Effects of (axial)vector mesons on the chiral phase transition: initial results

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Overview

1. **Introduction**
   - Motivation
   - QCD’s chiral symmetry, effective models

2. **The model**
   - Axial(vector) meson extended linear $\sigma$-model with constituent quarks and Polyakov-loops

3. **eLSM at finite $T/\mu_B$**
   - Polyakov loop
   - Equations of states
   - Parametrization at $T = 0$
   - $T$ dependence

4. **Summary**
**QCD phase diagram**

Phase diagram in the $T - \mu_B - \mu_I$ space

- At $\mu_B = 0$
  
  $T_c = 151(3) \text{ MeV}$
  

- Is there a CEP?

- At $T = 0$ in $\mu_B$ where is the phase boundary?

- Behaviour as a function of $\mu_I/\mu_S$

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)
Previous results (with linear $\sigma$-model)
Critical surface and the CEP

P. Kovács, Zs. Szép: Phys. Rev. D 75, 025015

The surface bends towards the physical point $\iff$ The CEP must exist
Previous results (with linear $\sigma$-model)
The CEP at the physical point of the mass plane

![Graph showing the CEP at the physical point of the mass plane with various lines and markers.]

- Effective model
  - $T_c(\mu_B = 0) = 154.84$ MeV
  - $\Delta T_c(x\chi) = 15.5$ MeV
  - $T_{CEP} = 74.83$ MeV
  - $\mu_{B,CEP} = 895.38$ MeV
  - $T_c \left. \frac{d^2 T_c}{d\mu_B^2} \right|_{\mu_B=0} = -0.09$

- Lattice
  - $T_c(\mu_B = 0) = 151(3)$ MeV
  - $\Delta T_c(x\bar{\psi}\psi) = 28(5)$ MeV
  - $T_{CEP} = 162(2)$ MeV
  - $\mu_{B,CEP} = 360(40)$ MeV
  - $-0.058(2)$
Addressed problems

- By adding more degrees of freedom to our model how does the phase boundary change?
- More specifically adding (axial)vector mesons to the model how does the position of the CEP change?
- What is the effect of the medium on the various masses?
- Results will be closer to the Lattice?
Chiral symmetry

If the quark masses are zero (chiral limit) $\Rightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

- $U(1)_V$ term $\rightarrow$ baryon number conservation
- $U(1)_A$ term $\rightarrow$ broken through axial anomaly
- $SU(3)_A$ term $\rightarrow$ broken down by any quark mass
- $SU(3)_V$ term $\rightarrow$ broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$
  $\quad \rightarrow$ totally broken if $m_u \neq m_d \neq m_s$ (realized in nature)

Since QCD is very hard to solve $\rightarrow$ low energy effective models can be set up $\rightarrow$ reflecting the global symmetries of QCD $\rightarrow$ degrees of freedom: observable particles instead of quarks and gluons

Linear realization of the symmetry $\rightarrow$ linear sigma model
(nonlinear representation $\rightarrow$ chiral perturbation theory (ChPT))
Axial(vector) meson extended linear $\sigma$-model with constituent quarks and Polyakov-loops

Lagrangian (2/1)

\[
\mathcal{L}_{\text{Tot}} = \text{Tr}[(D_\mu \Phi)^\dagger(D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2
\]

\[
- \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \text{Tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \text{Tr}[H(\Phi + \Phi^\dagger)]
\]

\[
+ c_2 (\det \Phi - \det \Phi^\dagger)^2 + i \frac{g_2}{2} \left( \text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\} \right)
\]

\[
+ \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R_\mu^\dagger \Phi^\dagger).
\]

\[
+ g_3 [\text{Tr}(L_\mu L_\nu L_\mu^\dagger L_\nu^\dagger) + \text{Tr}(R_\mu R_\nu R_\mu^\dagger R_\nu^\dagger)] + g_4 [\text{Tr}(L_\mu L_\mu^\dagger L_\nu L_\nu^\dagger)]
\]

\[
+ \text{Tr}(R_\mu R_\mu^\dagger R_\nu R_\nu^\dagger)] + g_5 \text{Tr}(L_\mu L_\mu^\dagger) \text{Tr}(R_\nu R_\nu^\dagger) + g_6 [\text{Tr}(L_\mu L_\mu^\dagger) \text{Tr}(L_\nu L_\nu^\dagger)]
\]

\[
+ \text{Tr}(R_\mu R_\mu^\dagger) \text{Tr}(R_\nu R_\nu^\dagger)] + \bar{\Psi} \left( i \slashed{\partial} - g_F \Phi_5 \right) \Psi + \mathcal{L}_{\text{Polyakov}}
\]

where

\[
D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ieA^\mu [T_3, \Phi]
\]
Axial(vector) meson extended linear $\sigma$-model with constituent quarks and Polyakov-loops

**Lagrangian (2/2)**

\[
\Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^{8} h_i T_i \quad T_i : U(3) \text{ generators}
\]

\[
R^\mu = \sum_{i=0}^{8} (\rho^\mu_i - b^\mu_i) T_i, \quad L^\mu = \sum_{i=0}^{8} (\rho^\mu_i + b^\mu_i) T_i
\]

\[
L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu [T_3, L^\mu]\}
\]

\[
R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu [T_3, R^\mu]\}
\]

\[
\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})
\]

**non strange – strange base:**

\[
\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8, \\
\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \quad \varphi \in (\sigma, \pi, h)
\]

broken symmetry: non-zero condensates $\langle \sigma_N \rangle, \langle \sigma_S \rangle \leftrightarrow \phi_N, \phi_S$
Axial(vector) meson extended linear $\sigma$-model with constituent quarks and Polyakov-loops

### Included fields - pseudoscalar and scalar meson nonets

\[ \Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \frac{\pi^+}{\sqrt{2}} & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & K^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j) \]

\[ \Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a^0_0}{\sqrt{2}} & a^+ & K^+_S \\ a^-_0 & \frac{\sigma_N-a^0_0}{\sqrt{2}} & K^0_S \\ K^-_S & K^0_S & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j) \]

### Particle content:
- **Pseudoscalars:** $\pi(138), K(495), \eta(548), \eta'(958)$
- **Scalars:** $a_0(980 \text{ or } 1450), K^*_0(800 \text{ or } 1430), \ (\sigma_N, \sigma_S) : 2 \text{ of } f_0(500, 980, 1370, 1500, 1710)$
Axial(vector) meson extended linear $\sigma$-model with constituent quarks and Polyakov-loops

**Included fields - vector meson nonets**

\[
V_\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{\omega_N + \rho_0^0}{\sqrt{2}} \\
\rho^- \\
K^*-
\end{pmatrix}
\begin{pmatrix}
\frac{\omega_N + \rho_0^0}{\sqrt{2}} \\
\rho^+ \\
K^{*+}
\end{pmatrix}
\begin{pmatrix}
K^{*0} \\
\omega_S
\end{pmatrix}
\]

\[
A^\mu_V = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{f_{1N} + a_1^0}{\sqrt{2}} \\
\frac{a_1^-}{\sqrt{2}} \\
K^-
\end{pmatrix}
\begin{pmatrix}
\frac{f_{1N} + a_1^0}{\sqrt{2}} \\
\frac{a_1^+}{\sqrt{2}} \\
K_1^+
\end{pmatrix}
\begin{pmatrix}
K_1^0 \\
f_{1S}
\end{pmatrix}
\]

**Particle content:**

Vector mesons: $\rho(770), K^*(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$
Polyakov loops in Polyakov gauge

Polyakov loop variables: \( \Phi(\vec{x}) = \frac{\text{Tr}_c L(\vec{x})}{N_c} \) and \( \bar{\Phi}(\vec{x}) = \frac{\text{Tr}_c \bar{L}(\vec{x})}{N_c} \) with
\[ L(x) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right] \]

\( \rightarrow \) signals center symmetry \((\mathbb{Z}_3)\) breaking at the deconfinement transition

low \( T \): confined phase, \( \langle \Phi(\vec{x}) \rangle , \langle \bar{\Phi}(\vec{x}) \rangle = 0 \)

high \( T \): deconfined phase, \( \langle \Phi(\vec{x}) \rangle , \langle \bar{\Phi}(\vec{x}) \rangle \neq 0 \)

