Complete next-to-next-to-leading order calculation of $NN \rightarrow NN\pi$ in chiral EFT

in collaboration with
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Outline:

- Introduction: chiral EFT and $NN \rightarrow NN\pi$
- Why is pion production interesting?
- High accuracy pion production operator
- Importance of Delta(1232)-resonance
- Summary and outlook
Method and goal

Approaches to QCD at low energies:

• Phenomenological models
• Lattice calculations
• Chiral effective field theory
Method and goal

Approaches to QCD at low energies:
- Phenomenological models
- Lattice calculations
- Chiral effective field theory

Chiral EFT - effective field theory of QCD below 1 GeV
- most general Lagrangian compatible with symmetries of QCD
- effective degrees of freedom: pions, nucleons, Delta(1232)-resonance
- systematic expansion in small momenta and small masses
- suited for investigations of $\pi\pi$, $\pi N$, $NN$ interactions and nuclear forces

- successful application to pion reactions on few-nucleon systems: $\pi A \rightarrow \pi A$ ($A=2,3,4$), $\pi d \rightarrow \gamma nn$, $\gamma d \rightarrow \pi NN$,...

Our goal is to study $NN \rightarrow NN\pi\pi$ within chiral EFT
Specifics of pion production

NN interactions are non-perturbative [deuteron]

Hybrid chiral EFT method:

1. Calculate irreducible production operator perturbatively in chiral EFT
2. Convolute it with non-perturbative NN wave functions

realistic phenomenological NN WF: CD-Bonn, CCF, AV18, ...

Large transferred momenta

• NN momenta in CMS are large enough to produce a pion

\[ |\vec{p}| \sim \sqrt{m_\pi m_N} \sim 360 \text{ MeV} – \text{new scale} \]

Special counting: Momentum Counting Scheme (MCS)

expansion parameter \( \chi_{\text{MCS}} \sim \sqrt{\frac{m_\pi}{m_N}} \)

• Explicit Delta(1232)-resonance \( m_\Delta - m_N \sim 280 \text{ MeV} \sim |\vec{p}| \)
Why is pion production interesting?

- **First inelastic process** in nucleon-nucleon interactions.
- **Several channels:**
  - \( pp \rightarrow pp\pi^0 \) and \( pp \rightarrow d\pi^+ \) cross sections differ by an order of magnitude.
  
  \[
  \sigma_{\text{tot}}(pp \rightarrow pp\pi^0) \approx 3 \, \mu b \quad \sigma_{\text{tot}}(pp \rightarrow d\pi^+) \approx 43 \, \mu b \quad T_{\text{lab}} = 293.5 \, \text{MeV}
  \]

- **Building block** for more complicated processes:
  - CSB in \( dd \rightarrow \alpha\pi^0 \)
  - 3N forces
  - Pionic deuterium \( \pi d \rightarrow NN \rightarrow \pi d \)

- **Charge symmetry breaking** in \( pn \rightarrow d\pi^0 \)
Why is pion production interesting?

Charge symmetry – invariance under interchange of u- and d-quarks

• Approximate symmetry of QCD
• Explicitly broken by quark mass difference and electromagnetic effects
• On the level of hadrons → invariance under interchange of $p$ and $n$
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Charge symmetry breaking in \( \text{pn} \rightarrow \text{d} \pi^0 \):


- Interchange of p and n changes differential cross section
- Forward backward-asymmetry \( A_{fb} \propto \left( \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) \right) / \frac{d\sigma}{d\Omega}(\theta) \)
- Experiment: \( A_{fb} = (17.2 \pm 8 \pm 5.5) \times 10^{-4} \) TRIUMF (2003)
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Charge symmetry breaking in $pn \rightarrow d\pi^0$:

Opper et al. (2003), v.Kolck et al. (2000), Bolton and Miller (2009), AF et al. (2009)

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• Forward backward-asymmetry $A_{fb} \propto \left( \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) \right) / \frac{d\sigma}{d\Omega}(\theta)$
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• Theory $A_{fb} \propto Re(M_{s-wave}^{CS} M_{p-wave}^{CS*}) / |M_{s-wave}^{CS}|^2 \propto (m_p - m_n)^{str} / |M_{s-wave}^{CS}|^2$
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**Charge symmetry** – invariance under interchange of u- and d-quarks

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Charge symmetry breaking in \( p n \rightarrow d \pi^0 \):

- Interchange of p and n changes differential cross section
- Forward backward-asymmetry \( A_{\text{fb}} \) \( \propto \frac{\frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta)}{\frac{d\sigma}{d\Omega}(\theta)} \)
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- Theory \( A_{\text{fb}} \propto \frac{\text{Re}(M_{\text{s-wave}}^{\text{CSB}} M_{\text{p-wave}}^{\text{CS*}})}{|M_{\text{s-wave}}^{\text{CS}}|^2} \propto \frac{(m_p - m_n)^{\text{str}}}{|M_{\text{s-wave}}^{\text{CS}}|^2} \)
- s-wave amplitude \( M_{\text{s-wave}}^{\text{CS}} \) is important prerequisite to extract \( (m_p - m_n)^{\text{str}} \) – strong part of p – n mass difference
s-wave pion production

