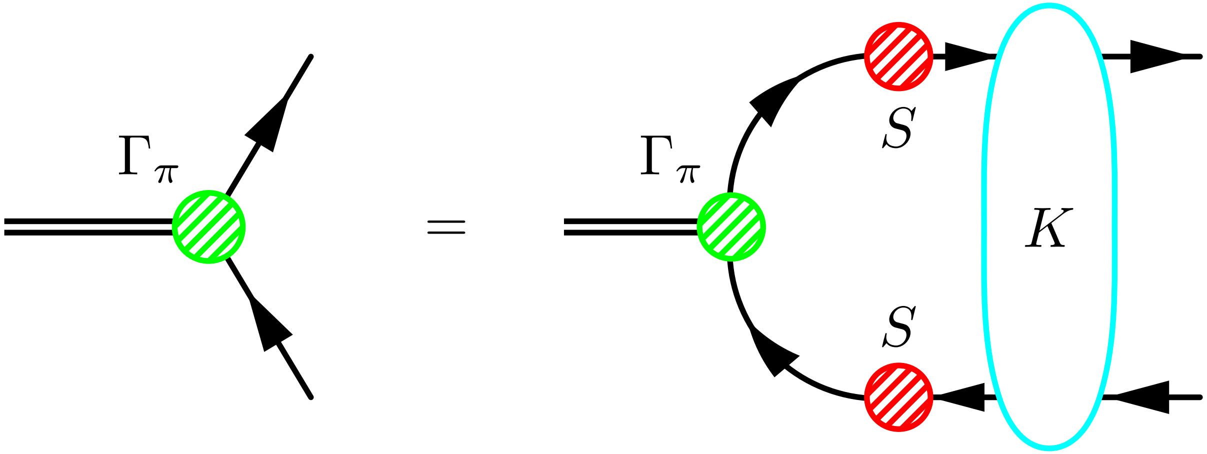
$\rightarrow\; D\chi SB$

Bethe-Salpeter Equation

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\,\Gamma_\pi\,S$)

$$\Gamma_{tu}(p;P) = \int d^4q\,[\chi(q;P)]_{sr}\,K^{tu}_{rs}(q,p;P)\,.$$

- Gap eq. output → BSE input



AV WTI

- Axial-vector Ward-Takahashi identity, equal mass case

$$
\begin{aligned}
P_\mu \Gamma^j_{5\mu}(k;P) \quad &= \quad S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-) \\
&\quad - 2i\, m(\zeta)\, \Gamma^j_5(k;P),
\end{aligned}
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satisfies BSE

satisfies DSE

$\Rightarrow$ Kernels of BSE and DSE must be related

- → Nontrivial constraint (chiral symmetry!)

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satisfies BSE

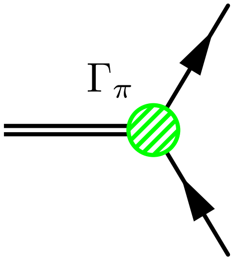
satisfies DSE

$\Rightarrow$ Kernels of BSE and DSE must be related

- → Nontrivial constraint (chiral symmetry!)
- Preserved at every step in truncation scheme
- At least one such scheme exists
- Lowest order: rainbow truncation in gap,
  ladder truncation in BS equation

Pseudoscalar Mesons

- Pseudoscalar meson BSA (${}_n = {}_{gr},\ {}_{exc1},\ \ldots$):



$$\Gamma^j_{\pi_n}(k;P) = \tau^j \gamma_5 \left[iE_{\pi_n}(k;P) + \gamma \cdot P F_{\pi_n}(k;P) \right.$$

$$\left. + \quad \gamma \cdot k\, G_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_{\pi_n}(k;P) \right]$$

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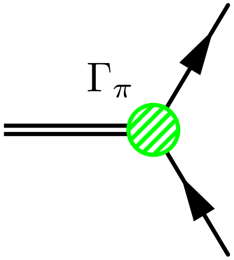

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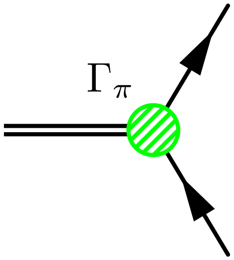

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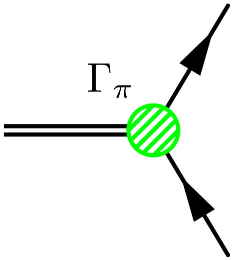

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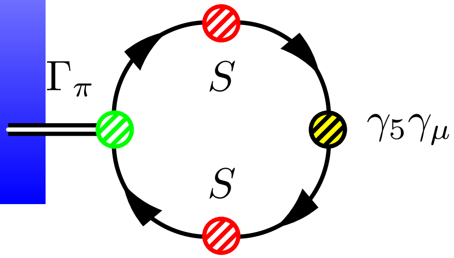
- P: total momentum, k: relative momentum
- pseudoscalar piece
- pseudovector pieces:
  - intrinsic orbital angular momentum
  - crucial for Lorentz invariance
  - preserving symmetries (AV WTI)

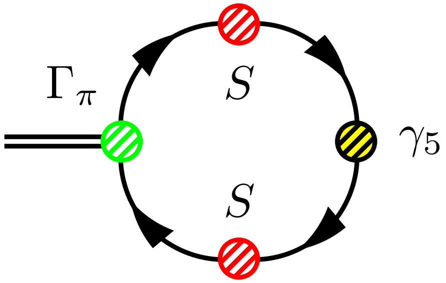
Some Facts

- Leptonic decay constant ($\chi = S\,\Gamma_\pi\,S$)



$$f_{\pi_n}\, P_\mu \;\sim\; \mathrm{tr}\int d^4q\; \gamma_5\gamma_\mu\, \chi_{\pi_n}(q;P)$$
$$\;\hat{=}\; \langle 0|\bar{q}\gamma_\mu\gamma_5 q|\Pi(P)\rangle$$

- Pseudoscalar decay constant



$$i\rho_{\pi_n}(\zeta) \;\sim\; \mathrm{tr}\int d^4q\; \gamma_5\, \chi_{\pi_n}(q;P)$$
$$\;\hat{=}\; \langle 0|\bar{q}\gamma_5 q|\Pi(P)\rangle$$

- Axial-vector and pseudoscalar vertex

$$\Gamma_{5\mu}(k;P), \qquad \Gamma_5(k;P)$$

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- Consequence (residues):

$$f_{\pi_n} m^2_{\pi_n} = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)\,;$$

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- Consequence (residues):

$$f_{\pi_n} m^2_{\pi_n} = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)\,;$$

- valid for every pseudoscalar meson (including radial excitations)
- valid for every current quark mass
- $\Rightarrow$ GMOR
- PCAC

Mass Formula

- Investigate the chiral limit of

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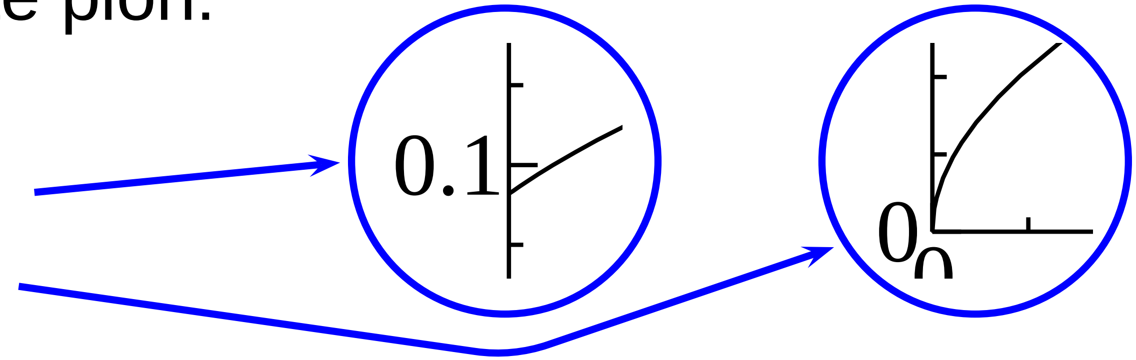
  - $m(\zeta) \to 0$
  - $f_{\pi_{gr}}$ finite
  - $m_{\pi_{gr}} \to 0$



