Interactions of light mesons with photons

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Collaborators

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- **Bonn**: Franz Niecknig, Martin Hoferichter (now Bern), Sebastian Schneider, Bastian Kubis
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1. Transition form factors and two-gamma physics

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Reactions of hadrons with (virtual) photons

Why is it interesting?

- explore intrinsic structure of hadrons
  - form factors
  - to which extent does vector meson dominance hold?

$\pi^0 \rightarrow e^- + e^+$

$\mu(p) \gamma(k) \rho_{had} + 5$ permutations of the $q_i$
Reactions of hadrons with (virtual) photons

Why is it interesting?
- explore intrinsic structure of hadrons
  - \( \rightarrow \) form factors
  - \( \rightarrow \) to which extent does vector meson dominance hold?
- background for physics beyond standard model
  - \( \rightarrow \) rare pion decay \( \pi^0 \rightarrow e^+ e^- \)
  - \( \rightarrow g - 2 \) of muon
Hadronic contribution to $g - 2$ of the muon

light-by-light scattering

- $\gamma^*\gamma^* \leftrightarrow \text{hadron(s)}$ is not directly accessible by experiment
  - need good theory with reasonable estimate of uncertainty (ideally an effective field theory)
  - need experiments to constrain such hadronic theories

true for all hadronic contributions:

- the lighter the hadronic system, the more important (though high-energy contributions not unimportant for light-by-light)
  - $\gamma^*\gamma^* \leftrightarrow \pi^0$ (you’ve seen this before for rare pion decay),
    $\gamma^*\gamma^* \leftrightarrow 2\pi, \ldots$
Shopping list for hadron theory and experiment

- transition form factors of pseudoscalars \( \gamma^(*) \gamma^(*) \leftrightarrow P \)
  with \( P = \pi^0, \eta, \eta', \ldots \)

\( \hookrightarrow \) several interesting kinematical regions \( \leadsto \) next slide (for pion)
\[ \pi^0 \rightarrow \gamma^*(q^2_v)\gamma^*(q^2_s) \] transition form factor
Shopping list for hadron theory and experiment

- transition form factors of pseudoscalars $\gamma^{(*)}\gamma^{(*)} \leftrightarrow P$
  with $P = \pi^0, \eta, \eta', \ldots$
- if invariant mass of dilepton around mass of a vector meson:
  - relation to
  - transition form factors of vector to pseudoscalar mesons $V \leftrightarrow P\gamma^{(*)}$ with $V = \rho^0, \omega, \phi, \ldots$
Shopping list for hadron theory and experiment

- transition form factors of pseudoscalars $\gamma^{(*)}\gamma^{(*)} \leftrightarrow P$
  with $P = \pi^0, \eta, \eta', \ldots$

- if invariant mass of dilepton around mass of a vector meson:
  $\Rightarrow$ relation to
  transition form factors of vector to pseudoscalar mesons $V \leftrightarrow P\gamma^{(*)}$ with $V = \rho^0, \omega, \phi, \ldots$

- “two-gamma physics” $\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0, \pi^0\eta, K\bar{K}, \ldots$
  (cross relation to polarizability of the pion)

$\Rightarrow$ has triggered a lot of experimental activity,
in particular MesonNet (WASA, KLOE, MAMI, HADES, \ldots)
Two complementary approaches

- Lagrangian approach
  - use only hadrons which are definitely needed
    (here: lowest nonets of pseudoscalar and vector mesons)
  - sort interaction terms concerning importance,
    essentially based on large-$N_c$
  - include causal rescattering/unitarization for reactions
  - long-term goal: obtain sensible estimates of uncertainties

- dispersive approach
  - include most important hadronic inelasticities
  - use measured (and dispersively improved) phase shifts (2-body)
  - use Breit-Wigner plus background for narrow resonances
    ($n$-body, $n > 2$)
  - error estimates from more vs. less subtracted dispersion relations
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Transition form factor $\omega \rightarrow \pi^0 + \text{dilepton}$