Introduction

At threshold only s-wave gives non-zero contribution

General s-wave production amplitude at threshold

\[
M_{\text{th}}(NN \rightarrow NN\pi) = A (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{p} \ (\tau_1 + \tau_2) \cdot \phi^* \\
+ B (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{p} \ (\tau_1 \times \tau_2) \cdot \phi^*
\]

Amplitudes A and B contribute to different reaction channels

- A contributes to \( pp \rightarrow pp\pi^0 \)
- B contributes to \( pp \rightarrow d\pi^+ \)

Goal: derive pion production operators A and B within chiral EFT
s-wave pion production operators

<table>
<thead>
<tr>
<th>$\chi_{\text{MCS}} \sim \sqrt{\frac{m_\pi}{m_N}}$</th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram LO]</td>
<td>![Diagram NLO]</td>
<td>![Diagram NNLO]</td>
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</tbody>
</table>

**For $pp \rightarrow pp\pi^0$ LO rescattering contribution is forbidden, NLO is zero**

$\Rightarrow$ effects of **NNLO** loops are very **important**
NNLO loop-diagrams

- **Topologies of NNLO diagrams:**

  \[
g_A^1: \quad \begin{array}{ccc}
  & 1 & \\
  \frac{1}{2} & \bullet & \\
  & \bullet & \\
  \end{array}
  \quad \text{Football}
  
  \begin{array}{c}
  \bullet \\
  \bullet \quad \bullet \\
  \bullet
  \end{array}
  \quad \text{Type Ia}
  
  \begin{array}{c}
  \bullet \quad \bullet \\
  \bullet \\
  \bullet
  \end{array}
  \quad \text{Type Ib}
  
  \begin{array}{c}
  \bullet \\
  \bullet \quad \bullet \\
  \bullet
  \end{array}
  \quad \text{Mini-Football}
  
  \begin{array}{c}
  \bullet \\
  \bullet \quad \bullet \\
  \bullet
  \end{array}
  \quad \text{Box a}
  
  \begin{array}{c}
  \bullet \\
  \bullet \quad \bullet \\
  \bullet
  \end{array}
  \quad \text{Box b}
\]

- \(\frac{1}{m_N}\) correction should be included in every vertex
  → **Lots of terms** and lengthy calculation

- Efficient method: collect \(\pi N\) → **\(\pi \pi N\) subgraphs**
NNLO loop-diagrams: calculation method

Collecting subgraphs – efficient way to calculate NNLO loop diagrams

\[ g_A: \]
\[ \frac{1}{2} \]
Football
\[ + \]
Type Ia
\[ + \]
Type Ib
\[ + \]
Mini-Football
\[ = \]
\[ \frac{1}{2} \]

\[ g_A^3: \]
Type II
\[ + \]
Type IIIa
\[ + \]
Type IIIb
\[ + \]
Type IV
\[ + \]
Box a
\[ + \]
Box b
\[ = \]

Operator \( \mathcal{T} \) is the sum of all \( \pi N \rightarrow \pi \pi N \) subgraphs

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NNLO loop-diagrams: results

• Put [few NNLO terms] into common structures

→ Get full NNLO loop operator
→ Very compact expression

• NNLO loop production operator (simplified in Dim.Reg.):

\[
iM_{\text{nucl}}^{\text{Sym.}} = \frac{g_A^3}{f_\pi^5} v \cdot q \tau^a_+ (i \varepsilon^{\alpha\mu\nu\beta} \gamma_\alpha k_{1\mu} S_{1\nu} S_{2\beta}) (-2I_{\pi\pi})
\]

\[
+ \frac{g_A^3}{f_\pi^5} v \cdot q \tau^a_\times (S_1 + S_2) \cdot k_1 \left( - \frac{19}{24} I_{\pi\pi} + \frac{5}{9} \frac{1}{(4\pi)^2} \right)
\]

\[
+ \frac{g_A f_\pi^5}{f_\pi^5} v \cdot q \tau^a_\times (S_1 + S_2) \cdot k_1 \left( \frac{1}{6} I_{\pi\pi} - \frac{1}{18} \frac{1}{(4\pi)^2} \right)
\]

with only one basic integral:

\[
I_{\pi\pi} = \frac{\mu^e}{i} \int \frac{d^{4-\epsilon} l}{(2\pi)^{4-\epsilon}} \frac{1}{(l^2 - m^2_\pi + i0)((l + k_1)^2 - m^2_\pi + i0)}
\]
Inclusion of Delta(1232) explicitly

Motivation

For $\text{NN} \rightarrow \text{NN}\pi$ Delta-resonance contribution is important

- Typical momenta in $\text{NN} \rightarrow \text{NN}\pi$ is about $p \approx 360 \text{ MeV}$
- Delta-nucleon mass difference $m_\Delta - m_N \approx 280 \text{ MeV}$ → same order as $p$

→ Delta(1232) should be included as a dynamic degree of freedom

Additional NNLO loop-diagrams with delta:
Explicit Delta: groups and cancellations

- **Same calculation method** as for pure nucleon case: selecting groups
- **Cancellation patterns** for s-wave pion production:

\[ \Delta \text{II} + \Delta \text{IIIa} + \Delta \text{IIIb} + \Delta \text{IV} + \Delta \text{Box a} + \Delta \text{Box b} = 0 \]

\[ \Delta \text{V} + \Delta \text{VIa} + \Delta \text{VIIa} + \Delta \text{VIIIa} + \Delta \text{IXa} + \Delta \text{IV} = 0 \]

\[ \Delta \text{V} + \Delta \text{VIb} + \Delta \text{VIIb} + \Delta \text{VIIIb} + \Delta \text{IXb} + \Delta \text{IV} = 0 \]

- **Finite remainder survives like in NN case**
Explicit inclusion of Delta in NN → NNπ

NNLO loop-corrections to NN → NNπ operator due to explicit Δ(1232)

\[ iM_{\Delta\text{-loops}}^{\text{NNLO}} = \frac{g_A g_\pi N A}{f_\pi^5} v \cdot q \tau^a (i\epsilon^{\alpha\mu\nu\beta}v_{\alpha}k_{1\mu}S_{1\nu}S_{2\beta}) \]

\[ \times \left\{ \frac{2}{9} \left( I_{\pi\pi} + \frac{1}{2} \frac{J_{\pi\Delta}}{\Delta} + \Delta J_{\pi\pi\Delta} + \frac{2}{(4\pi)^2} \right) + \frac{1}{18} k_1^2 J_{\pi\pi N \Delta} \right\} \]

\[ + \frac{g_A g_\pi N A}{f_\pi^5} v \cdot q \tau_x (S_1 + S_2) \cdot k_1 \]

\[ \times \left\{ \frac{5}{9} \left( I_{\pi\pi} + \frac{1}{2} \frac{J_{\pi\Delta}}{\Delta} + \Delta J_{\pi\pi\Delta} + \frac{2}{(4\pi)^2} \right) + \frac{1}{18} k_1^2 J_{\pi\pi N \Delta} \right\} \]

\[ + \frac{8 \delta^2}{9 k_1^2} \left( I_{\pi\pi} + \frac{1}{2} \frac{J_{\pi\Delta}}{\Delta} + \Delta J_{\pi\pi\Delta} + \frac{2}{(4\pi)^2} \right) - \frac{2}{27} \left( I_{\pi\pi} + \frac{1}{2} \frac{J_{\pi\Delta}}{\Delta} + \frac{1}{3} \frac{2}{(4\pi)^2} \right) \] .

**Result:**

A

B

Correct analytic behavior:
If \( m_\Delta \to \infty \) then the contribution of Delta vanishes (decoupling of Delta)

Appelquist, Carazzone (1975)

No additional unknown LECs
Comparison of Delta- and nucleon-loops

Ratio of long-range loop-contributions: nucleon / Delta

- Explicitly proves MCS counting estimation: delta~p
- Sum of Delta and nucleon-loop contributions:
  - In A: net NNLO effect ~ A_N – of natural size in MCS
  - In B: net NNLO effect is smaller than MCS expectations due to cancellations
  → Both facts are consistent with indications from data:
    - For B there is already a good description of data at NLO
    - For A we probe NNLO contributions directly (LO + NLO ≈ 0)
Summary and outlook

The reaction $NN \to NN\pi$ in Chiral EFT

- Tool to study charge symmetry breaking ($pn \to d\pi^0$)
- Building block for more complicated reactions ($dd \to \alpha\pi^0$, 3NF, ...)
- Cross section puzzle (different cross sections in different channels)

Current results:

- s-wave pion production operator at threshold up to $\mathcal{N}^2$LO MCS (6%) including explicit Delta(1232)

Next step

- Convolution with nucleon-nucleon wave functions and calculation of the observables
spares
\[ V_{\pi N \rightarrow \pi N} = \frac{1}{4f_\pi^2} \epsilon_{abc} \tau^c (\frac{k}{q} + \phi) \]

\[ k = q - p + p' \]

can be identically rewritten as:

\[ V_{\pi N \rightarrow \pi N} = \frac{1}{4f_\pi^2} \epsilon_{abc} \tau^c (2\phi - \phi + \phi') \]

\[ = \frac{1}{4f_\pi^2} \epsilon_{abc} \tau^c \left( 2\phi - (\phi - m_N) + (\phi' - m_N) \right) \]

Parts of \( \pi N \rightarrow \pi N \) vertex can cancel nucleon propagators