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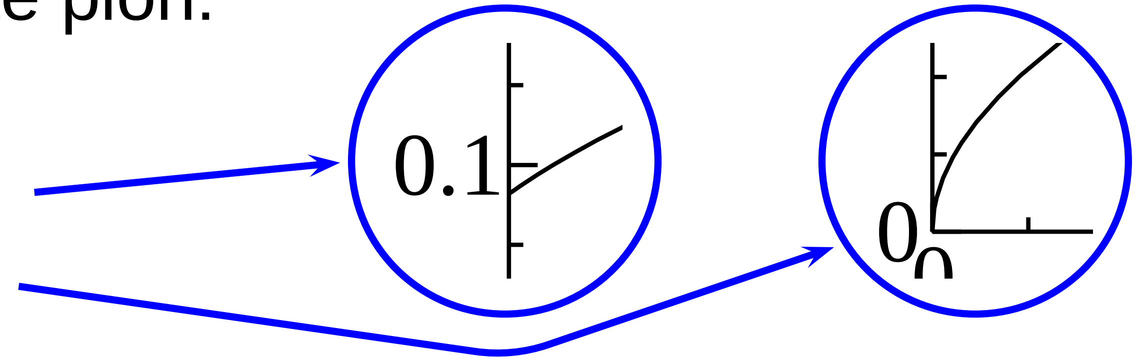

- Excited state pion:
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  - $m_{\pi_{exc1}}$ finite
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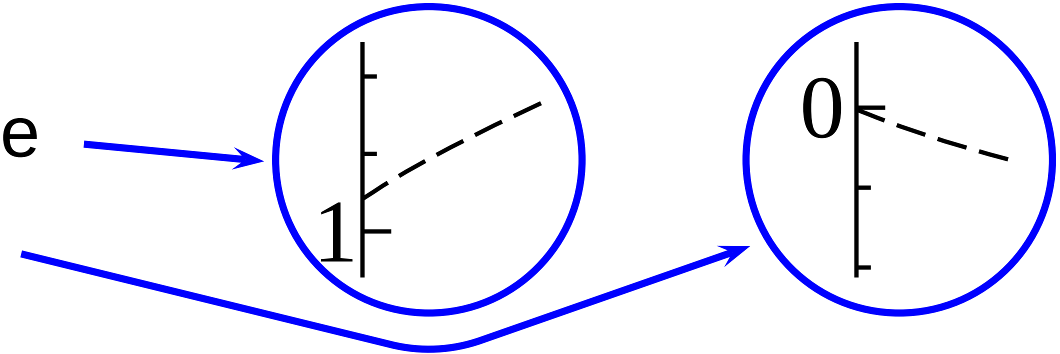
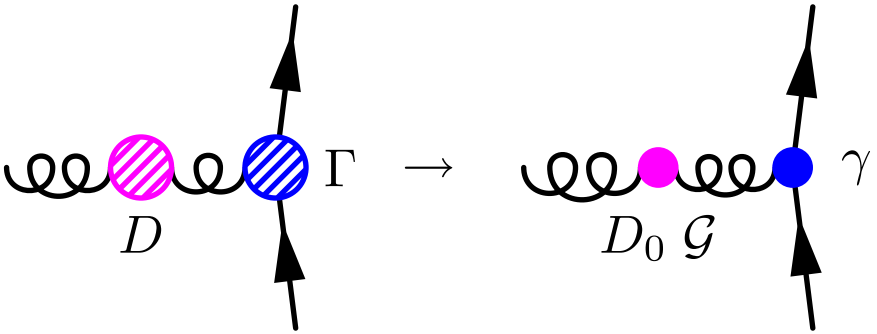


Rainbow-Ladder (RL) Truncation

- Rainbow approximation for gap equation
- Ladder approximation for BSE



- Effective coupling $\mathcal{G}$
- Bare quark-gluon vertex γ_ν
- Bare gluon propagator $D_{\mu\nu}^{\text{free}}(p-q)$
- How good is this?

Possibilities — Ansätze

- Various couplings $\mathcal{G}$
- For some: beyond RL
- Ansatz $\mathcal{G}(q^2) \sim \delta(q^2)$: algebraic equations
  $\rightarrow$ perform truncation at any order in scheme
- $\Rightarrow$ get reliable error estimates for each step

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- More sophistication:
  - UV-finite Ansatz
  - RG-improved Ansatz
  - Results from DSEs for gluon propagator and quark-gluon vertex
- More general scheme?

Model Building

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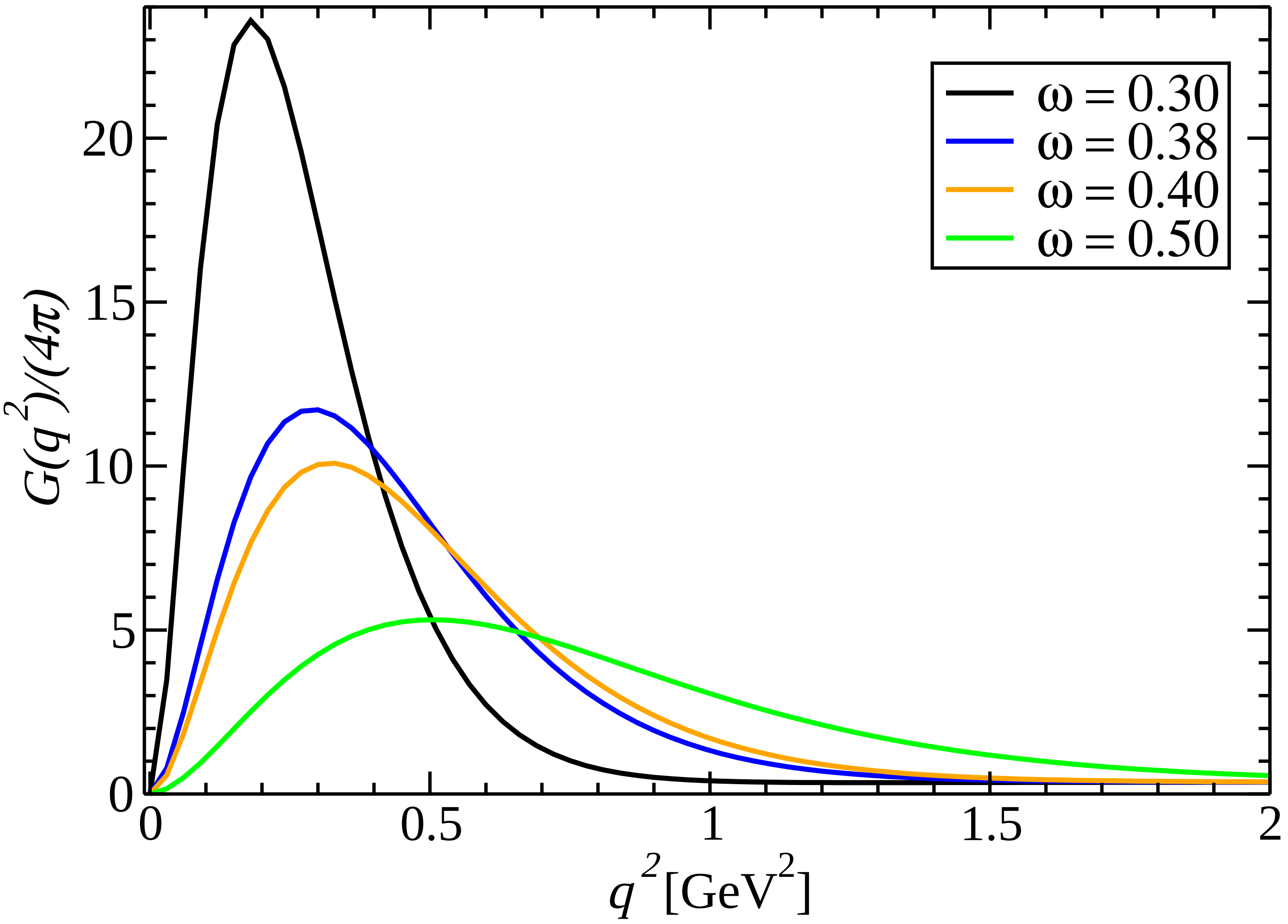
- Precise form at low $Q^2 \rightarrow$ model
- IR: two-parameters via Gaussian: strength D and width ω
- perturbative α in the UV region

Model Details

- Effective coupling $\mathcal{G}(Q^2)$: $\omega\, D =$const.



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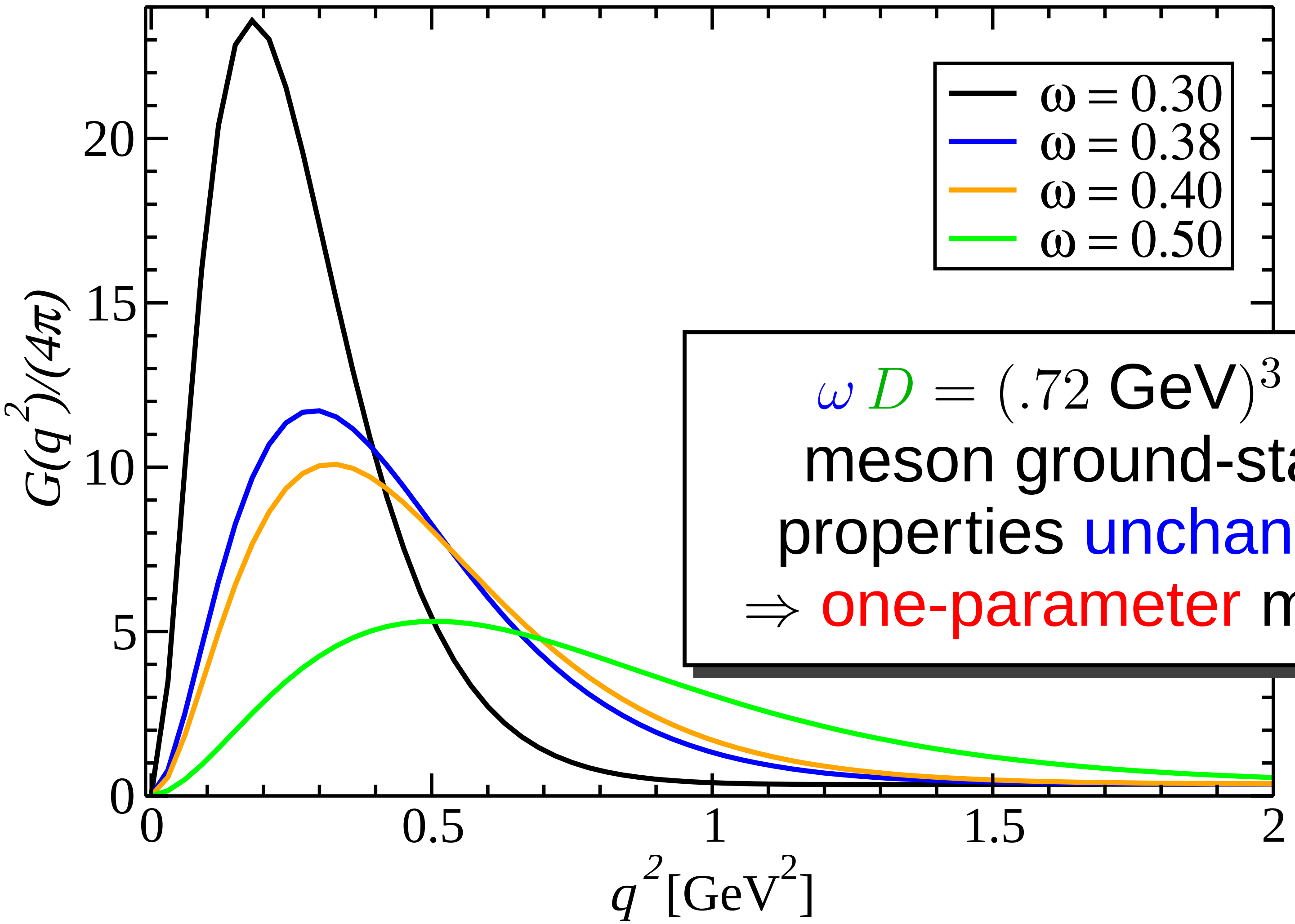

$\omega\, D = (.72\ \text{GeV})^3 \Rightarrow$
meson ground-state
properties unchanged
$\Rightarrow$ one-parameter model

So Far – Ground States

- P. Maris, P. C. Tandy: series of papers following

  P. Maris and P. C. Tandy, Phys. Rev. C **60**, 055214 (1999).

- Successful description of light pseudoscalar and vector mesons:
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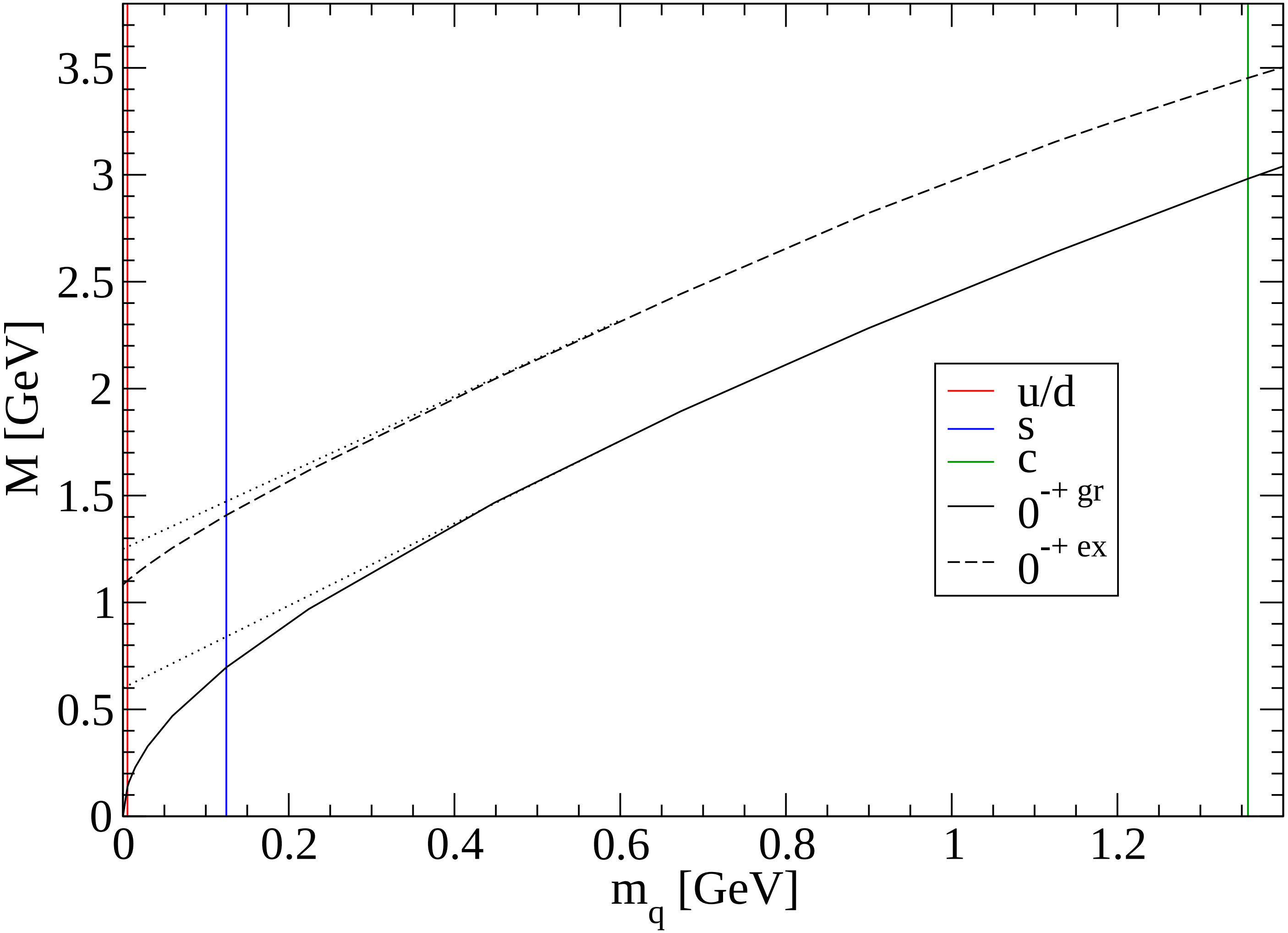
- Successful description of light pseudoscalar and vector mesons: 31 masses/couplings/radii with r.m.s. error of 15% including prediction of $F_\pi(Q^2)$ measured in Hall A.
- Now:
  - Radial excitaions
  - Scalar mesons
  - Axial vector mesons
- Study long range part of the strong interaction between light quarks