- data and our Lagrangian approach show strong deviations from vector-meson dominance (VMD)
- our approach describes data fairly well except for large invariant masses close to phase-space limit (log plot!)
- second experimental confirmation desirable

Transition form factor $\phi \rightarrow \eta +$ dilepton

- our Lagrangian approach deviates from VMD
- new data from KLOE will come soon

$\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0$

- dashed black lines: tree level,
- blue lines: with coupled-channel rescattering of two pseudoscalar mesons
- overall good description, room for improvement concerning $f_0(980)$
- at high energies spin-2 mesons are missing

I.V. Danilkin, M.F.M. Lutz, S.L., C. Terschlüsen,
\[ \gamma \gamma \rightarrow \pi^0 \eta \]

- dashed black line: tree level,
- blue line: with coupled-channel rescattering of two pseudoscalar mesons
- \(a_0(980)\) dynamically generated

I.V. Danilkin, M.F.M. Lutz, S.L., C. Terschlüsen,
\( \gamma \gamma \rightarrow K^+ K^-, K^0 \bar{K}^0, \eta \eta \) (pure predictions)
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\[ \pi^0 \rightarrow \gamma^* (q_V^2) \gamma^* (q_s^2) \] transition form factor

\[ \pi^0 \rightarrow e^+ e^- e^+ e^- \]

(figures from Bastian Kubis)
\[ \pi^0 \rightarrow \gamma^*(q^2_v)\gamma \text{ transition form factor} \]
\( \pi^0 \rightarrow \gamma\gamma^* (q_s^2) \) transition form factor

(Original paper by Bastian Kubis)
Pion transition form factor — dispersive approach

- want prediction for $e^+e^- \rightarrow \pi^0\gamma$ (up to $\approx 1$ GeV)

→ dominant inelasticities:
  - $I = 1$: $e^+e^- \rightarrow \pi^+\pi^- \rightarrow \pi^0\gamma$
  - $I = 0$: $e^+e^- \rightarrow \pi^0\pi^+\pi^- \rightarrow \pi^0\gamma$

- required input for $I = 1$:
  - pion phase shift and pion form factor $\rightsquigarrow$ measured
  - strength of amplitude $\pi^+\pi^- \rightarrow \pi^0\gamma$ $\rightsquigarrow$ chiral anomaly

- input for $I = 0$ (three-body!):
  - dominated by narrow resonances $\omega, \phi$
  - use Breit-Wigners plus background for amplitude
  - fit to $e^+e^- \rightarrow \pi^+\pi^-\pi^0$
Pion transition form factor \((e^+ e^- \rightarrow \pi^0 \gamma)\)

- unsubtracted dispersion relation
- uncertainty estimate from quality of \(\omega/\phi \rightarrow \pi^0 \gamma\)
  
  Schneider et al., PRD86, 054013

- can be extended to decay region \(\pi^0 \rightarrow \gamma e^+ e^-\) and to spacelike region

- final aim: double virtual transition form factor
  
  relevant for \(g - 2\) and \(\pi^0 \rightarrow e^+ e^-\)

M. Hoferichter, B. Kubis, S.L., F. Niecknig and S. P. Schneider, in preparation
Summary

- meson (transition) form factors and two-photon reactions allow access to intrinsic structure of hadrons
- quark structure, polarizabilities, ...
- in addition input for standard-model baseline calculations for rare decays ($\pi^0$) and high-precision determinations (muon’s $g - 2$)
- we are sharpening our theory tools to improve the accuracy of predictions
Instead of an outlook

From two- to three-gamma physics

- yet another contribution to light-by-light scattering:

\[ \gamma^* \rightarrow \omega \rightarrow 3\gamma(\ast) \]

- related to scattering amplitude (dispersion theory)

\[ \gamma \omega \rightarrow \pi \pi \rightarrow \gamma \gamma \]

i.e. to decays

\[ \omega \rightarrow \gamma \pi^+ \pi^- , \quad \omega \rightarrow \gamma \pi^0 \pi^0 \]