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space
\[ A_{4,d}(\vec{x}) = \phi_3(\vec{x}) \lambda_3 + \phi_8(\vec{x}) \lambda_8; \quad \lambda_3, \lambda_8 : \text{Gell-Mann matrices.} \]

In this gauge the Polyakov loop operator is
\[ L(\vec{x}) = \text{diag}(e^{i\beta \phi_+(\vec{x})}, e^{i\beta \phi_-(\vec{x})}, e^{-i\beta (\phi_+(\vec{x}) + \phi_-(\vec{x}))}) \]

where \( \phi_{\pm}(\vec{x}) = \pm \phi_3(\vec{x}) + \phi_8(\vec{x})/\sqrt{3} \)
Polyakov loop potential

“Color confinement”
\[ \langle \Phi \rangle = 0 \rightarrow \text{no breaking of } \mathbb{Z}_3 \]
one minimum

“Color deconfinement”
\[ \langle \Phi \rangle \neq 0 \rightarrow \text{spontaneous breaking of } \mathbb{Z}_3 \]
minima at \(0, 2\pi/3, -2\pi/3\)
one of them spontaneously selected

from H. Hansen et al., PRD75, 065004 (2007)
Form of the potential

I.) Simple polynomial potential invariant under $\mathbb{Z}_3$ and charge conjugation: R.D. Pisarski, PRD 62, 111501

$$\frac{U_{\text{poly}}(\Phi, \bar{\Phi})}{T^4} = -\frac{b_2(T)}{2} \Phi \bar{\Phi} - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\Phi \bar{\Phi})^2$$

with

$$b_2(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2} + a_3 \frac{T_0^3}{T^3}$$

II.) Logarithmic potential coming from the $SU(3)$ Haar measure of group integration


$$\frac{U_{\text{log}}(\Phi, \bar{\Phi})}{T^4} = -\frac{1}{2} a(T) \Phi \bar{\Phi} + b(T) \ln \left[ 1 - 6 \Phi \bar{\Phi} + 4 (\Phi^3 + \bar{\Phi}^3) - 3 (\Phi \bar{\Phi})^2 \right]$$

with

$$a(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2}, \quad b(T) = b_3 \frac{T_0^3}{T^3}$$

$U(\Phi, \bar{\Phi})$ models the free energy of a pure gauge theory

→ the parameters are fitted to the pure gauge lattice data
Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

\[
\begin{align*}
f(E_p - \mu_q) &\rightarrow f_{\Phi}^+(E_p) = \frac{\left(\Phi + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}{1 + 3 \left(\Phi + \Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}} \\
nf(E_p + \mu_q) &\rightarrow f_{\Phi}^-(E_p) = \frac{\left(\Phi + 2\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}{1 + 3 \left(\Phi + \Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}
\end{align*}
\]

\[\Phi, \bar{\Phi} \rightarrow 0 \implies f_{\Phi}^\pm(E_p) \rightarrow f(3(E_p \pm \mu_q)) \quad \Phi, \bar{\Phi} \rightarrow 1 \implies f_{\Phi}^\pm(E_p) \rightarrow f(E_p \pm \mu_q)\]

three-particle state appears: mimics confinement of quarks within baryons

the effect of the Polyakov loop is more relevant for \( T < T_c \)

at \( T = 0 \) there is no difference between models with and without Polyakov loop:

\[\Theta(3(\mu_q - E_p)) \equiv \Theta((\mu_q - E_p))\]

H. Hansen et al., PRD75, 065004
### Equations of states

**$T/\mu_B$ dependence of the Polyakov-loops (EoS)**

By deriving the grand canonical potential for Polyakov loops ($\Omega$) according to $\Phi$ and $\bar{\Phi}$

\[
- \frac{d}{d\Phi} \left( \frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3 p}{(2\pi)^3} \left( \frac{e^{-\beta E_q^-}(p)}{g_q^-(p)} + \frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)} \right) = 0
\]

\[
- \frac{d}{d\bar{\Phi}} \left( \frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3 p}{(2\pi)^3} \left( \frac{e^{-\beta E_q^+(p)}}{g_q^+(p)} + \frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)} \right) = 0
\]

\[
g_q^+(p) = 1 + 3 \left( \bar{\Phi} + \Phi e^{-\beta E_q^+(p)} \right) e^{-\beta E_q^+(p)} + e^{-3\beta E_q^+(p)}
\]

\[
g_q^-(p) = 1 + 3 \left( \Phi + \bar{\Phi} e^{-\beta E_q^-(p)} \right) e^{-\beta E_q^-(p)} + e^{-3\beta E_q^-(p)}
\]

\[
E_q^\pm(p) = E_q(p) \mp \mu_B/3, \quad E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2}, \quad E_s(p) = \sqrt{p^2 + m_s^2}
\]
Equations of states

$T/\mu_B$ dependence of the condensates ($\phi_{N/S}$)

Equation of state: $\left\langle \frac{\partial L_{\text{Tot}}}{\partial \sigma_{N/S}} \right\rangle_T = 0$

Hybrid approach at $T = 0$: fermions at one-loop, mesons at tree-level (their effects are much smaller)

At $T \neq 0$: first approximation $\rightarrow$ only fermion thermal loops

\begin{align*}
m_0^2 \phi_N &+ \left( \lambda_1 + \frac{1}{2} \lambda_2 \right) \phi_N^3 + \lambda_1 \phi_N \phi_S^2 - h_N + \frac{g_F}{2} N_c \left( \langle u\bar{u} \rangle_T + \langle d\bar{d} \rangle_T \right) = 0 \\
m_0^2 \phi_S &+ \left( \lambda_1 + \lambda_2 \right) \phi_S^3 + \lambda_1 \phi_N^2 \phi_S - h_S + \frac{g_F}{\sqrt{2}} N_c \langle s\bar{s} \rangle_T = 0
\end{align*}

\[\langle q\bar{q} \rangle_T = -4m_q \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_q(p)} \left( 1 - f^-_\phi(E_q(p)) - f^+_\phi(E_q(p)) \right)\]
Determination of the parameters of the Lagrangian

14 unknown parameters → Determined by the min. of $\chi^2$:

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[ \frac{Q_i(x_1, \ldots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,$$

where $(x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1, \ldots, x_N)$ calculated from the model, while $Q_i^{\text{exp}}$ taken from the PDG

multiparametric minimalization → MINUIT

- PCAC → 2 physical quantities: $f_\pi, f_K$
- Tree-level masses → 16 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_\eta', m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_S}, m_{f_0^L}, m_{f_0^H}$
- Decay widths → 12 physical quantities:
  $\Gamma_\rho \to \pi\pi, \Gamma_\Phi \to KK, \Gamma_{K^*} \to K\pi, \Gamma_{a_1} \to \pi\gamma, \Gamma_{a_1} \to \rho\pi, \Gamma_{f_1} \to KK^*, \Gamma_{a_0}, \Gamma_{K_S} \to K\pi, \Gamma_{f_0^L} \to \pi\pi, \Gamma_{f_0^L} \to KK, \Gamma_{f_0^H} \to \pi\pi, \Gamma_{f_0^H} \to KK$
Behaviour of the order parameters

- $v$ vs. $T$ for different values of $\mu_B$:
  - $\mu_B = 0$
  - $\mu_B = 200$ MeV
  - $\mu_B = 300$ MeV

- $\Phi$ vs. $T$ for different values of $\Phi^*$:
  - $\Phi^* = 0$
  - $\Phi^* = 0.3$ GeV

The graphs show the dependence of the order parameters on temperature and chemical potential.
An extended linear $\sigma$-model was shown with constituent quarks and Polyakov-loops.

We used hybrid approach at $T = 0$: only fermion loops, since it has the largest contribution.

At finite $T/\mu_B$ there was 4 coupled equations for the 4 order parameters.
An extended linear $\sigma$-model was shown with constituent quarks and Polyakov-loops.

We used hybrid approach at $T = 0$: only fermion loops, since it has the largest contribution.

At finite $T/\mu_B$ there was 4 coupled equations for the 4 order parameters.

→ To do ...
→ Finalize program code
→ Explore the phase diagram especially the CEP
→ Investigate the effect of meson thermal loops
→ Calculate medium dependence of the meson masses
Thank you for your attention!