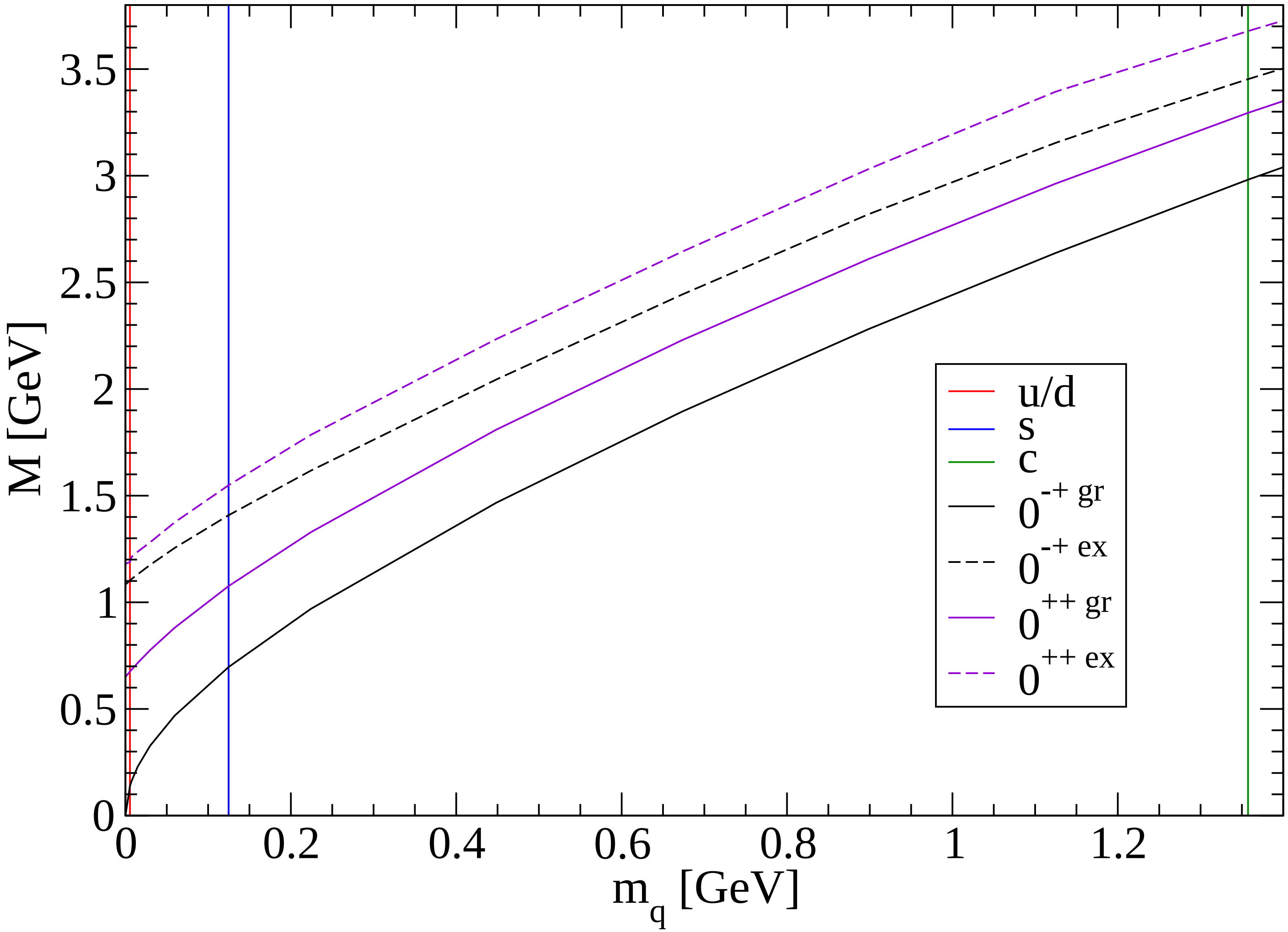
Masses

- $m_{\pi_{gr}}$ and $m_{\pi_{exc1}}$ as functions of current quark mass



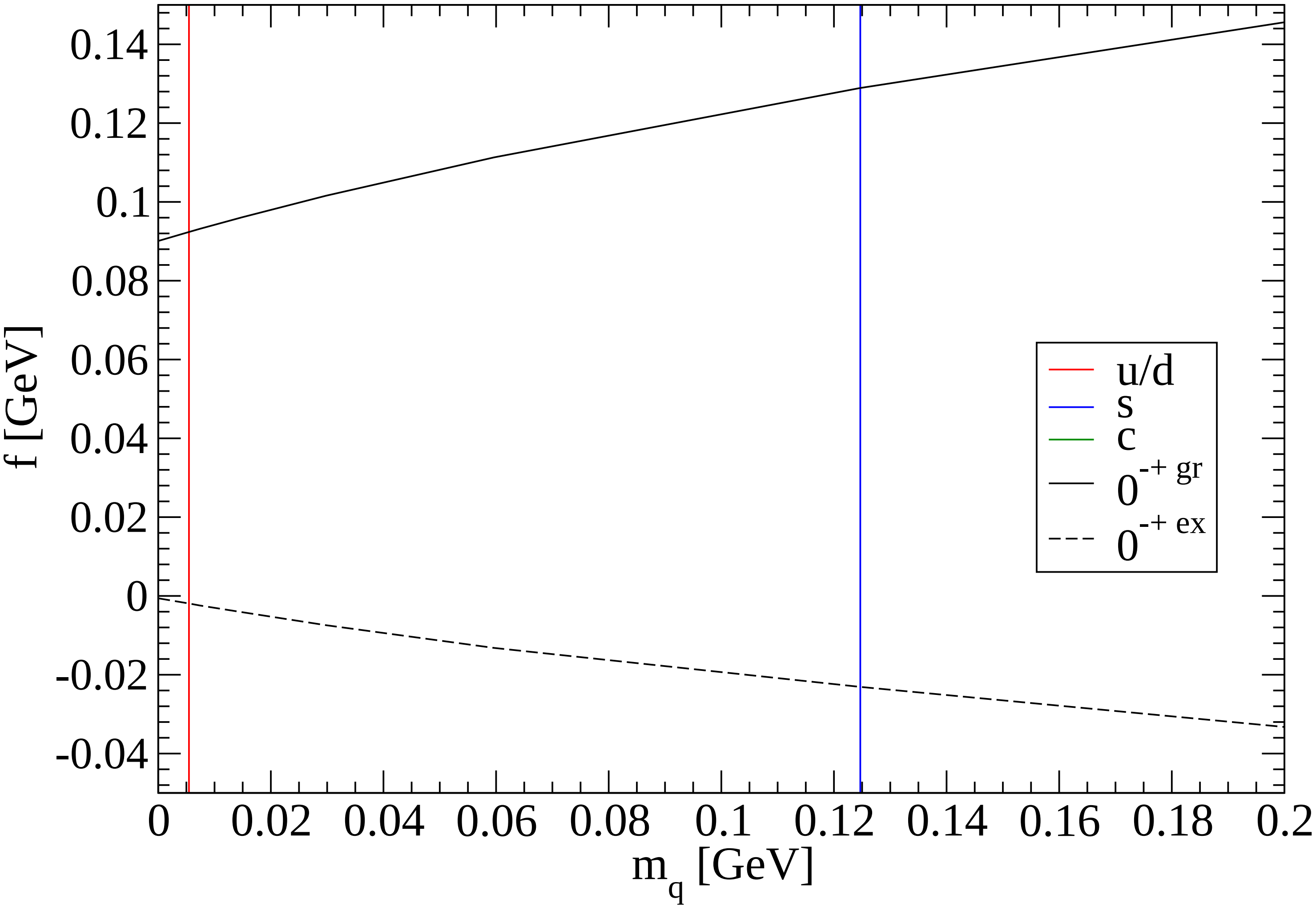
Masses

- $m_{0^{-+}}$ and $m_{0^{++}}$ as functions of current quark mass



Leptonic Decay Constants

- $f_{\pi_{gr}}$ and $f_{\pi_{exc1}}$ as functions of current quark mass



Selection of Results

- Results of calculations (in GeV, $\omega = 0.38$ GeV)

m_q	$m_{\pi_{gr}}$	$m_{\pi_{exc1}}$	$f_{\pi_{gr}}$	$f_{\pi_{exc1}}$
0	0	1.08	0.088	0
0.0054	0.14	1.10	0.092	-0.002
0.125	0.70	1.41	0.129	-0.023
1.34	2.98	3.45	$0.22(1)$	$-0.15(4)$

- Chiral limit, u/d, s, and c quark masses
- A. Höll, A. Krassnigg and C. D. Roberts, Phys. Rev. **C70**, 042203 (R) (2004)

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$\pi\,(138) \longrightarrow$	0.14	1.10	$\longleftarrow \pi$ (1300),η (1294)	
0.125	0.70	1.41	$\longleftarrow \eta$ (1476)	-0.023
$\eta_c(1S)\,(2980) \longrightarrow$	2.98	3.45	$\longleftarrow \eta_c(2S)$ (3654)	$.15(4)$

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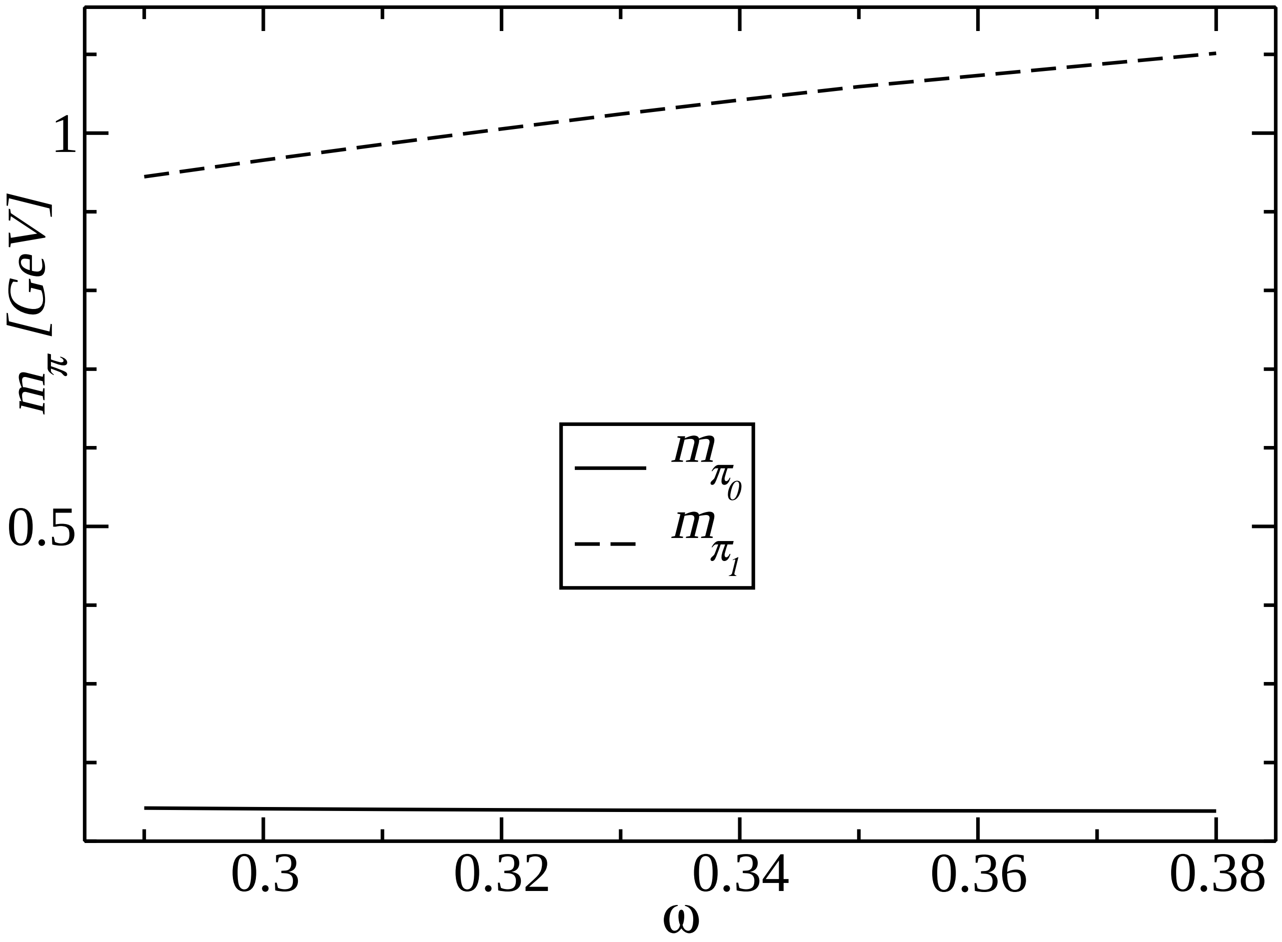
Model parameter dependence

- $m_{\pi_{gr}}$ and $m_{\pi_{exc1}}$ as a function of ω



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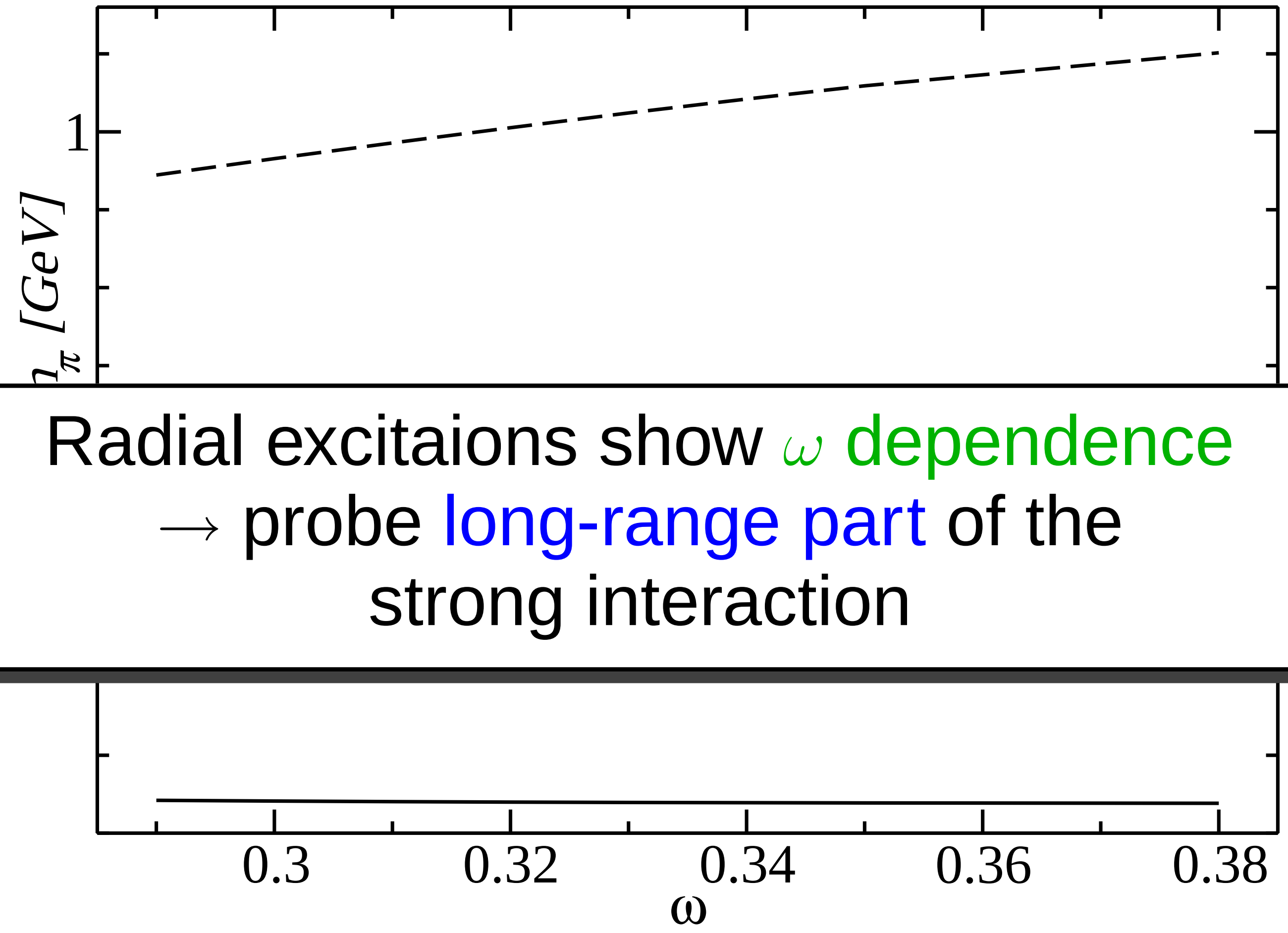
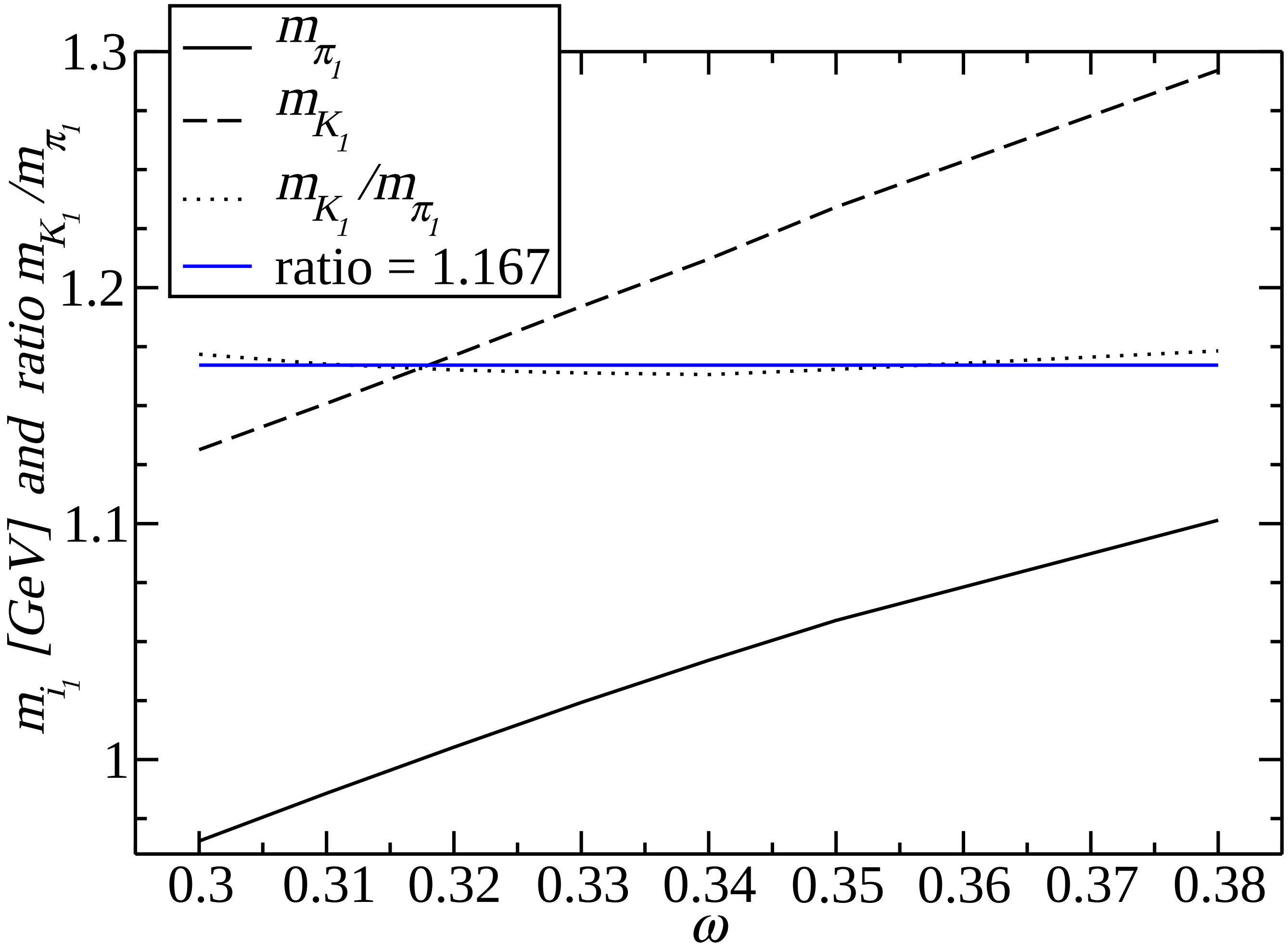


Radial excitaions show ω dependence
→ probe long-range part of the
strong interaction

Excited Kaon

- $m_{K_{exc1}}/m_{\pi_{exc1}}$ as a function of ω



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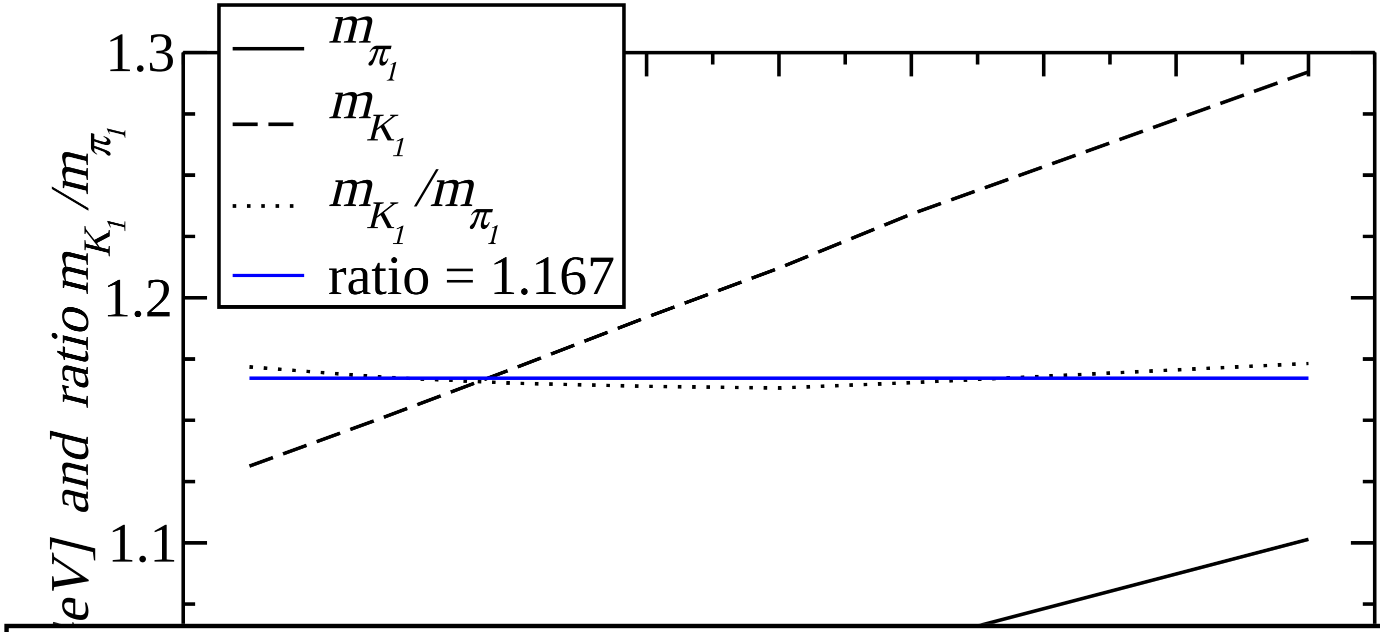
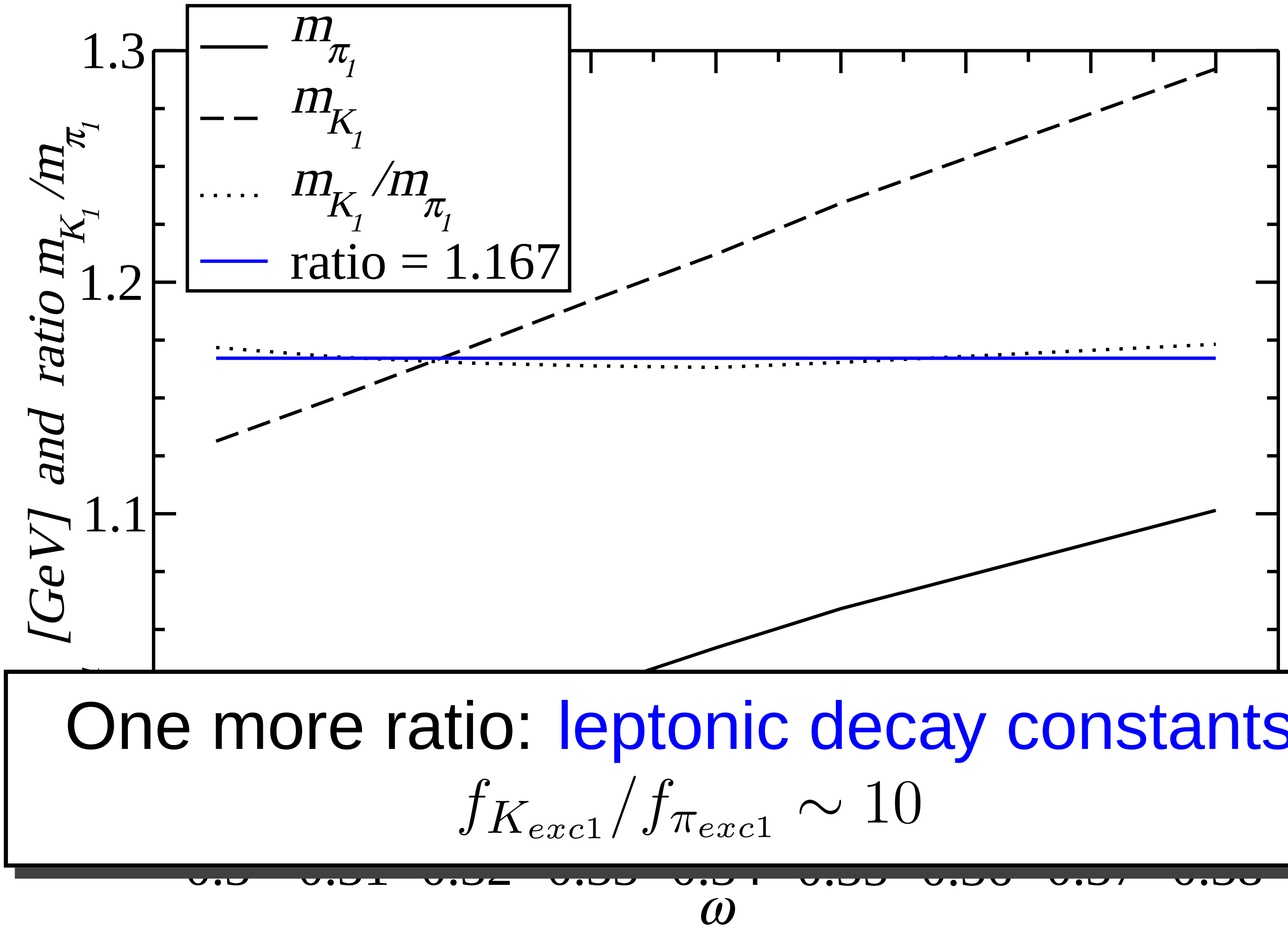


Consider ratio: $m_{K_{exc1}}/m_{\pi_{exc1}} = const \ \rightarrow$
estimate for $m_{K_{exc1}}$
via experimental mass of excited pion:
$m_{K_{exc1}} \approx \ 1520$ MeV (Exp: K(1460))

Excited Kaon

- $m_{K_{exc1}}/m_{\pi_{exc1}}$ as a function of ω



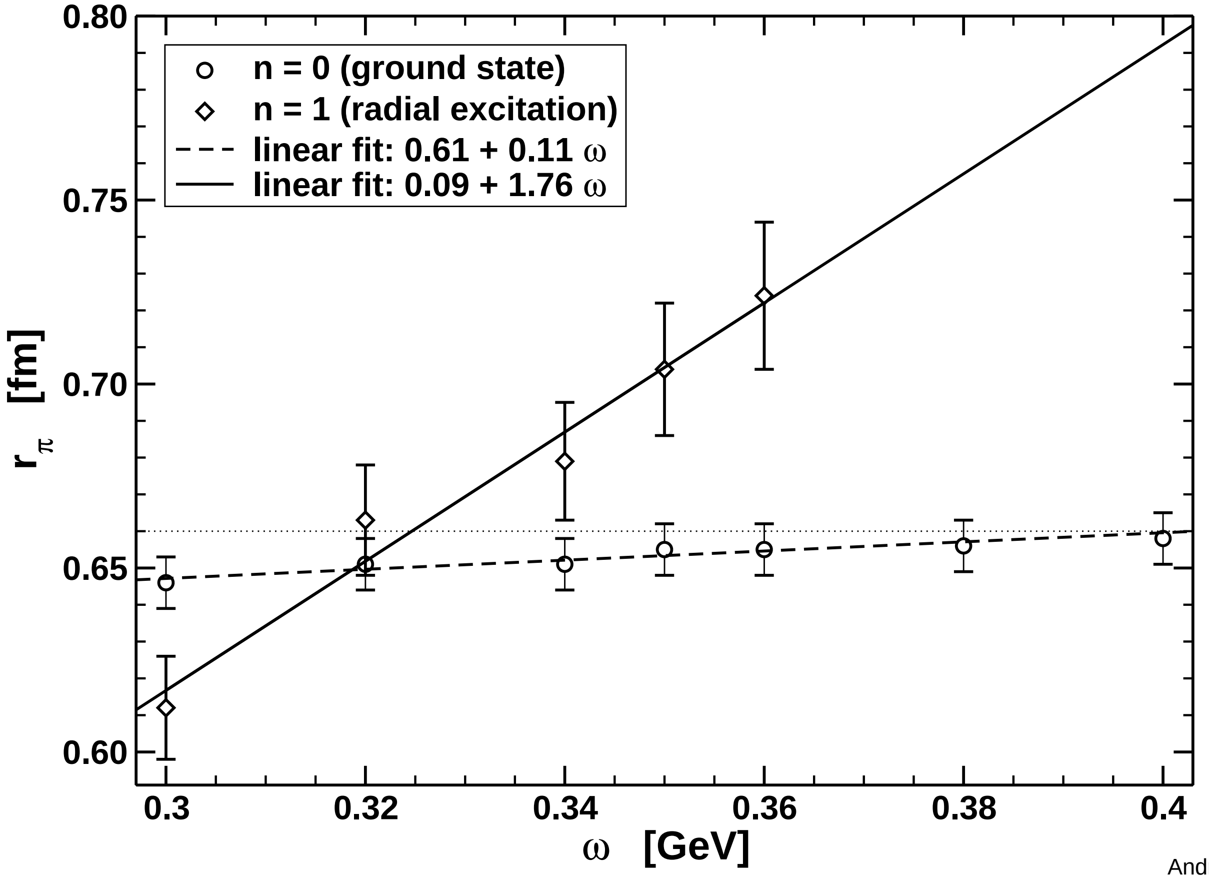
One more ratio: leptonic decay constants:

$$f_{K_{exc1}}/f_{\pi_{exc1}} \sim 10$$

Electromagnetic Properties

Estimates:

- π_{exc1} charge radius: $r_{\pi_{exc1}} \approx 0.93\,\mathrm{fm} \;\approx\; 1.4\,r_{\pi_{gr}}$
- radiative decay $\pi_{exc1} \to \gamma\,\gamma$: $\Gamma_{\pi^0_{exc1}\gamma\gamma} \;\approx\; 240$ eV



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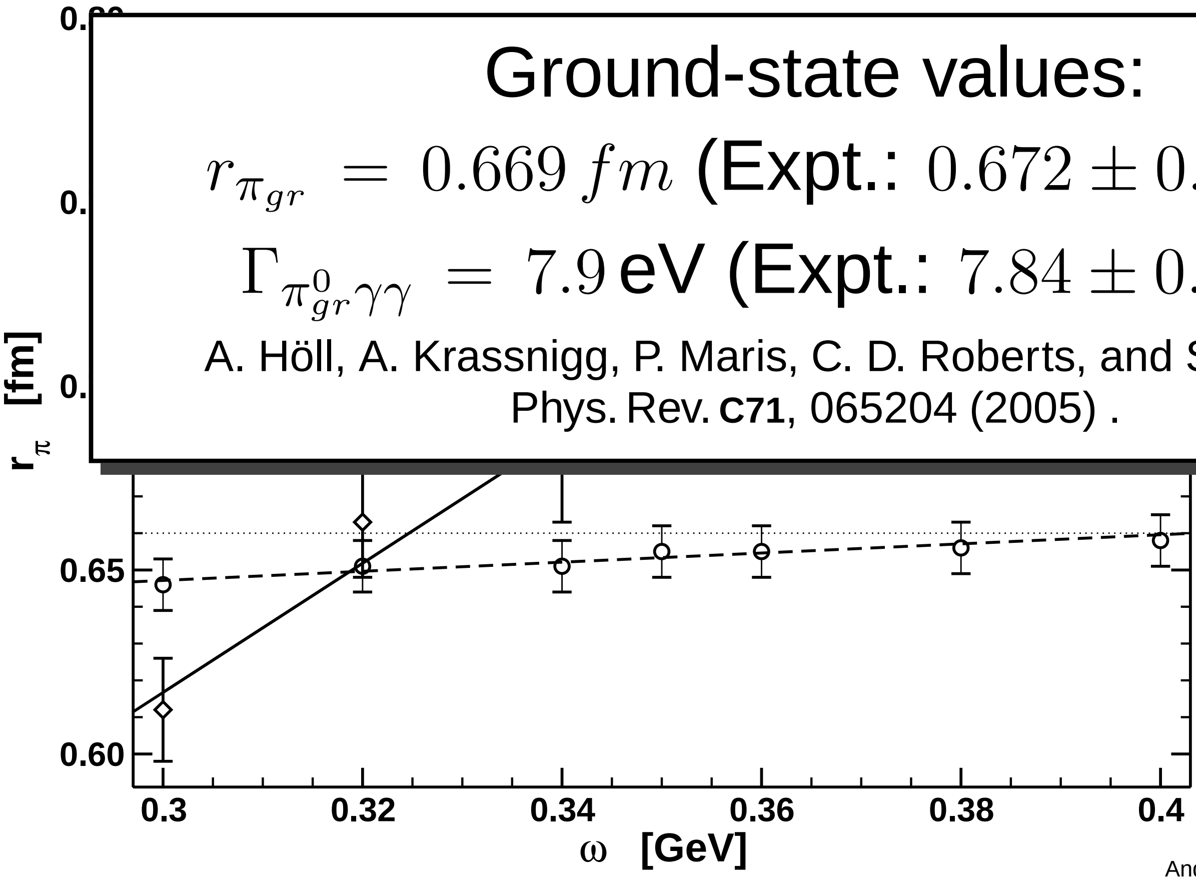

Ground-state values:

$$r_{\pi_{gr}} = 0.669\,fm \text{ (Expt.: } 0.672 \pm 0.008 \text{ fm)}$$

$$\Gamma_{\pi^0_{gr}\gamma\gamma} = 7.9\,\text{eV (Expt.: } 7.84 \pm 0.56\,\text{eV)}$$

A. Höll, A. Krassnigg, P. Maris, C. D. Roberts, and S. V. Wright,
Phys. Rev. **C71**, 065204 (2005) .

Other Things and Elsewhere

- Estimates for mesons with exotic QN, e. g. π_1 (1^{-+})
- Finite temperature and density
- Heavy-meson observables
- Diquark confinement (model-independent)
- Confinement mechanism for q and g
- Gluon propagator and quark-gluon vertex
- Comparison to lattice gauge QCD
- Baryon studies via quark-diquark Ansatz
- Exploratory steps beyond RL with sophisticated interaction
- Other QFTs than QCD

WIP - Wish List

- Work in progress
  - Hadronic decays, e. g. $\pi_{exc1} \longrightarrow \varrho\,\pi_{gr}$
  - Radial excitations of vector, etc. mesons
  - Higher radial excitations
  - Heavy-light mesons and radial excitations

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- Wish list
  - Sophisticated meson model beyond RLT
  - Good description of axial-vector mesons
  - Study states with “exotic” quantum numbers
  - . . .

Summary and Conclusions

- Dyson-Schwinger equations provide a nonperturbative continuum approach to QCD
- Symmetry-preserving truncation scheme enables proof of exact results and reliable studies of hadron properties
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Summary and Conclusions

- Dyson-Schwinger equations provide a nonperturbative continuum approach to QCD
- Symmetry-preserving truncation scheme enables proof of exact results and reliable studies of hadron properties
- Excited states provide means to study the long-range behavior of the strong interaction
- Step beyond Rainbow-Ladder truncation needed to go for axial vectors, scalars, exotics
- A lot of effort is being made
- Progress will be made

http://physik.uni-graz.at/itp/sicqft/

The End

Thank you!