- more (differential) data needed

- and also \( \phi \) instead of \( \omega \) (better data situation)
Rare $\omega$ decays into $2\pi\gamma$

- $\omega \rightarrow \pi^+\pi^-\gamma$: only upper limit
- $\omega \rightarrow \pi^0\pi^0\gamma$:
  - branching ratio: $6.6 \cdot 10^{-5}$

Histories are simulations with an intermediate rho (full) or sigma meson (dotted)
backup slides
How we sort interactions/diagrams

- without assigning importance to anything:
  - infinitely many interaction terms
    (with more and more derivatives)
  - infinitely many loop diagrams
- large-$N_c$ framework ($N_c =$ number of colors)
  - loops are suppressed
  - note: we resum loops from rescattering, $s$-channel
  - sorting scheme applies to scattering kernel (potential),
    not to scattering amplitude
- for interaction terms:
  - ensure appropriate $N_c$ scaling by
    dimensionful decay constant $f \sim \sqrt{N_c}$
  - to ensure pertinent dimension of interaction term in Lagrangian:
    - assume large scale $\Lambda_{\text{hard}} \gg m_V$ in denominator
    - expansion in derivatives/momenta over $\Lambda_{\text{hard}}$
  - depends on chosen representation
Examples for interaction terms

- relevant, e.g., for $\omega \to 3\pi$ and $\omega \to \pi \gamma^*$
- both can proceed directly or via $\pi \rho^*$
- some unsuppressed interaction terms

$$\varepsilon_{\mu\nu\alpha\beta} \operatorname{tr} \left( \{ V^{\mu\nu}, \nabla_\lambda V^\lambda{}^\alpha \} u^\beta \right),$$

$$i f \operatorname{tr}(V_{\mu\nu} [u^\mu, u^\nu]), \quad f \operatorname{tr}(V^{\mu\nu} f^{+}_{\mu\nu})$$

- some suppressed interaction terms (the direct ones)

$$\frac{f}{\Lambda^2_{\text{hard}}} \varepsilon_{\mu\nu\alpha\beta} \operatorname{tr}(\nabla^\lambda V_{\lambda\mu} u_\nu u_\alpha u_\beta),$$

$$\frac{f}{\Lambda^2_{\text{hard}}} \varepsilon_{\mu\nu\alpha\beta} \operatorname{tr}(\{ \nabla^\lambda V_{\lambda\mu}, f^{+}_{\nu\alpha} \} u_\beta).$$

- $\Lambda_{\text{hard}}$: hadrogenesis gap or (here also O.K.) mass of excited vector mesons
hadro genesis conjecture

\[
\begin{align*}
\Lambda_{\text{hard}} & \quad \text{spectrum at large } N_c \\
\vdots & \quad J^P = 0^\pm, 1^\pm, \ldots \\
J^P = 1^- & \\
J^P = 0^- & 
\end{align*}
\]

other observed mesons below \( \Lambda_{\text{hard}} \) are supposed to be dynamically generated, i.e. meson molecules

Rare pion decay — status

- $B(\pi^0 \rightarrow e^+ e^-) = (6.46 \pm 0.33) \cdot 10^{-8}$ (KTeV, 2007)
- 3 $\sigma$ deviation between experiment and standard model
  - (but controversial among theorists!)

- for point-like pion QED loop is divergent
- process is sensitive to hadronic transition form factor of pion $\pi^0 \leftrightarrow \gamma(\ast)\gamma(\ast)$
$g - 2$ of the muon — status

$g - 2$ of the muon — theory

Largest uncertainty of standard model: hadronic contributions

vacuum polarization
\[ \sim \alpha^2 \]

light-by-light scattering
\[ \sim \alpha^3 \]
Transition form factor $\omega \rightarrow \pi^0 + \mu^+\mu^-$

corresponding differential decay rate: